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Linear Programming Optimization of a Gas-Oil Blend*

Substantial savings achieved through judiciously combining the constituents of a blend fully warrant the utilization of computers in this field.

Starting with the paper by Charnes [1] referring to the possibility of mathematically solving by programming methods the problems raised by gasoline, gas-oil and mineral oil blending within oil refineries, this subject has been approched in a number of other specialty papers [2], [3], [4], [5], [6], substantially contributing to the clarification of the blending problems. As an exemple, it was established that not all limiting conditions interfering in blending problems are linear, as was initially assumed [4], [6]. Therefore, the efforts have been directed towards deriving mathematical functions to reflect more accurately the chemical processes taking place in the gasoline blends with differing octane number, a procedure which was initially approximated by means of a linear function. In gas-oil blends, the specific features *viscosity index* and *freezing point* required by the State Standards are not additive linear functions; e.g., if two equal amounts having viscosity indexes of 1.3 and 1.5 respectively are mixed together, the resulting blend will not have the viscosity index 1.4, as could be assumed, but one lower than the arithmetic mean of the corresponding two values.

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Based on the existing mathematical methods [4], [6], we set out to specifically solve some gas-oil blending problems, at the Brazi oil refinery. In view of this purpose, we initially started from the possibility of optimizing

* The mathematical solution has been achieved by co-operating with M. Oprescu and V. Ionescu, research workers at the Centre of Economic Computation and Economic Cybernetics in Bucharest.

a single blend (an order) designed for a specific type of gas-oil, performed in a single tank and taking into account the specific conditions prevailing in the refinery at a given time, i.e. the available quantities of the various constituents to be introduced in the blend and the total final amount of gas-oil required. We did not try to optimize the full range of possible blends prepared during a certain time interval, as there exists a great number of instances when a blend prepared in a great capacity tank is not entirely delivered to the user. The residual quantity of blend is frequently used for the preparation of some other blends. Hence, the final blends do not always originate from the original constituents, this fact somewhat confusing the solving of the problem, if the latter is extended to a longer time interval.

In the first stage we aimed at minimizing the blend costs. We considered that at least three groups of limitations should be taken into account, namely:

a. availabilities;

b. requirements (the required amount of final blend);

c. limitations referring to the final product specifications required by the State, Standards, or other special requirements.

As the first two restriction groups do not raise any problem, we shall endeavour a detailed examination of the last group of limitations. The following specifications required by the State Standards were thought to be essential:

1. Diesel-fuel index number;
2. 15°C specific gravity;
3. colour index expressed in union degrees;
4. viscosity index, expressed in Engler degrees;
5. freezing point, in °C;
6. percent distillation at 300°C (or 350°C);
7. flammability point;
8. organic acidity.

Other specifications required by the State Standards, which could be included in an improved model are: the iodine number, percentage of sulphur, the coke number and percentage of ash. However, these are less important specifications and their variation may be approximated by additive linear relationships. They could be introduced in the extended model without whatever essential difficulties of a theoretical or practical kind.

The viscosity restriction, as well as the closely related freezing point restriction, may be expressed as logarithmic functions, such as

$$\Sigma (\log e_i)\, x_i = \log\lambda \, \Sigma \, x_i$$

where e_i is the viscosity index of each component and λ is the viscosity index of the final mixture. As for the restriction referring to the freezing point, in which negative figures are included, we started from —275°C, considered in physics as "absolute zero". Thus, all figures referring to the freezing point may be written as positive values.

Generally speaking, in effecting the blends (the admixing process being compulsory, because as a result of instrumentary processes, the refineries get products which cannot possibly be delivered as such) two main problems are raised, a technical problem — meeting the quantitative and qualitative requirements — and an economical problem — choosing the most favourable solution — [6].

Mathematically, the two problems may be expressed by the following matrix:

The maximum (minimum) value is to be determined for the linear function

$$f(x) = Cx$$

within the restrictive equations (or inequations):

$$AX = B \quad \text{and} \quad X \geqslant 0$$

where:

A = the matrix of technical coefficients determined through observing the phenomenon under study;
B = the matrix of free terms;
C = the matrix of coefficients, which may be considered as costs attached to various activities; and
X = the level at which the various activities are to be carried out.

The problem may be solved by using one of the simplex method algorithms: primal, dual, primal-dual, the algorithm based on the Dantzig-Wolfe decomposition principle.

