Dr. E. ŢIGĂNESCU,
G. OPRESCU
Department of Economic Cybernetics
Academy of Economic Studies

Generalized Analysis of the Economic-Cybernetic Systems

Two trends for the substantiation and analysis of the complex dynamic systems are mainly outlined in the technical literature, trends concerning the two ways of cybernetically simulating the operation of these systems [1]: the first methodological step consists in identifying the input-output transformation operators for each elementary system by applying the black box method; the second one, by identifying the state vectors and specifying the input-state and state-output equations in the sense of the factorization introduced by Kalman [2].

Each of the two tendencies for simulating the dynamic systems behaviour has some advantages as compared to the other one. Thus, the step substantiated on the black box method, although sensibly lower as regards the contents of display for the operation of each elementary system constituent of the system, explicitly emphasizes the part played by the system structure and by the structural transformations on the system behaviour as a whole; this quality is the more significant as the part of the structure and the progress of the management degree in the economic systems are tendencies of primary importance for the increase of the economic efficiency, for the optimizing of their evolution. The second step brings an indisputable information input by the identification of the essential variables describing at any moment the state of the system and the mathematical simulation of its dynamics by means of sets of differential equations or of difference equations.

The merging of the two methods allows the substantiation of new ways of approaching the analysis of the behaviour of the economic cybernetic systems.

1. RESULTS APPEARING FROM THE ANALYSIS SUBSTANTIATED ON THE BLACK BOX METHOD

We consider the dynamic system $S = \{\mathcal{F}_t^{(s)}, \mathcal{E}_t^{(s)}, C_t^{(s)}\}$ is represented by the specification of the functions $\mathcal{F}_t^{(s)}$ and of its structure $\mathcal{S}_t^{(s)} = \{\mathcal{E}_t^{(s)}, C_t^{(s)}\}$, where $\mathcal{E}_t^{(s)} = \{e_1, ..., e_n\}$ is the set of elementary systems, which are limited through the known methods in the economic cybernetics and $C^{(s)}$ is the matrix of the connections between the component elementary systems and between these and the environment of the system (M) at the t moment.

The set of connections between the elementary systems may be described by starting from the known coupling matrix relation [4], [5]:

$$u^{(s)} = C_{sr} \cdot y^{(r)} \tag{1}$$

where $u^{(s)}$ is the vector of the elementary system $e^{(s)}$ inputs, and $y^{(r)}$ is the vector of the output variable values from the elementary system $e^{(r)}$; C_{sr} is the Boolean matrix having the elements 1 — for the existing flows between the two elementary systems (from $e^{(r)}$ to $e^{(s)}$) and 0 — for the rest.

On the assumption that the environment of the system S is considered as an elementary system (M), the matrix of the connections may be written

$$C^{(S)} = \begin{bmatrix} 0 & C^{(s)} \\ C_I^{(s)} & C_{int}^{(s)} \end{bmatrix} \text{ with } C_{int}^{S} = \begin{bmatrix} 0 & C_{12} & C_{13} \dots C_{1n} \\ C_{21} & 0 & C_{22} \dots C_{2n} \\ \vdots & & \vdots \\ C_{n1} & C_{n2} & C_{n1} \quad 0 \end{bmatrix}$$

C_{int}^{S} represents the matrix of intrinsic connections, straight (below the main diagonal) and feedback (over it), and $C_I^{(s)}$ and $C_E^{(s)}$ reflects the input connections from the environment and of output from the system S

$$C_E^{(S)} = [C_{M1}, C_{M2}, ..., C_{Mn}]; \qquad C_I^{(S)} = [C_{1M}, C_{2M}, ..., C_{nM}]^T$$

(the symbol "T" signifying the transposition operation).

The black box method applied to each elementary system $e^{(r)}$, $r = 1, 2... n$, allows the writing of the input-output equations $y^{(r)} = \mathcal{T}_M, u^{(r)}$ by the identification of the operators $\mathcal{T}_r$; a similar equation may be written for the environment on the assumption that: $y^{(M)} = \mathcal{T}_M \cdot u^{(M)}$.

