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A MACRO MODEL FOR FINANCIAL POLICY OF A CENTRAL BANK

The paper contains the description and analysis of a linear-quadratic optimisation model for the policy of a central bank.

The main state variables are money supply, the short-term interest rate and the price index; there is only one contriol variable, representing the "call" rate, i.e. the interest rate of the central bank.

The objective function represents the weighted sum of the state variables' deviations from certain imposed values (targets),and of the "cost of control", cumulated over the whole horizon of the model.

Keywords: linear-quadratic optimisation, monetary policy, targeting

1. INTRODUCTION

We consider here a highly aggregate model for the financial policy of a central bank. We were stimulated by [2], where the specific situation of the Bank of Japan was studied. As in [2], our approach here combines econometrics with optimal control; the econometric connections are also inspired from [2].

Our model contains four state variables, all of them expressed as logarithmic differences of cumulative indices; these states correspond to

indices of financial transactions, broad money, current interest rate and prices. The only control variable is the "call rate" as the main instrument of the central bank to influence the financial market. There are also two exogenous inputs corresponding to the indices of world financial markets activity and of the prices of raw materials, including oil.

Since we have in mind the specific situation of Romania and of other countries in transition, where there is no real share market, we do not consider shares in our model.

Considering logarithmic differences allows to use regressions to build a linear discrete – time model of the form

$x_{t+1} = A_t\, x_t + B_t\, u_t + C_t\, f_t$, where f_t are the exogenous inputs.

In fact, the matrices A_t, B_t, C_t are constant; since there is a possibility to introduce some time variations when considering inflation rate, we discuss the mathematics of the problem in time varying setting.

The above model can be used for simulation in order to study the way in which decisions of the central bank concerning the "call rate" influence the financial market.

But the model allows also for the study of an optimal policy, based on linear – quadratic optimization on finite horizon.

We shall concern ourselves, in this paper, with the deterministic approach, although some considerations can be developed in a stochastic framework. For instance, it is easy to prove that the same feed – back control gives the optimum both in the deterministic case and in the one in which the exogenous inputs are independent random variables. It is also possible to consider a case in which inflation rate or expected price are looked at as being Markov processes with finite number of states, corresponding, for example, to pessimistic, medium or optimistic attitudes.

The paper is organized as follows. In section II, we describe the content of the variables involved and the regression procedure used in building the model. We also bring the model to a form convenient for the theory at hand. Section III contains the mathematical results to be used to get the optimal control, the proof being sketched in an Appendix. Section IV contains some conclusions.

2. THE THEORETICAL MACRO MODEL. CONTROL METHODOLOGY

One of the most important state variables, which we shall denote by Z_t, represents the total amount of financial transactions in the time interval indexed by t. This total amount gives a highly aggregate picture of the economic activity, in the sense that the supply reflects sedentary money resulting from successful activities and the demand reflects the tendency for investment. The equality between demand and supply reflects an active economy, where the flow of capital (money) goes from saving to investment.

Starting from suggestions in [2], we accept for constructing the econometric model that the demand for Z will depend upon previous demand, broad money, price index and short – term interest rate; we also admit the influence of an external factor, Z^*, which is a corresponding index for external financial activity. This exogenous factor, as well as the price P^* for raw material can be assumed to allow for independent random fluctuations, but in the present paper we consider them as given for the whole period considered (e.g. as a result of a prognosis exogenous to this model).

The other state variables, as it can be seen from the above considerations, are: broad money M, the price index P and the short – term interest rate, R.

The broad money, considered as the expression of the wealth participating in the financial transactions, is regarded, for econometric connections, as demand and supply.

On the supply side, it will depend only upon the short – term interest rate and upon the interest rate of the central bank, which will be the only ***control variable*** in the model.

On the demand side, there is a dependence on the short – term interest rates, on Z and on the lagged values of M and P.

As concerns P, it is assumed to depend on Z, on the expected price P^e and on the external price P^*. In this deterministic framework, we consider $P_t^e = \Pi P_{t-1}$, where Π is constant. In a stochastic framework, we could consider $\Pi = \Pi_t(\sigma_t)$ where Π_t is a Markov process with a finite number of states (e.g. three states, corresponding to pessimistic, medium and optimistic attitudes).

