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On the Simulation of a System Reliability

A simulation algorithm of the reliability of a given system is presented in this paper, assuming the reliability functions of the respective system components are known. In the end, an example is presented, on which the algorithm was tested.

0. INTRODUCTION

The problem of carrying out some high reliability systems is a technical condition of the first importance in the designing and manufacturing process.

The accelerated rate of development of the contemporary science and technics makes impossible the application of the old methods: designing, testing, redesigning and testing again, and so on until the desired reliability is obtained.

The determination of the reliability is a difficult problem when considering the probabilistic and statistical character of the reliability characteristic properties and the fact that their determination with a certain accuracy supposes long and expensive tests, a very good organization of the collection, transfer and processing of the information about the behaviour of the installation its maintenance and repair and about the causes of its disturbances.

Although we have no possibility to get very exact results, the simulation is one of the modern methods for studying a system before carrying it out.

The use of simulation allows the quick detection of the elements with low reliability contributing to the system damaging and determining the decrease of the safety in operation.

The simulation allows the quantitative evaluation of the system reliability and the results of the simulation are very important in determining the structure and pre-calculated reliability of its components, so that they comply with the requirements it was designed for.

The carrying out of some high reliability systems during the designing may be obtained by the simulation of the various structures (versions of the system) and then by selecting the one which corresponds to the user's requirements.

For making it easier to be studied, the system should be figured as a graph, the nodes representing the system elements, and the curves their interconnections.

The system elements may be connected in series, paralleled bridged or their combination.

In the following we consider that we have a system with n components and designate by $R_i (i = 1, 2, ..., n)$ the reliability of the component i, and with R_S the system reliability.

For the sake of simplicity, we associate a binary variable x_i to each component $i (i = 1, 2, ..., n)$, as follows:

$$x_i = \begin{cases} 1 \text{ when the component } i \text{ operates} \\ 0 \text{ when the component } i \text{ is defective} \end{cases}$$

$$i = 1, 2, ..., n.$$

We designate by $x = (x_1, x_2, ..., x_n)$ the state vector

$$R_i = P(x_i = 1).$$

Similarly, we associate a binary variable (state variable) to the system as follows:

$$S = \begin{cases} 1 \text{ when the system operates} \\ 0 \text{ when the system is defective} \end{cases}$$

$$R_S = P(S = 1).$$

With the above notations, the relation between the system state and the component state may be expressed by a Boolean function, referred to as structure function, as follows:

$$S = \Phi(x_1, x_2, ..., x_n)$$

A matrix $A = (a_{ij})\ i, j = 1, 2, ..., m$, referred to as the connection matrix, is attached to the system

$$a_{ij} = \begin{cases} x_k \text{ when connecting the nodes } i \text{ and } j \\ 0 \text{ if otherwise} \end{cases}$$

$$i, j = 1, 2, ..., m, \qquad k = 1, 2, ..., n.$$

In the following we assume the system components are independent from the failure point of view.

Allowing for the above mentioned notations, the simulation algorithm is as follows:

Step 0. RND initialization (algorithm for generating a random number uniform on (0, 1).
Initialization of the algorithms for generating the reliability functions of the components.
Step 1. Calculates the structure function of the system.
Step 2. Generates the individual reliability of the system components R_i.
Step 3. Calculates the system reliability function.

2. CALCULUS ALGORITHM OF THE STRUCTURE FUNCTION

It is a well known fact that for a system with n components $x_1, x_2, ..., x_n$ which are connected in series, the system structure function is:

$$\Phi(x) = x_1 \wedge x_2 \wedge ... \wedge x_n = min\ (x_1, x_2, ..., x_n)$$

and for a system with paralleled components it is

$$\Phi(x) = x_1 \vee x_2 \vee ... \vee x_n = max\ (x_1, x_2, ..., x_n)$$

i.e. it has the form of a product or a logical sum of its components respectively.

A system whose components are not in series or paralleled, is of the bridge type, and its structure is a logical sum of logical products.

The systems, whose structure cannot be, as a rule, reduced to combinations of the series or paralleled type (elementary structures) will be referred to as complex systems.

With complex systems, the determining of the structure function becomes a difficult problem; this difficulty has resulted in researches and in different algorithms that are met in the technical literature.

Such an algorithm, we have referred to as FIAB 1 for a system with n components, figured as a graph with k nodes, is as follows [6]:

FIAB 1

Step 0. Input connection matrix attached to graph: $[C]_{k \times k}$.
Step 1. A matrix with 1 on the main diagonal is added to the connection matrix.
Step 2. The first column and the last line of the matrix C are eliminated and with the remaining lines and columns a determinant of the $k-1$ order is formed, designated by $|D|$.
Step 3. The determinant $|D|$ is expanded by using the sum and boolean product operations and the structure function is obtained.

3. SIMULATION OF INDIVIDUAL RELIABILITIES

On basis of the data obtained following the long lasting researches on components regarding the variation of their parameters with the stress and time, the probability densities associated to the parameters characterizing the system component elements are determined.