We consider that the economic-cybernetic system is a closed system, integrated into the environment, $S = (S, M)$. Its behaviour may be described by combining the input-output equations with the coupling matrix equations (1). Thus, on the example of the system in Fig. 1, we may deduce:

A matrix equational representation of the inputs is obtained in a certain production cycle in terms of the inputs from the previous cycle, analogous for output:

$$u_{t+\theta}^{(\bar{s})} = C_t^{(s)} \cdot \mathcal{T}_t^{(\bar{s})} \cdot u_t^{(\bar{s})} \tag{2}$$

$$y_{t+\theta}^{(\bar{s})} = \mathcal{T}_t^{(\bar{s})} \cdot C_t^{(s)} \cdot y_t^{(\bar{s})} \tag{3}$$

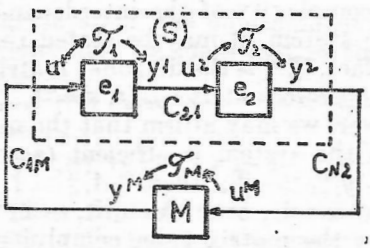

Fig. 1

where $C_t^{(s)}$ is the matrix of the connections at the t moment,

$$C_t^{(s)} = \begin{pmatrix} 0 & 0 & C_{M2} \\ 0 & 0 & 0 \\ C_{1M} & C_{21} & 0 \end{pmatrix}; \quad \mathcal{T}_t^{(\bar{s})} = \begin{pmatrix} \mathcal{T}_M & 0 & 0 \\ 0 & \mathcal{T}_1 & 0 \\ 0 & 0 & \mathcal{T}_2 \end{pmatrix}$$

$$u_t^{(\bar{s})} = \begin{pmatrix} u_t^M \\ u_t^1 \\ u_t^2 \end{pmatrix} \quad y^{(\bar{s})} = \begin{pmatrix} y_t^M \\ y_t^1 \\ y_t^2 \end{pmatrix}$$

and θ is the time of a complete cycle in the operation of the $\theta = \theta_s + \theta_M + + \theta_{MS} + \theta_{SM}$.

The relations (2) and (3) are true for any system S with n elementary systems and no matter how complex the matrix of the connections $C^{(s)}$, so that we will further refer to the general case only.

This representation is adequate to the structure simulation and the behaviour of the economic systems; thus, when S is an enterprise, its elementary systems may be the sections or shops; when S is an industrial plant the elementary systems are the constituent enterprises, etc. [6].

The analysis of the system operation is based on the remark that the equation (3) commensurabling the outputs as a result of the system activity as a whole, and it is a matrix equation of the class of discreet models. Thus, by agreeing to designate the time of the cycle θ and the time unit ($\theta = 1$), we have

$$y_{t+1}^{(\bar{s})} = A_t \cdot y_t^{(\bar{s})} \tag{3'}$$

where $A_t = \mathcal{T}_t^{(\bar{s})} \cdot C_t^{(s)}$ is the matrix reflecting simultaneously the individual operation of every elementary system and the structure of the cybernetic system as a whole. Obviously, the model is useful for logistically studying the system evolution, for the analysis of its stability, the substantiation of the equilibrium conditions, for the analysis of the system quantic properties [1], [6], etc.

We will only expand a result regarding the conditions of the evolution on the trajectory of efficient and balanced growth for a cybernetic and economic system (section, enterprise, plant, branch or territory, etc.), the input and output flows being evaluated in money units.

Allowing for the complexity of the interdependences in the conditions of an actual production system, it may be stated that the matrix A_t is irreducible. As a matter of fact, $\mathcal{T}^{(\bar{s})}$ is the diagonal matrix of the $\mathcal{T}_r$ operators, and $C^{(s)}$ has at least a non-zero matrix C_{rs} $(r, s = 1,..., n, M)$ on each line and on each column. Moreover, we may affirm that the matrix A_t is semipositive also; even more, when the system is efficient (according to K. Lankaster [3]: "productive"), i.e. $y_{t+1}^{(\bar{s})} > y_t^{(\bar{s})}$, then $\| A_t \| > 1$. Indeed, the transformation operators are, as a rule, over the unit, — in the scalar case or of an over unitary norm — in the matrix case, complying to the requirement of achieving in each system a strictly positive volume of a newly created value (more generally, of added value). The matrix $C^{(s)}$ is Boolean, and hence $\| \mathcal{T}^{(\bar{s})} C^{(s)} \| > 1$.

