Eng. T. HĂNCIULESCU
V. DUMITRU

Application of linear programming to the choice and distribution of investments in new structures for oil extraction

This paper defines certain economic and technical parameters to be considered in studying the choice and distribution of investments in oil extraction on new structures. Subsequently two simplified models of optimization are set forth and solved, one having the object of bringing costs down to a minimum and the other of bringing investment funds down to a minimum.

Linear programming as a scientific method for establishing optimum results in the guiding of activities has developed considerably in the last few years. It is applied in all domains of activity : optimization of production, optimization of use of investment funds, optimization of profits, etc.

We shall endeavour hereunder to show some of the elements required for the application of linear programming in the oil industry for the optimum distribution of investment funds to the structures where oil wells will be drilled in the succeeding years.

Generally the investment funds allotted to a branch or to an enterprise are limited. The problem which may arise in oil extraction is of using the funds in such a way that :

— the greatest production be obtained within the limits of a given investment fund;

— the lowest cost price of planned production or the highest profit be achieved;

If production and investment funds are pre-established by the plan, only the costs of extraction remain to be optimized.

Obviously the problem should also be considered in relation with the period required for writing off the fixed funds created by means of the invesments, and this ultimately appears as a problem of dynamic programming in which the profit obtained from the exploitation of the structures must be optimized over a certain period of time.

The determination of all the conditions required for the application of such complex programming is under study, and this paper only confines itself to setting forth certain elements for the calculation of some of the

parameters and the optimization of two simplified models, which may, however, provide useful indications in the problem.

Production as established under plan is obtained from the oil wells in operation on January 1st and from the wells newly drilled during the year.

If we note by P the production of the preceding year and by k the coefficient of production decrease, the year's production will be:

$$P + \Delta P = P(1 - k) + N_n \cdot D_n$$

where:

ΔP — production increase
N_n — number of new wells
D_n — average output of the wells

Hence, the number of new wells will be:

$$N_n = \frac{P + \Delta P - P(1 - k)}{D_n} = \frac{\Delta P + kP}{D_n} \tag{1}$$

Considering the average investment per new well as I_{sn}, the volume of investments will be shown by

$$I = \Sigma N_n I_{sn} = \text{constant}, \tag{2}$$

Both I_{sn} and N_n are variable according to the structure where the new wells are located; in the structure with deep deposits, I_{sn} will be higher. It is, however, assumed that deeper wells yield a higher production.

If we have n structures, where exploitation is carried out, we shall have to choose the structures in such a way as to have

$$I_{s1}N_1 + I_{s2}N_2 + I_{s3}N_3 + \ldots + I_{sn}N_n = I \tag{3}$$

This is a condition in choosing the structures where work will be carried out. It is obvious that drilling may be caried out in all the structures, or only in certain structures, leaving some in reserve. However, a necessary condition is that the costs should be at a minimum. Costs are, in their turn, a consequence of the costs of the old and the new structures.

The costs for the older structures will be:

$$p_v = a_v + \frac{C}{D_v} + \frac{A}{P(1 - k)} \tag{4}$$

where:

a_v — represents fixed costs per extracted ton
C — average cost per well-month, excluding writing off of the older structures
D_v — average monthly output of older structures
A — yearly writing off of investments in older structures

In calculations, P should be taken as a constant.

The costs for new structures will be :

$$p_n = a_n + \frac{C_s}{D_n} + \frac{I_s \cdot N_n}{n_a \cdot N_n \cdot D_n} \tag{5}$$

where :

n_a — period of writing off the new investments or, by introducing the values of N or Is :

$$p_n = a_n + \frac{C_s}{D_n} + \frac{I}{n_a(\Delta P + kP)} \tag{6}$$

Since p_v is the same in all cases, minimum costs will be given by relation (6).

In this relation, a_n may be taken as a constant, and the 3rd term is likewise constant since I, n_a and $(\Delta P + kP)$ are constant.