The mathematical model for gas-oil blending optimization, aiming at the cost minimization, is as follows:

I. The function $f(x) = \sum_{i=1}^{n} c_i x_i$

where:

x_i = amount of constituent i
c_i = unit cost of constituent i

is to be minimized, taking into account the following conditions:

II. 1. $\sum_{i=1}^{n} a_i x_i \geqslant a \sum_{i=1}^{n} x_i$ — a condition referring to the Diesel-fuel index, a;

2. $\sum_{i=1}^{n} x_i \leqslant d \sum_{i=1}^{n} \frac{x_i}{d_i}$ — a condition referring to specific gravity, d;

3. $\sum_{i=1}^{n} x_i b_i \leqslant b \sum_{i=1}^{n} x_i$ — a condition referring to Union colour index, b;

4. $\sum_{i=1}^{n} (\log \lambda_i)\, x_i \leqslant \log \lambda \sum_{i=1}^{n} x_i$ — a condition referring to viscosity, λ;

5. $\sum_{i=1}^{n} (\log k_i) x_i \leqslant \log k \sum_{i=1}^{n} x_i$ — a condition referring to freezing point, k;

6. $\sum_{i=1}^{n} x_i g_i \geqslant g \sum_{i=1}^{n} x_i$ — a condition referring to percent distillation at 300°C. (or 350°C.), g;

7. $\sum_{i=1}^{n} x_i s_i \geqslant s \sum_{i=1}^{n} x_i$ — a condition referring to flammability point, s;

8. $\sum_{i=1}^{n} x_i r_i \leqslant r \sum_{i=1}^{n} x_i$ — a condition referring to organic acidity, r;

9. $\sum_{i=1}^{n} x_i \geqslant Q$ — a condition referring to final amount required, Q;

10. $x_i \leqslant q_i$ — a condition referring to non-exceeding existing availabilities, q_i; $i = 1 \ldots n$.

III. $x_i \geqslant 0$; $i = 1 \ldots n$.

The above model has been applied to the following actual conditions in the refinery, synthetically shown in Table I.

Table I

Quantities	Notations	Diesel-fuel index number	15° C. Specific gravity	Union colour index	20° C. Viscosity index	Freezing point, °C.	Percent distillation at 300 °C.	Flamability point	Organic acidity	Available quantity
Supply	x_1	53	0.853	2.5	1.58	(263) —10	66	74	10	200
Tank No. 1	x_2	55	0.859	1	1.5	(254) —19	65	92	1.68	1,500
Tank No. 2	x_3	48	0.880	3	1.63	(274) +1	28	102	3.68	600
Tank No. 3	x_4	55	0.820	1	1.02	(225) —48	98	72	0.56	830
Tank No. 4	x_5	49	0.867	5	1.55	(265) —8	59	81	12.3	1,000
State Standards specifications		*min* 49	*max* 0.860	*max* 4	1.2— 1.7	(263) —10	50	60	8	

The following restrictive system resulted:

1. For restriction referring to the Diesel-fuel index number:

$$53x_1 + 55x_2 + 48x_3 + 55x_4 + 49x_5 \geqslant 49\,(x_1 + x_2 + x_3 + x_4 + x_5)$$

2. For restriction referring to 15°C. specific gravity:

$$(x_1 + x_2 + x_3 + x_4 + x_5) \leqslant 0.860\left(\frac{x_1}{0.853} + \frac{x_2}{0.859} + \frac{x_3}{0.880} + \frac{x_4}{0.826} + \frac{x_5}{0.867}\right);$$

3. For restriction referring to colour index:

$$2.5x_1 + x_2 + 3x_3 + x_4 + 5x_5 \leqslant 4\,(x_1 + x_2 + x_3 + x_4 + x_5);$$

4. For conditions referring to viscosity and freezing, which have been approximated by logarithmic functions:

$$\log 1.58x_1 + \log 1.5x_2 + \log 1.63x_3 + \log 1.02x_4 + \log 1.55x_5 \leqslant$$
$$\leqslant \log \lambda (x_1 + x_2 + x_3 + x_4 + x_5)$$

(the variation interval of λ is 1.2 to 1.7; in the above computation the upper limit 1.7 has been considered)

5. The transformed values referring to the freezing point may be read in the freezing point column, in Table I, being given by the figures placed within brackets:

$$(\log 263)\, x_1 + (\log 254)\, x_2 + (\log 274)\, x_3 + (\log 225)\, x_4 + (\log 265)\, x_5 \leqslant$$
$$\leqslant \log 263(x_1 + x_2 + x_3 + x_4 + x_5);$$