According to the theorem of Perron-Frobenius, there is a positive, singular, proper value λ^* whose module is greater than that of any other eigenvalue and attached to it, the characteristic vector ξ^*, strictly positive and singular, so that the efficient and balanced growth trajectory of the system $\bar{S}$ is stated by

$$y^*(t) = (\lambda^*)^t \cdot \xi^* \tag{4}$$

where ξ^* is defined from the homogeneous system $(A - \lambda^* \cdot I) \cdot \xi^* = 0$, and the equation $\xi^* \cdot e = y_0^{(\bar{s})} \cdot e$ being the column vector $(1, ..., 1)$. The maximum growth rate, $r^* = \lambda^* - 1$, is positive, since $\| A \| > 1$, and hence, as a consequence of the Perron-Frobenius theorem, the prevalent eigenvalue λ^* is over unit.

The system actual evolution takes place in the vicinity of the balanced and efficient growth trajectory, when the initial state wherefrom the system $y_0^{(\bar{s})}$ is started is characterized by the proportions defined by the eigen-vector ξ^*. Then, the deviation $y_t^* - y_t^{(\bar{s})} = (\lambda^*)^t \cdot \xi^* - A^t \cdot y_0^{\bar{s}}$ is the greater the $| \xi^* - y_0^s |$ is greater. The limits of the interval wherein there is the prevalent eigenvalue are easily deduced:

$$max \{min\ e^T \cdot \mathcal{T}^{(\bar{s})} \cdot C^s ;\ min\ \mathcal{T}^{(\bar{s})} C^s \cdot e\} \leqslant \lambda^* \leqslant$$
$$min \{max\ e^T \cdot \mathcal{T}^{(\bar{s})} \cdot C^{(s)} ;\ max\ \mathcal{T}^{(\bar{s})} \cdot C^{(s)} \cdot e\}.$$

A number of other propert es can be emphasized [7].

2. GENERALIZED SIMULATION OF THE EVOLUTION BY MEANS OF DYNAMIC EQUATIONS AND THE REFLECTION OF THE ECONOMIC CYBERNETIC SYSTEM STRUCTURE

Let us introduce for each elementary system $e^{(r)}$ the state vector $X_t^{(r)}$ and let us assume that $e^{(r)}$ is (according to [2]) a plane, dimensionally finite system, described by the octet $e^{(r)} = \{T,\ U^{(r)},\ \Gamma^{(r)},\ X^{(r)},\ Y^{(r)},\ \Omega^{(r)},\ \varphi^{(r)},\ \psi^{(r)}\}$ where $X^{(r)}$, $U^{(r)}$ and $Y^{(r)}$ represent the state space, that of the inputs and of the outputs, respectively for the elementary system $e^{(r)}$, $\Gamma^{(r)} = \{\gamma_r : T^{(r)} \to U^{(r)}\}$, $\Omega^{(r)} = \{\omega_t : T \to Y^{(r)}\}$ represent the set of input functions (γ_r) and of output ones (ω_r), and $\varphi^{(r)} : T \times T \times X^{(r)} \times \Gamma^{(r)} \to X^{(r)}$ and $\psi^{(r)} : T \times X^{(r)} \to Y^{(r)}$ are the transition and the output functions of the elementary system $e^{(r)}$; $T \subset R$ is the continual set of the time moments wherein the system evolution develops.

For each elementary system, the environment included, the input-state and state-output equations may be identified in the stated conditions:

$$\begin{cases} \dot{X}_r(t) = f_r(x_r(t),\, u_r(t),\, t), & r = 1,\ \ldots,\ n;\ M \end{cases} \tag{5}$$

$$\begin{cases} Y_r(t) = g_r(x_r(t),\, t), & r = 1, \ldots, n;\ M. \end{cases} \tag{6}$$

The system operation depends both on the individual operation of every elementary system, reflected by the equations (5) and (6), and on its. structure, on the mode of element intercoupling, and their intercoupling with the environment.