The only variable term is $\frac{C_s}{D_n}$

The average output of the new wells (D_n) will be given by the relation :

$$D_n = \frac{N_1 D_1 + N_2 D_2 + N_3 D_3 + \ldots}{N_1 + N_2 + N_3 + \ldots} = \frac{\Delta P + kP}{\Sigma N_n} \tag{7}$$

Taking into consideration indices 1, 2, 3, ... for the structures exploited, formula (6) becomes

$$p_n = a_n + \frac{N_n}{\Delta P + kP}\left(C_s + \frac{I_s}{n_a}\right) \tag{8}$$

where :

I_s — average investment per well

C_s — average cost per well

Both C_s and I_s are, however, variable according to the number of wells that will be drilled on the new structures to be exploited.

If a_n is assumed to be constant, the minimum will be given by the 2nd term of the formula.

$$p_{min} = \frac{N_n}{\Delta P + kP}\left(C_s + \frac{I_s}{n_a}\right) \tag{9}$$

But we consider total investments as constant, hence $I_s \cdot N_a = I$, and formula (9) then becomes :

$$p_{min} = \frac{I}{\Delta P + kP}\left(C_s N_n + \frac{I}{n_a}\right) \tag{10}$$

The minimum of this function will therefore be given by

$$p_{min} = \frac{C_s \cdot N_n}{\Delta P + kP} = k_1 C_s \cdot N_n \tag{11}$$

where the variables are C_s and N_n, determined by the number of wells to be drilled in each structure.

Consequently, we must so choose the structures that $y = N_n \cdot C_s$ be at a minimum. Hence, we shall have as a minimum

$$y = N_1C_1 + N_2C_2 + N_3C_3 + \ldots \tag{12}$$

C_1, C_2, C_3 may be considered as constant.

Likewise, the number of wells will be determined by the relation :

$$N_1I_{s1} + N_2I_{s2} + N_3I_{s3} + \ldots = I \tag{13}$$

I_{s1}, I_{s2}, I_{s3} are constant.

On the other hand the output of the wells per structure is constant, and we shall consequently have :

$$N_1D_1 + N_2D_2 + N_3D_3 + \ldots = \Delta P + kP. \tag{14}$$

Relations (12), (13) and (14) will show us the number of wells which will yield production at the lowest costs.

But the number of new structures to be exploited is higher than 3. Thus, another relation will also have to be found. As is known, the number of wells per structure is limited by the number of locations existing in the area of exploitation. Hence, we shall have :

$$N_1 \leqslant N_{G1}$$

$$N_2 \leqslant N_{G2} \text{ etc.}$$

where : N_G = number of wells in the area.

In these calculations, I_s and C_s were considered with an approximation, as being constant. Actually, however, average investments per well vary according to the number of wells drilled in the structure, on account of the general investments of the structure. Hence

$$I_{s1} = I_c + \frac{I_G}{N_n}$$

I_c is constant and represents drilling costs and the cost of production installations in wells and separator tanks, while I_G represents general investments. $\frac{I_G}{N_n}$ amounts to about 20 per cent of I_s.

The same may also be said of C_s — exploitation costs per well-year — whose variation does not, however, depend very much on the number of wells, but on their depth.

Hence, in order to determine the minimum costs per well it is necessary to know C, I_s, D_m for each structure to be put into exploitation.

Let us consider two practical examples in order to show how the problem of optimum distribution of investments among the structures may be solved.

We shall assume the existence of 5 structures having the following characteristics :

Structure	Production thousand t per well-year	Investments million lei per well	Yearly costs of exploitation Million lei/well	Maximum number of locations
1	5	2	0.4	70
2	6	2.5	0.43	50
3	6.5	3.0	0.46	40
4	7	3.5	0.48	30
5	8	4.2	0.52	25

I. SIMPLIFIED MODEL OF MINIMIZING PRODUCTION COSTS UNDER CONDITIONS OF FIXED INVESTMENT FUND AND PRODUCTION GIVEN BY PLAN.

$min\ F = 0.4x_1 + 0.43x_2 + 0.46x_3 + 0.48x_4 + 0.52x_5$ — function for optimizing the x_i variables satisfying the limitation :

$5x_1 + 6x_2 + 6.5x_3 + 7x_4 + 8x_5 = 700$ thousand tons — limiting condition regarding the acccomplishment of the production plan.