In order to use standard econometric procedures, we shall work with logarithmic differences, making in this way all our dependences to be linear.

Thus, we use the following notations:

$$z_t = \ln\left(Z_t/Z_{t-1}\right);\ r_t = \ln\left(R_t/R_{t-1}\right);\ m_t = \ln\left(M_t/M_{t-1}\right);$$

$$p_t = \ln\left(P_t/P_{t-1}\right);\ z_t^* = \ln\left(Z_t^*/Z_{t-1}^*\right);\ p_t^* = \ln\left(P_t^*/P_{t-1}^*\right).$$

The regressions expressing the dependence between the above variables are the following:

$$z_t = \gamma_1 z_{t-1} + \gamma_2 r_{t-1} + \gamma_3 (m-p)_{t-1} + \gamma_4 z_t^* \qquad (1)$$

We assume that the interest rate has negative effects on z, so that $\gamma_2 < 0$, while real wealth has positive effects, so that $\gamma_3 < 0$.

For the supply of broad money, we consider the following regression:

$$m_t = \mu_1 r_{t-1} + \mu_2 i_t \qquad (2)$$

We assume that a decrease in the short – term interest rate would allow commercial banks to expand their supply of loans, which increases money supply, so that $\mu_1 < 0$. An increase in the "call rate" should decrease supply of loans, which decreases money supply, so that $\mu_2 < 0$.

On the demand side, we have the following regression for real money balances:

$$(m-p)_t = \lambda_1 r_t + \lambda_2 z_t + \lambda_3 (m-p)_{t-1}, \qquad (3)$$

with $\lambda_2 > 0$; $\lambda_2 > 0$; $\lambda_3 < 0$.

As concerns the regression equation for p, it will be of the form:

$$p_t = \Gamma_1 z_t + \Gamma_2 p_{t-1} + \Gamma_3 p_t^* \text{ with } \Gamma_3 > 0 \qquad (4)$$

To obtain a standard discrete time dynamic control system of the form

$$x_{t+1} = A\, x_t + B\, u_t + C\, f_t,$$

we shall transform equations (1) - (4) as follows.

Substituting (1) into (4), we obtain

$$p_t = a_{41} z_{t-1} + a_{42} m_{t-1} + a_{43} r_{t-1} + a_{44} p_{t-1} + c_{41} z_t^* + c_{42} p_t^* \quad (4')$$

where

$$a_{41} = \Gamma_1 \gamma_1, \quad a_{42} = \Gamma_1 \gamma_3, \quad a_{43} = \Gamma_1 \gamma_2,$$
$$a_{44} = \Gamma_2 - \Gamma_1 \gamma_3, \quad c_{41} = \Gamma_1 \gamma_4, \quad c_{42} = \Gamma_3.$$

We use then (3) to express r_t as

$$r_t = \frac{1}{\lambda_1}\left[m_t - p_t - \lambda_2 z_t - \lambda_3 m_{t-1} - \lambda_3 p_{t-1}\right].$$

Substituting in the right-hand side of the above relation the expressions for z_t, m_t and p_t, given by (1), (2) and (4), respectively, we get

$$r_t = a_{31} z_{t-1} + a_{32} m_{t-1} + a_{33} r_{t-1} + a_{34} p_{t-1} + b_3 i_t + c_{31} z_t^*, \quad (3')$$

where

$$a_{31} = -\frac{\lambda_2 \gamma_1}{\lambda_1}, \quad a_{32} = -\frac{\lambda_3 + \lambda_2 \gamma_3}{\lambda_1}, \quad a_{33} = -\frac{\mu_1 - \lambda_2 \gamma_1}{\lambda_1},$$

$$a_{34} = \frac{\lambda_2 \gamma_3 - \lambda_3}{\lambda_1}, \quad b_3 = \frac{\mu_2}{\lambda_1}, \quad c_{31} = -\frac{\lambda_2 \gamma_4}{\lambda_1}.$$