In this sense, the inputs will be described by the coupling equations (1).

Thus, in the example illustrated in Fig. 1, in addition to the three input-state equations of dynamics and the other three of state-output, we will take into consideration the other three coupling equations, $u^{(r)} = C_{ri}\, y^{(i)}$; we obtain:

$$\dot{X}_M(t) = f_M(X_M(t),\ C_{M2} \cdot g_2(x_2\ (t))$$
$$\dot{X}_1(t) = f_1(x_1(t),\ C_{1M}\, g_M\,(X_M(t))$$
$$\dot{X}_2(t) = f_2(x_2(t),\ C_{21}\, g_1\,(x_1(t)).$$

Agreeing to designate the operation $f_j(x_j(t),\ W) = f_j(x_j)) * W$; we find, for instance, $\dot{x}_1(t) = f_1(x_1(t)) * C_{1M} \cdot g_M(x_n(t))$, etc. With this notation we may have a matrix description of the behaviour of any system $\bar{S}$ by means of the continuous dynamic model:

$$\begin{cases} \dot{X}^{(\bar{s})}_{(t)} = \mathcal{F}^{(\bar{s})}_t\ (x^{(\bar{s})}(t)) * C^{(s)}_t \cdot g^{(\bar{s})}_t(x^{(\bar{s})} - (t)) \end{cases} \tag{7}$$

$$\begin{cases} y^{(\bar{s})}_{(t)} = g^{(\bar{s})}(x^{(\bar{s})}(t)) \end{cases} \tag{8}$$

where

$$\mathcal{F}^{(\bar{s})}_{(t)}(s)\ (x^{(\bar{s})}(t) = \text{diag}\ [f_M(x_M(t)), f_1(x_1(t), \ldots, f_n(x_n(t)]$$

is the diagonal matrix of the input-state functions characterizing the environment and the elementary systems at the moment t, and $g^{(\bar{s})}_t(x)^{(\bar{s})}_{(t)}$ is the column vector of the state-output relevant functions. The relations (7), (8) generalize the known input-state and state-output relations describing in the Kalman's sense the behaviour of a complex dynamic system, also emphasizing the role of the structure in the dynamics of the analysed processes.

When each elementary system is a dynamic linear system with input-state and state-output equations of the following type:

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r u_r(t) \end{cases} \tag{5'}$$
$$\begin{cases} y_r(t) = D_r x_r(t), & r = 1, \ldots, n,\ M \end{cases} \tag{6'}$$

we have the explicit expression of the behaviour overall matrix equation (7) (the $*$ operation becomes $+$) of the type:

$$\begin{cases} \dot{X}^{(\bar{s})}(t) = [A^{(\bar{s})}_t + B^{(\bar{s})}_t \cdot C^{(s)}_t \cdot D^{(\bar{s})}_t] \cdot X^{(\bar{s})}(t) \end{cases} \tag{7'}$$

$$\begin{cases} Y^{(\bar{s})}(t) = D^{(\bar{s})} \cdot X^{(\bar{s})}(t) \end{cases} \tag{8'}$$

where:

$$X^{(\bar{s})}(t) = \begin{bmatrix} X^M(t) \\ X_1(t) \\ \vdots \\ X_n(t) \end{bmatrix}, \quad A^{(\bar{s})} = \begin{bmatrix} A_M & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & A_n \end{bmatrix}$$

and, analogically, $B^{(\bar{s})}$, $D^{(\bar{s})}$, are the diagonal matrices containing the submatrices B_r, $D_r (r = M, 1, ..., n)$.

The differential matrix equation (7') has the peculiarity of being homogeneous, so that the analysis of the behaviour is made in relation to its own component. The behaviour characteristic features result from the examination of the matrix eigenvalues:

$$M^{(\bar{s})} = A^{(\bar{s})} + B^{(\bar{s})} \cdot C^{(\bar{s})} \cdot D^{(\bar{s})}$$

leading to the information required for evaluating the efficiency of the system operation (maximum possible growth rate, proportions of balanced development, etc.) depending on the changes in each elementary system behaviour, those of the environment included, as well as on the structural transformations, thus allowing a better organization of the system, in the sense of increasing the economic efficiency, at overall level.

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