6. For restriction referring to percent distillation at 300°C.:

$$66\, x_1 + 65\, x_2 + 28x_3 + 98x_4 + 59x_5 \geqslant 50\, (x_1 + x_2 + x_3 + x_4 + x_5)$$

7. For restriction referring to flammability point

$$74x_1 + 92x_2 + 102x_3 + 72x_4 + 80x_5 \leqslant 60(x_1 + x_2 + x_3 + x_4 + x_5);$$

8. For restriction referring to organic acidity:

$$10x_1 + 1.68x_2 + 3.68x_3 + 0.56x_4 + 12.3x_5 \leqslant 8(x_1 + x_2 + x_3 + x_4 + x_5);$$

9. The total amount of the blend is to be 3,200 tons, a restriction expressed as:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 3{,}200;$$

10. The restriction referring to not exceeding existing availabilities is written:

$$x_1 = 200$$
$$x_2 \leqslant 1{,}500$$
$$x_3 \leqslant 600$$
$$x_4 \leqslant 830$$
$$x_5 \leqslant 1{,}000$$

The variables non-negativity restriction will also have to be accomplished:

$$x_i \geqslant 0 \quad ; \quad i = 1, \ldots, 5$$

After effecting the necessary computations (logarithmation and reduction of similar terms), the entire model is as follows:

I. The function

$$f(x) = 355.61x_1 + 372.34x_2 + 330.81x_3 + 391.04x_4 + 351.43x_5 \rightarrow min.$$

is to be minimized, with the following restrictions:

II. $4x_1 + 6x_2 - x_3 + 6x_4 \geqslant 0,$

$-0.0082x_1 - 0.0012x_2 + 0.0227x_3 - 0.0412x_4 + 0.0081x_5 \leqslant 0,$

$-1.5x_1 - 3x_2 - x_3 - 3x_4 + x_5 \geqslant 0,$

$-0.03179x_1 - 0.05436x_2 - 0.01826x_3 - 0.22185x_4 - 0.04012x_5 \leqslant 0,$

$-0.01513x_2 + 0.01779x_3 - 0.06778x_4 + 0.00329x_5 \leqslant 0,$

$14x_1 + 32x_2 + 42x_3 + 12x_4 + 20x_5 \geqslant 0,$

$2x_1 - 6.32x_2 - 4.32x_3 - 7.44x_4 + 4.3x_5 \leqslant 0,$

$16x_1 + 15x_2 - 22x_3 + 48x_4 + 9x_5 \geqslant 0,$

$x_1 + x_2 + x_3 + x_4 + x_5 = 3\,200,$

$x_1 = 200,$

$x_2 \leqslant 1{,}500,$

$x_3 \leqslant 600,$

$x_4 \leqslant 830,$

$x_5 \leqslant 1{,}000.$

The comparative study of the blend prepared according to the empirical formula and of that prepared according to the formula produced by the computer, may be accomplished by referring to the data shown in Table II.

Table II

Constituent	Cost per ton	Amounts according to		Monetary worth of	
		empirical formula	optimal formula	empirical formula	optimal formula
Supply	355.61	200	200	71,122	71,122
Tank No. 1	372.34	1,150	940	428,191	349,999.60
Tank No. 2	330.81	600	600	198,486	198,486
Tank No. 3	391.09	600	460	234,654	179,901
Tank No. 4	351.43	650	1,000	228,429.50	351,430
		3,200	3,200	1,160,882.50	1,150,938.60

The difference of monetary worth between the empirical and optimal formulas is:

$$1{,}160{,}882.50 - 1{,}150{,}938.60 = 9{,}943.90.$$

Consequently, the potential economy of this blend, which could have been achieved if the optimal formula obtained with the computer had been used, would have amounted to about 10,000 lei. This sum of money would have been spared in only 10 hours (the approximate necessary time for preparing the 3,200 tons of blend) without any additional effort or cost, and only by judiciously using the constituents available at that time in the refinery.

In view of a higher reliability to be obtained in applying the model developed in this study, it was checked for another case, namely the finding of an optimal formula for a gas-oil with the freezing point —5°C., for which the constituents having the technological specifications shown .in Table III, were available.