We rewrite equation (1) in the form

$$z_t = a_{11} z_{t-1} + a_{12} m_{t-1} + a_{13} r_{t-1} + a_{14} p_{t-1} + c_{11} z_t^* \quad (1')$$

where

$$a_{11} = \gamma_1, \quad a_{12} = \gamma_3, \quad a_{13} = \gamma_2,$$

$$a_{14} = -\gamma_3, \quad c_{11} = \gamma_4.$$

Finally, denoting i_t by u_{t-1} and writing a_{23} for μ_1 and b_2 for μ_2, we have

$$m_t = a_{23} r_{t-1} + b_2 u_{t-1}. \quad (2')$$

Denoting by x the column-vector consisting of the variables z, m, r and p, and by f – the column-vector formed by the exogenous variables z^* and p^*, the system of equations (1') – (4') can be written in the following matrix form:

$$x_{t+1} = A x_t + B u_t + C f_{t+1}, \; t = 0, 1, \ldots, T-1,$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & 0 & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}; \quad B = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix}; \quad C = \begin{pmatrix} c_{11} & 0 \\ 0 & 0 \\ c_{31} & 0 \\ c_{41} & c_{42} \end{pmatrix}.$$

Denoting $C f_t = F_{t-1}$, $t = 1, \ldots, T$, we obtain the standard form

$$x_{t+1} = A x_t + B u_t + F_t. \quad (5)$$

We attach to this system a cost-function of the central bank

$$\sum_{t=0}^{T-1} \rho\, i_t^2 + \sum_{t=1}^{T} \left[(m_t - \hat{m}_t)^2 q_2 + (r_t - \hat{r}_t)^2 q_3 + (p_t - \hat{p}_t)^2 q_4 \right].$$

We can rewrite the above cost-function in matrix form, as

$$J(u) = \sum_{t=0}^{T-1} u_t^* K u_t + \sum_{t=1}^{T} \left[(x_t - \hat{x}_t)^* Q (x_t - \hat{x}_t) \right],$$

where

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{pmatrix} \text{ and } K = \rho.$$

The sequences $\hat{m}_t, \hat{r}_t, \hat{p}_t$ represent targets; $\hat{r}_t, \hat{p}_t$ may be negative, if we want interest rates and prices to decrease, while $\hat{m}_t > 0$, if we want broad money to increase. The weights q_2, q_3, q_4 are positive, and so is ρ.

As such, the problem corresponds to the theory at hand.

3. SOLUTION OF THE OPTIMAL CONTROL PROBLEM

Our model can be used either as a simulation model or as an optimization model. For simulation, we use equation (5) above, whereas the optimization problem consists in minimizing the cost – function, subject to (5).

In what follows, we present the main results concerning the solution of the optimization problem. The proof can be found in the Appendix. Although the matrices A, B, Q, S are constant in our specific case, we give the theoretical results for a wider framework, considering the matrices as being time – dependent. Indeed, if we divide the identification horizon into several parts and perform identification separately in each part, we obtain a time – dependence for the past, which can be continued by periodicity for the optimization horizon. In such situation, a time varying model will be obtained.

We consider the dynamic system

$$x_{t+1} = A_t x_t + B_t u_t + F_t,$$

where x is the n–dimensional column–vector of state variables, u is the

m–dimensional column–vector of control variables, F is the p–dimensional column–vector of exogenous variables, A is an $n \times n$ matrix and B is an $n \times m$ matrix.

We associate the cost–function

$$J(u) = (x_T - \hat{x}_T)^* Q_T (x_T - \hat{x}_T) + \sum_{t=0}^{T-1} \left[(x_t - \hat{x}_t)^* Q_t (x_t - \hat{x}_t) + u_t^* K_t u_t \right],$$

where $\hat{x}_t$ are of the same dimension as x_t and stand for *targets*, Q_t are symmetric, positive definite $n \times n$ matrices and K_t are symmetric, strictly positive definite $m \times m$ matrices.

The optimal control problem consists in finding a control $u = (u_0, u_1, \ldots, u_{T-1})$ which minimizes $J(u)$.