When the probability densities are known, the generating algorithm of the reliability functions (FIAB 2) is as follows:

FIAB 2

Step 0. R N D initialization (the algorithm for generating a random number uniform on (0, 1).
Parameter input.
Step 1. Call of the algorithm relevant to the respective law of distribution.
Step 2. A selection value R is obtained.

The fact should be specified that there are performing algorithms for programmed generation in [1], [4] for all the random variables whose distributions have practicability in reliability.

4. CALCULUS ALGORITHM OF THE SYSTEM RELIABILITY

A system whose components are in series, meaning from the reliability point of view that the system operates if, and only if all its components operate, has the reliability given by

$$R_S = P(S = 1) = \prod_{i=1}^{n} P(x_i = 1) = \prod_{i=1}^{n} R_i.$$

A system whose components are paralleled, meaning from the reliability point of view that the system operates if, and only if at least one of its components operates, has the reliability given by

$$R_S = 1 - \prod_{i=1}^{n} (1 - R_i.)$$

But, in fact, the systems have complex structures and their reduction to elementary (series, paralleled) structures is a difficult problem, resulting in a great volume of calculations and difficult to be algorithmized.

In such cases, a digital evaluation of the reliability function may be obtained by means of an experimental method, based on the Monte Carlo simulation, by means of the computer [2], [5].

In the hypothesis of the system independence from component failure, each component may be in one of the two states: operation or defective, and the algorithm is as follows:

FIAB 3

Step 0. Initializes the R N D algorithm for generating some random numbers uniform on (0, 1).
Inputs: structure function Φ and parameters of the probability densities $k \leftarrow 1$.

Step 1. Generating with the FIAB 2 a selection value for a reliability function R_k.

Step 2. Generating of a random number U_k on (0, 1).

Step 3. When $U_k < R_k$, then $x_k = 1$, otherwise $x_k = 0$.

Step 4. $k \leftarrow k + 1$
When $k \leqslant n$, pass on to step 1.
Otherwise, to step 5.

Step 5. The status vector X is formed.
Let us calculate a state of the system,

$$S = \Phi(X).$$

Following the carrying out of this algorithm, we obtain a state of the system; the evaluation of the reliability function of the system is given by

$$\hat{R}_S = \frac{\sum_{i=1}^{N} S_i}{N}$$

where N depends on the required accuracy and signifies the number of simulations carried out with the above algorithm.

5. EXAMPLE

Let us assume that we have a system with 9 elements whose structure is as follows:

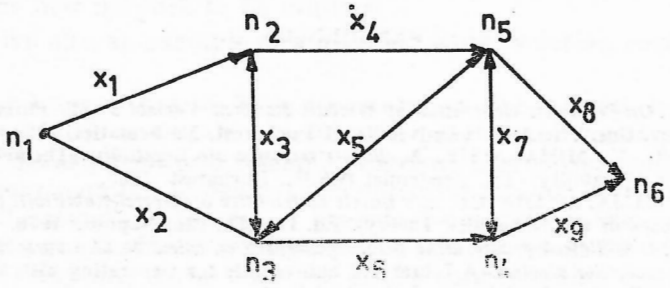

Fig. 1

and the reliability functions have a Weibull distribution of the parameters

$$\nu_1 = 0.65;\ \nu_2 = 0.86;\ \nu_3 = 0.64;\ \nu_4 = 0.72;\ \nu_5 = 0.83;$$

$$\nu_6 = 0.68;\ \nu_7 = 0.85;\ \nu_8 = 0.70;\ \nu_9 = 0.91.$$

Following the call of the FIAB 1 algorithm we obtain the structure function:

$$\Phi = x_1 x_3 x_6 x_7 x_8 \vee x_1 x_3 x_6 x_9 \vee x_1 x_3 x_5 x_8 \vee x_1 x_3 x_5 x_7 x_9 \vee$$
$$\vee x_1 x_4 x_8 \vee x_1 x_4 x_7 x_9 \vee x_1 x_4 x_5 x_6 x_9 \vee x_2 x_6 x_7 x_8 \vee$$
$$\vee x_2 x_6 x_9 \vee x_2 x_5 x_8 \vee x_2 x_5 x_7 x_9 \vee x_2 x_3 x_4 x_8 \vee x_2 x_3 x_4 x_7 x_9.$$

The above described simulation model was programmed on computer with the adequate data for this example and for the number of steps $N = 5\,000$ the reliability $R_s = 0.471$ was obtained, and the computing time was of 54 seconds on the FELIX C-256 system. The low reliability of the system is explained by the system complexity and by the fact that the parameters ν_i, $1 \leqslant i \leqslant 9$ are small (i.e. the components have low reliabilities).

6. CONCLUSIONS

When we vary the parameters ν_i and give them higher values we may find a set of values for these parameters to lead to a system pre-set reliability, and this would state which values should have the Weibull reliabilities for providing the R_S reliability.

The above described simulation algorithm has the advantage to be easily programmed in the FORTRAN IV language and implemented on any computer.

The computing time is reduced due to the existence of some performing algorithms for generating random variables whose distributions describe the reliability of the system components.

With the above algorithm different versions of the structure of a system may be simulated with a view to selecting the one which meets the user's requirements.

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