Table III

Supply	Notation	Diesel-fuel index number	Union colour index	20°C. Viscos'ty index	—15°C. Specific gravity	Freezing point, °C	Percent distillation at 350°C*	Flammability point	Organic acidity	Available quantity	Prices
Tank No. 1	x_1	50	0.872	2.5	1.6	(265) —8	83	82	1.68	170	351.45
Tank No. 2	x_2	47	0.886	4.0	2.5	(274) +1	66	48	5.20	860	350.81
Tank No. 3	x_3	53	0.865	2.5	1.68	(263) —10	86	74	9.52	800	355.61
Tank No. 4	x_4	52	0.866	1.5	1.7	(259) —14	91	104	0	1,600	372.34
Tank No. 5	x_5	55	0.824	1.0	1.02	(225) —48	98	54	1.68	800	391.09
State Standards specifications		*min* 49	*max* 0.870	4	1.2— 1.7	(268) —5	*min* 88	*min* 60 C	*max* 8		

** In order to check the validity of this model for the per cent distillation at 350 C°., we applied the characteristics at this temperature to the computation for this formula.*

The system of restrictions regarding this blend is as follows:

1. $50\ x_1 + 47\ x_2 + 53\ x_3 + 52\ x_4 + 55\ x_5 \geqslant 49\ (x_1 + x_2 + x_3 + x_4 + x_5)$ (for the Diesel-fuel index number);

2. $(x_1 + x_2 + x_3 + x_4 + x_5) \leqslant 0.870 \left(\frac{x_1}{0.872} + \frac{x_2}{0.880} + \frac{x_3}{0.865} + \frac{x_4}{0.866} + \frac{x_5}{0.824}\right)$ (for specific gravity);

3. $2.5\ x_1 + 4.0\ x_2 + 2.5\ x_3 + 1.5\ x_4 + 1.0\ x_5 \leqslant 4(x_1 + x_2 + x_3 + x_4 + x_5)$

(for Union colour index);

4. $(\log 1.6)\, x_1 + \log 2.5)\, x_2 + (\log 1.68)\, x_3 + (\log 1.7)\, x_4 + (\log 1.02)\, x_5 \leqslant$
$\leqslant \log 1.7(x_1 + x_2 + x_3 + x_4 + x_5)$

(for viscosity);

5. $(\log 265)\, x_1 + (\log 274)\, x_2 + (\log 263)\, x_3 + (\log 259)\, x_4 + (\log 225)\, x_5 \leqslant$
$\leqslant \log 268(x_1 + x_2 + x_3 + x_4 + x_5)$

(for freezing points, by taking $-273°C. = 0$);

6. $83\, x_1 + 66\, x_2 + 86\, x_3 + 91\, x_4 + 98\, x_5 \geqslant 86(x_1 + x_2 + x_3 + x_4 + x_5)$

(for distillation at 350°C.);

7. $82\, x_1 + 48\, x_2 + 74\, x_3 + 104\, x_4 + 54\, x_5 \geqslant 60(x_1 + x_2 + x_3 + x_4 + x_5)$

(for the flammability point);

8. $1.68\, x_1 + 5.2\, x_2 + 9.52\, x_3 + 0.0\, x_4 + 1.68\, x_5 \leqslant 8(x_1 + x_2 + x_3 +$
$x_4 + x_5)$

(for organic acidity).

The limiting values for the variation of the final blend were 2,500 and 3,500. This restriction may be expressed as:

9. $$2{,}500 \leqslant x_1 + x_2 + x_3 + x_4 + x_5 \leqslant 3{,}500$$

For the restrictions regarding the highest amounts of each constituent available in the refinery, at the time of admixing, the following equations were written:

$$x_1 = 170 \qquad x_4 \leqslant 1{,}600$$
$$x_2 \leqslant 860 \qquad x_5 \leqslant 800$$
$$x_3 \leqslant 800$$

As a result of effecting the necessary transformations — logarithmation, reduction of similar terms and introduction of the function objective — the following classical problem of linear programming was obtained:

The function:

I. $f(x) = 351.43\, x_1 + 330.81\, x_2 + 355.61\, x_3 + 372.34\, x_4 + 391.09\, x_5.$

is to be minimized, with the restrictions

II. $x_1 - 2x_2 + 4\, x_3 + 3\, x_4 + 6\, x_5 \geqslant 0.$

$0.0023\, x_1 + 0.0181\, x_2 - 0.0058\, x_3 - 0.0046\, x_4 - 0.0558\, x_5 \leqslant 0,$