To solve this problem, we consider the auxiliary system

$$\eta_{t+1} = A_t \eta_t + F_t; \ \eta_0 = 0.$$

Using the solution of this system, we define the vectors

$$y_t = x_t - \eta_t; \ \hat{y}_t = \hat{x}_t - \eta_t$$

and obtain the linear control problem

$$y_{t+1} = A_t y_t + B_t u_t \qquad \text{(I)}$$

$$\min_u J(u) = (y_T - \hat{y}_T)^* Q_T (y_T - \hat{y}_T) + \sum_{t=0}^{T-1} \left[(y_t - \hat{y}_t)^* Q_t (y_t - \hat{y}_t) + u_t^* K_t u_t \right]. \qquad \text{(II)}$$

We define the system of nonlinear matrix equations

$$\begin{cases} K_t + B_t^* S_{t+1} B_t = V_t^2 \\ A_t^* S_{t+1} B_t = W_t^* V_t \\ A_t^* S_{t+1} A_t - S_t + Q_t = W_t^* W_t, \end{cases} \qquad \text{(III)}$$

with final condition $S_T = Q_T$, as well as the linear system

$$g_t = (A_t + B_t H_t)^* g_{t+1} + Q_t \hat{y}_t, \quad t = 0, 1, \ldots, T-1, \qquad \text{(IV)}$$

with final condition $g_T = Q_T \hat{y}_T$.

In (IV), we denoted $H_t = -V_t^{-1}W$, where V_t, W_t are solutions of (III).

It can be proved (by backward induction) that the system (III) has a solution (V_t, W_t, S_t), $t = 0,1,\ldots,T-1$, such that V_t is strictly positive definite (hence invertible) and S_t is symmetric and positive definite.

It can be proved (see Appendix) that the optimal control is given by:

$$\tilde{u}_t = H_t y_t + \left(K_t + B_t^* S_{t+1} B_t\right)^{-1} B_t^* g_{t+1},\ t = 0,1,\ldots,T-1 \quad \text{(V)}$$

It is also easy to prove that, although V_t, W_t may not be unique, the matrices S_t and H_t which occur in formula (IV) for g_t and in formula (V) for the optimal control are unique, so that the optimal control u_t will result unique, for each t.

Remark

In order to solve system (IV), we need to know all the values for f_t, $t = 1,\ldots,T$.

4. CONCLUSIONS

The present paper, inspired by [2], differs from [2] by two main aspects: one the one hand, some elements specific for Japan are removed from the model and, on the other hand, we consider here the deterministic problem on finite horizon. In this way, we hope that we succeeded to give a clearer explanation of the model's specificity, a simplification of the identification process and a computational procedure, which is easy to implement.

Thus, the computational procedure comprises the following steps:

1. For $t = 1, 2, \ldots, T$ we compute η_t from system (1), as well as the new variables y_t and $\hat{y}_t$.

2. For each t (from T to 0 backwards), we compute V_t (one – dimensional), W_t (four – dimensional row – vector), S_t (four – dimensional symmetric square matrix, i.e. 10 unknown elements), H_t (four – dimensional row – vector) and g_t (four – dimensional column – vector); by means of these elements we compute u_t, as well as $V_{t-1}, W_{t-1}, S_{t-1}, g_{t-1}$.

3. For $t = 1, 2, \ldots, T$ we compute the optimal value of y_t, using the dynamic equation from system II and the values of the optimal control, determined at step 2. Then we compute the optimal value of x_t, using the relation $x_t = y_t + \eta_t$.

APPENDIX

We shall prove that

$$\tilde{u}_t = H_t\, y_t + \left(K_t + B_t^*\, S_{t+1}\, B_t\right)^{-1} B_t^*\, g_{t+1},\ t = 0, 1, \ldots, T-1$$

is optimal for problem II.