$2x_1 + 2.5x_2 + 3.0x_3 + 3.5x_4 + 4.2x_5 = 300$ million lei — limiting condition regarding the investment fund

$x_1 \leqslant 70$
$x_2 \leqslant 50$
$x_3 \leqslant 40$
$x_4 \leqslant 30$
$x_5 \leqslant 25$

limiting conditions regarding the maximum number of locations per structure

$x_i \geqslant 0$

The number of wells per structure i was noted by x_i.

To facilitate the calculations, which were carried out manually, the model was solved by the simplex method for continuous variables. The optimum solution is

$$x_1 = \frac{125}{4}, \quad x_2 = 50, \quad x_3 = \frac{75}{2}$$

This solution satisfies the imposed conditions and indicates 51.25 million lei as the minimum value of exploitation costs in the first year. The solution is, however, unacceptable, owing to its fractionary character. Obvious-

ly, for such problems an algorythm for a solution in integer numbers should be applied. Owing, however, to the fact that the number of variables is small, reasonings may be made, leading to a solution close to the optimum integer solution. Even though the optimum in integer numbers has not been found, we may estimate what, in the technical literature, is described as the "regret" of the solution — the maximum loss with respect to the optimum.

We suggest the following solution :

$$x_1 = 32, \quad x_2 = 50, \quad x_3 = 37$$

For this solution we have

Production :

$$5 \cdot 32 + 6 \cdot 50 + 6.5 \cdot 37 = 700.5 \text{ thousand tons}$$

Required investment fund

$$2 \cdot 32 + 2.5 \cdot 50 + 3 \cdot 37 = 300.0 \text{ million lei}$$

Exploitation costs in the first year, minimum

$$0.4 \cdot 32 + 0.43 \cdot 50 + 0.46 \cdot 37 = 51.32 \text{ milion lei}$$

There results that, should this not be the optimum integer solution the regret would be 70,000 lei at most, which is a negligible amount compared with total costs and the total accuracy itself anticipated with respect to the probability with which the coefficients used in the calculations can be guaranteed.

Obviously, other close solutions may also be taken into consideration. Thus, the penultimate simplex programme shows :

$$x_1 = 40, \quad x_2 = 40, \quad x_3 = 40$$

For this solution we have :

Production :

$$5 \cdot 40 + 6 \cdot 40 + 6.5 \cdot 40 = 700 \text{ thousand tons}$$

Investments :

$$2 \cdot 40 + 2.5 \cdot 40 + 3 \cdot 40 = 300 \text{ million lei}$$

Exploitation costs :

$$0.4 \cdot 40 + 0.43 \cdot 40 + 0.46 \cdot 40 = 51.6 \text{ million lei/year}$$

Or else the solution :

$$x_1 = 32, \quad x_2 = 51, \quad x_3 = 36$$

considering that the number of wells on structure 2 may be exceeded by one unit.

This solution leads to :

Production :

$$5 \cdot 32 + 6 \cdot 51 + 6.5 \cdot 36 = 700 \text{ thousand tons}$$

Investments :

$$2 \cdot 32 + 2.5 \cdot 51 + 3 \cdot 36 = 299.5 \text{ million lei}$$

Exploitation costs :

$$0.4 \cdot 32 + 0.43 \cdot 51 + 0.46 \cdot 36 = 51.29 \text{ million lei}$$

Total costs for the following year may be estimated by means of formulas (4) and (6).

II. SIMPLIFIED MODEL FOR MINIMIZING THE INVESTMENT FUND UNDER CONDITIONS OF A PRODUCTION GIVEN BY PLAN.

As already mentioned, the problem should be set and solved under its complex aspect, in relation with the technical limitations and the economical policy, but in any case, with a view to obtain a maximum profit under the given conditions, a profit which should be a maximum for a period, e.g. for a period of 10 years, the span of time over which it is considered that the fixed funds should be written off is already stated, we have in view, to this end, a model of dynamic programming as a future study.

Nevertheless, even within this simplified framework, we believe that a plan for minimizing the necessary investment fund would be of importance. In fact, the granting of the fund also proceeds from the suggestion of the Ministry and it is obviously interesting to know what the required minimum is.