$1.5\, x_1 - 1.5\, x_3 - 2.5\, x_4 - 3\, x_5 \leqslant 0,$

$-0.02633\, x_1 + 0.16749\, x_2 - 0.00514\, x_3 - 0.22188\, x_5 \leqslant 0,$

$-0.00488\, x_1 + 0.00962\, x_2 - 0.00817\, x_3 - 0.01483\, x_4 - 0.07595 x_5 \leqslant 0$

$-3\, x_1 - 20\, x_2 + 5\, x_4 + 12\, x_5 \geqslant 0.$

$$22\,x_1 - 12\,x_2 + 14\,x_3 + 44\,x_4 - 6\,x_5 \geqslant 0,$$

$$-6.32\,x_1 - 2.8\,x_2 - 1.52\,x_3 - 8\,x_4 - 6.32\,x_5 \leqslant 0,$$

$$2{,}500 \leqslant x_1 + x_2 + x_3 + x_4 + x_5 \leqslant 3{,}500.$$

$$x_1 = 170, \qquad x_4 \leqslant 1{,}600,$$

$$x_2 \leqslant 860, \qquad x_5 \leqslant 800$$

$$x_3 \leqslant 800,$$

III. $x_i \geqslant 0, \qquad i = 1 \ldots 5.$

By interpreting the results obtained with the computer "Economist CIFA 102", the following optimal formula is obtained:

$$x_1 = 170$$

$$x_2 = 348.4$$

$$x_3 = 800$$

$$x_4 = 957.3$$

$$x_5 = 224.3$$

2,500 tons

the minimum value of the function amounting to 887,647.86 lei.

Both examples corroborate the ability of realizing considerable savings (which during one year may amount to millions of lei) with gas-oil blends. However, owing to the restricted storing capacities available in the refinery, which do not allow the storage of some constituents for a period of at least 24 hours, as well as owing to the wide daily qualitative variations of the constituents resulting from the existing equipment, a variability induced by the lack of stability of the raw materials technological qualities, it is not possible to determine in advance an optimal blend. This is also due to the fact that there are many instances, where the blending operations are initiated in a tank and only a few hours later the incomplete blend is, for some reasons, utilized to the preparation of another blend.

In order to overcome these difficulties in the application of mathematical methods, in actual practice, there is a sole reasonable solution, namely to equip the refinery with a computer, or at least, to ensure its permanent access to a computer allowing the immediate working out of formulas designed for various specific blends. (The preparation and computation of a model require about 2 to 3 hours.) In this way, it becomes possible to perform the admixing only after receiving the optimal formulas.

In our opinion, the necessary investments in view of acquiring a computer large enough to solve the problems referring to optimal formulas for the various blends, are realy efficient. This efficiency will increase when the model needed for gasoline blends, which is being elaborated, will be achieved.

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L'OPTIMISATION DES MELANGES DE GAS-OIL GRACE A LA PROGRAMMATION LINEAIRE

Le modèle mathématique formulé pour un seul mélange (dans une première étape de la recherche des possibilités d'obtenir de bons résultats dans le domaine des mélanges de gas-oil à l'aide de la programmation linéaire) confirme la perspective d'importantes économies dan le cas où il serait appliqué au calcul des recettes de mélange.

Pour ce modèle on a tenu compte des restrictions technologiques suivantes du mélange final:

— Indice Diesel
— Couleur Union
— Densité à 15°C
— Viscosité
— Congélation
— Acidité organique
— Point d' inflamabilité

L'auteur a également calculé les restrictions concernant les quantitès disponibles dans la quantité finale demandée.

Le but proposé est d'obtenir le prix de revient minimum du mélange final.

ОПТИМИЗАЦИЯ СМЕСЕЙ МОТОРИНА ПРИ ПОМОЩИ ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ

Математическая модель, составленная для одной смеси (на первом этапе исследования возможностей получения благоприятных результатов в области смесей моторина при помощи линейного программирования) подтверждает перспективу значительной экономии в случае ее применения для составления рецептов смесей.

При этом имелись в виду следующие технологические ограничения:

— Показатели Дизель,
— Цвет Унион
— Плотность при 15° С
— Вязкость
— Замораживаемость
— Кислотность
— Точка воспламеняемости

В расчет были также взяты ограничения, касающиеся наличного количества в требуемом конечном количестве.

В качестве заданной функции была предложена минимизация себестоимости конечной смеси.