To do so, we evaluate the cost $J(u)$, using systems (III) and (IV). We have

$$J(u) = y_T^*\, Q_T\, y_T + \sum_{t=0}^{T-1} \left(y_t^*\, Q_t\, y_t + u_t^*\, K_t\, u_t\right) + \hat{y}_T^*\, Q_T\, \hat{y}_T -$$

$$- y_T^*\, Q_T\, \hat{y}_T - \hat{y}_T^*\, Q_T\, y_T + \sum_{t=0}^{T-1} \hat{y}_t^*\, Q_t\, \hat{y}_t - \sum_{t=0}^{T-1} \left(\hat{y}_t^*\, Q_t\, y_t + y_t^*\, Q_t\, \hat{y}_t\right).$$

We denote

$$\Sigma_1 = y_T^*\, Q_T\, y_T + \sum_{t=0}^{T-1} \left(y_t^*\, Q_t\, y_t + u_t^*\, K_t\, u_t\right)$$

and

$$\Sigma_2 = y_T^*\, Q_T\, \hat{y}_T + \hat{y}_T^*\, Q_T\, y_T + \sum_{t=0}^{T-1} \left(\hat{y}_t^*\, Q_t\, y_t + y_t^*\, Q_t\, \hat{y}_t\right).$$

Then we can write

$$J(u) = \Sigma_1 - \Sigma_2 + \sum_{t=0}^{T-1} \hat{y}_t^*\, Q_t\, \hat{y}_t$$

We remark that the last term is independent of $\tilde{u}$, so that the optimal control $\tilde{u}$ is that which minimizes $\Sigma_1 - \Sigma_2$.

For Σ_1 we find the following evaluation

$$\Sigma_1 = y_T^* S_T y_T + \sum_{t=0}^{T-1} \left[y_t^* \left(W_t^* W_t + S_t - A_t^* S_{t+1} A_t \right) y_t + u_t^* K_t u_t \right] =$$

$$= y_T^* S_T y_T + \sum_{t=0}^{T-1} \left[y_t^* W_t^* W_t y_t + y_t^* S_t y_t - \right.$$

$$\left. y_t^* A_t^* S_{t+1} A_t y_t + u_t^* K_t u_t \right] =$$

$$= y_T^* S_T y_T + \sum_{t=0}^{T-1} \left[y_t^* W_t^* W_t y_t + y_t^* S_t y_t - \right.$$

$$\left. - (y_{t+1} - B_t u_t)^* S_{t+1} (y_{t+1} - B_t u_t) + u_t^* K_t u_t \right] =$$

$$= y_T^* S_T y_T + \sum_{t=0}^{T-1} \left(y_t^* W_t^* W_t y_t + y_t^* S_t y_t - y_{t+1}^* S_{t+1} y_{t+1} - \right.$$

$$\left. - u_t^* B_t^* S_{t+1} B_t u_t + u_t^* B_t^* S_{t+1} y_{t+1} + y_{t+1}^* S_{t+1} B_t y_t + u_t^* K_t u_t \right) =$$

$$= y_0^* S_0 y_0 + \sum_{t=0}^{T-1} \left[y_t^* W_t^* W_t y_t + u_t^* B_t^* S_{t+1} (A_t y_t + B_t u_t) + \right.$$

$$\left. + \left(y_t^* A_t^* + u_t^* B_t^* \right) S_{t+1} B_t u_t - u_t^* B_t^* S_{t+1} B_t u_t + u_t^* K_t u_t \right] =$$

$$= y_0^* S_0 y_0 + \sum_{t=0}^{T-1} \left[y_t^* W_t^* W_t y_t + u_t^* V_t^* W_t y_t + \right.$$

$$\left. y_t^* W_t^* V_t u_t + u_t^* \left(B_t^* S_{t+1} B_t + K_t \right) u_t \right] =$$

$$= y_0^* S_0 y_0 + \sum_{t=0}^{T-1} \left[y_t^* W_t^* W_t y_t + u_t^* V_t^* W_t y_t + y_t^* W_t^* V_t u_t + u_t^* V_t^* V_t u_t \right] =$$

$$= y_0^* S_0 y_0 + \sum_{t=0}^{T-1} (V_t u_t + W_t y_t)^* (V_t u_t + W_t y_t) .$$