The model taken into consideration is :

$$\min F = 2x_1 + 2.5x_2 + 3.0x_3 + 3.5x_4 + 4.2x_5$$

with the following limiting conditions :

$5x_1 + 6x_2 + 6.5x_3 + 7x_4 + 8x_5 = 700$ thousand tons — condition regarding the fulfilment of the production plan

$x_1 \leqslant 70$
$x_2 \leqslant 50$
$x_3 \leqslant 40$ conditions regarding the maximum number of locations
$x_4 \leqslant 30$ per structure
$x_5 \leqslant 25$
$x_i \geqslant 0$

The following solution was obtained after calculation :

$$x_1 = 70, \quad x_2 = 50, \quad x_3 = \frac{100}{13} \approx 8$$

This solution leads to :

Production :

$$5 \cdot 70 + 6 \cdot 50 + 6.5 \cdot 8 = 702 \text{ thousand tons}$$

Investments :

$$2 \cdot 70 + 2.5 \cdot 50 + 3 \cdot 8 = 289 \text{ million lei}$$

Exploitation costs, first year :

$$0.4 \cdot 70 + 0.43 \cdot 50 + 0.46 \cdot 8 = 55.18 \text{ million lei}$$

In comparison with the first solution yielded by the minimization of operating costs, for approximately the same production, savings of 11 million lei are achieved for the investment fund, but costs in the first year of operation increase by $53.18 - 51.32 = 1.86$ million lei.

We believe that this variant is also important and may be adopted in some situations.

Although, as already mentioned, the problem set is considered soluble by a dynamic model, we nevertheless consider that the results obtained by means of these simplified models may prove useful, particularly when the number of structures is very great.

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L'APPLICATION DE LA PROGRAMMATION LINÉAIRE AU CHOIX ET À LA DISTRIBUTION DES INVESTISSEMENTS PAR STRUCTURES NOUVELLES DANS L'EXTRACTION DU PÉTROLE

L'analyse du choix et de la distribution des investissements par structures nouvelles dans l'extraction du pétrole, à l'aide de la programmation linéaire, est un problème important par ses applications pratiques possibles, avec des conséquences économiques considérables.

L'étude constitue une introduction à cette analyse, en définissant certains des paramètres économiques et techniques du problème et en présentant ensuite deux modèles simplifiés d'optimisation. L'objectif du premier modèle est la minimisation du prix de revient de la production dans la première année, dans les conditions d'une production et d'un fond d'investissements fixes, établis dans le plan. L'objectif du second modèle est la minimisation du fond d'investissements dans les conditions d'une production fixe planifiée.

Bien que les modèles présentés soient des modèles simplifiés, les résultats obtenus à l'aide de la programmation linéaire peuvent être particulièrement utiles pour un nombre important de structures

Les auteurs envisagent la possibilité future du solutionnement du problème au moyen d'un modèle de programmation dynamique, en tenant compte de la période d'amortissement des investissements dont le but serait l'optimisation du bénéfice net.

ПРИМЕНЕНИЕ ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ ПРИ ВЫБОРЕ И РАСПРЕДЕЛЕНИИ КАПИТАЛЬНЫХ ВЛОЖЕНИЙ ПО НОВЫМ СТРУКТУРАМ В ДОЪЫЧЕ НЕФТИ

Исследование выбора и распределения капитальных вложений по новым структурам в добыче нефти при помощи линейного программирования является важным вопросом вследствие большого экономического эффекта, который могло бы дать его практическое применение.

В статье дается введение в эту работу путем определения отдельных экономических и технических параметров задачи, а затем приводятся и решаются две упрошенные модели оптимизации. Первая модель преследует цель минимизации себестоимости продукции в первом году, когда планом задаются неизменный выпуск продукции и неизменный фонд капитальных вложений. Вторая модель преследует цель минимизации фонда капитальных вложений в условиях заданного планом неизменного выпуска продукции.

Хотя представленные модели являются упрошенными, получаемые при помощи линейного программирования результаты могут быть особенно полезными для большого числа структур.

Авторы видят в будущем возмохность разрешения вопроса и при помощи модели динамического программирования применительно к периоду амортизации капитальных вложений, в целях оптимизации чистой прибыли.