As concerns Σ_2, we have

$$\Sigma_2 = g_T^* y_T + y_T^* g_T + \sum_{t=0}^{T-1} \left[\left(g_t^* - g_{t+1}^* (A_t + B_t H_t) \right) y_t + \right.$$

$$+ y_t^* \left(g_t - \left(A_t^* + H_t^* B_t^*\right) g_{t+1}\right)\Big] =$$

$$= g_T^* y_T + y_T^* g_T + \sum_{t=0}^{T-1} \left(g_t^* y_t - g_{t+1}^* A_t y_t - g_{t+1}^* B_t H_t y_t + \right.$$

$$\left. + y_t^* g_t - y_t^* A_t^* g_{t+1} - y_t^* H_t^* B_t^* g_{t+1}\right) =$$

$$= g_T^* y_T + y_T^* g_T + \sum_{t=0}^{T-1} \left(g_t^* y_t + y_t^* g_t\right) -$$

$$- \sum_{t=0}^{T-1} \left[g_{t+1}^* \left(y_{t+1} - B_t u_t\right) + \left(y_{t+1}^* - u_t^* B_t^*\right) g_{t+1}\right] -$$

$$\sum_{t=0}^{T-1} \left(g_{t+1}^* B_t H_t y_t + y_t^* H_t^* B_t^* g_{t+1}\right) =$$

$$= g_T^* y_T + y_T^* g_T + \sum_{t=0}^{T-1} \left(g_t^* y_t + y_t^* g_t\right) - \sum_{t=0}^{T-1} \left(g_{t+1}^* y_{t+1} + y_{t+1}^* g_{t+1}\right) +$$

$$+ \sum_{t=0}^{T-1} \left(g_{t+1}^* B_t u_t - u_t^* B_t^* g_{t+1}\right) - \sum_{t=0}^{T-1} \left(g_{t+1}^* B_t H_t y_t + y_t^* H_t^* B_t^* g_{t+1}\right) =$$

$$= g_0^* y_0 + y_0^* g_0 + \sum_{t=0}^{T-1} \left(g_{t+1}^* B_t u_t + u_t^* B_t^* g_{t+1}\right) -$$

$$\sum_{t=0}^{T-1} \left(g_{t+1}^* B_t H_t y_t + y_t^* H_t^* B_t^* g_{t+1}\right).$$

Then

$$\Sigma_1 - \Sigma_2 = y_0^* S_0 y_0 + \sum_{t=0}^{T-1} \left[\left(V_t u_t + W_t y_t\right)^* \left(V_t u_t + W_t y_t\right)\right] -$$

$$- g_0^* y_0 - y_0^* g_0 - \sum_{t=0}^{T-1} \left(g_{t+1}^* B_t u_t + u_t^* B_t^* g_{t+1}\right) +$$

$$\sum_{t=0}^{T-1} \left(g_{t+1}^* B_t H_t y_t + y_t^* H_t^* B_t^* g_{t+1}\right) =$$

$$= y_0^* S_0 y_0 - g_0^* y_0 - y_0^* g_0 +$$

$$+ \sum_{t=0}^{T-1} \left[V_t u_t + W_t y_t - \left(V_t^*\right)^{-1} B_t^* g_{t+1} \right]^* \left[V_t u_t + W_t y_t - \right.$$

$$\left. - \left(V_t^*\right)^{-1} B_t^* g_{t+1} \right] - \sum_{t=0}^{T-1} \left[g_{t+1}^* B_t V_t^{-1} \left(V_t^*\right)^{-1} B_t^* g_{t+1} \right] .$$

The only term which depends on u is

$$\sum_{t=0}^{T-1} \left[V_t u_t + W_t y_t - \left(V_t^*\right)^{-1} B_t^* g_{t+1} \right]^* \left[V_t u_t + W_t y_t - \left(V_t^*\right)^{-1} B_t^* g_{t+1} \right].$$

We remark that it is non-negative, so that its minimum value is obtained if

$$V_t u_t + W_t y_t - \left(V_t^*\right)^{-1} B_t^* g_{t+1} = 0 , \quad t = 0,1,\ldots,T-1 .$$

It follows that

$$\tilde{u}_t = -V_t^{-1} W_t y_t - V_t^{-1} \left(V_t^*\right)^{-1} B_t^* g_{t+1} =$$

$$= H_t y_t + \left(K_t + B_t^* S_{t+1} B_t\right)^{-1} B_t^* g_{t+1} .$$

REMARK

We used in this proof the fact that V_t is invertible. As we mentioned it in section III, this fact is easy to prove by backward induction.

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