G. BOLDUR

Certain Remarks on the Concept of Utility and Its Part in the Decision Making Process

In the first section of this article, certain basic notions of decision making and utility theory are recalled (typical elements of the decision making process, utility estimation according to von Neumann-Morgenstern, utility additivity). The second section deals with the analysis of the main factors involved in utility estimation. The problem of the objectives (criteria), the states of nature and the individuals estimating utility is discussed. In the final section a complex model of decision making process is outlined.

1. BASIC NOTIONS

1.1. *Utility in the Decision Making Process*

The process of decision making, usually defined as a choice of the alternative (strategy) endowed with maximum utility, out of the set of possible alternatives, is characterized by the following typical factors:

a. The objective (goal), or objectives aimed at.

b. The decision maker, i.e. the individual or group of individuals which are about to make a decision concerning the accomplishment of the objective, or objectives, set forth.

c. The set of alternatives (strategies) including all possible variants of action involved in the achievement of the specific objectives.

d. The set of consequences induced by the alternatives, which may include either as many consequences as alternatives (certainty conditions), or a number of potential consequences for each alternative (risk of uncertainty conditions).

e. The utility associated to each consequence, in other words, the benefit expected by the decision maker as a result of the accomplishment of a given consequence.

In order to choose the alternative endowed with maximum utility, it is obviously necessary to mutually compare all utilities, hence to find a measure of utility. This essential part in the process of decision making is at the same time one of the most subtle and controversial problems in the recent literature dealing with this subject [1], [3], [4], [6], [11].

It is frequently easy to estimate utility, especially when it refers to "quantitative" objectives such as maximizing financial profits. If, for instance, the decision maker has the choice among the alternatives A_1, A_2, ...,A_m, having well known and definitely determined consequences in conditions of certainty and leading to the corresponding profits P_1, P_2,...,P_m, he will no doubt choose alternative $A^* \in \{A_1\, A_2,...,A_m\}$ corresponding to $P^* = max\ [P_1,\ P_2,...,P_m]$; hence it might be inferred that utility is proportional to the monetary value (this is true for the example mentioned above, while it is not, however, a general rule, as will be shown later).

In the case of "qualitative" objectives, difficulties associated to utility estimation are bound to appear. As an illustration, we shall consider a manufacturer whose main objective, during a certain time interval, is to acquire as high a commercial prestige as possible. If a number of alternatives are available regarding future trends of manufacturing and sales management, he will choose the alternative returning *maximal utility in terms of the specific objective.* In this case, however, the objective factor is an important element in measuring utility.

Particularly complex facets of this problem begin to arise *in case we consider a number of objectives* (goals, criteria). By merging the two examples given above in a single one and assuming the manufacturer to be interested both in profit maximization over the current period (objective number 1) and in commercial prestige maximization (objective number 2), it may be easily visualized that alternative A_1, leading to maximal utility as to objective number 1, will not coincide with alternative A_2, which is optimal for objective number 2. What is the decision maker to do in such circumstances? Will he ignore one of the two objectives, while substantiating his decision by considerig only the objective viewed as essential? The modern viewpoint of utility and decision theory states that it is reasonable to take into account both objectives, by considering a single "objective-synthesis" [2], [3], [4], [12]. In this case, however, the addition of utilities associated to both objectives, that is the finding of a common unit, will be required. This is obviously quite a difficult problem.

Utility, when a quantitative objective, a qualitative one, or even an "objective-synthesis" is considered, is affected by the attendant circumstances of the decision maker, consequently by the moment the estimation occurs [7]. Let us consider a decision maker that has the choice among three possible alternatives for making an investment, one of the objectives being least possible currency expenses. If the three possible alternatives stand for currency expenses amounting to 100 million, 105 million and 110 million, respectively, the utility point of view will not be indifferent toward the state of currency stocks available to the investor. In case the investor is in need of currency, the utility

associated to the alternative representing 100 million expenses will be superior to the utility of the same alternative, when considerable currency stocks are in store.

Consequently, there is not always a direct relation between utility and the monetary value corresponding to expected outcomes [1], [6].

Another important element in utility estimation is the probabilistic side of the matter. If the information available to the decision maker is such as to enable the definite knowledge of the outcomes associated to possible alternatives, the probabilistic aspect is not involved. If some specific alternative involves a number of consequences, this being a situation very frequently encountered in actual practice, the calculation includes the probability of occurrence of these consequences. Consider some objective, the strategies S_1, S_2,...,S_m and the possible circumstances*) for the occurrence of events N_1, N_2,...,N_n with associated probabilities p_1, p_2,...,p_n.

Let u_{ij} be the utility of strategy $S_i, i \in \{1,2, \dots ,m\}$ in the state of nature N_j, $j \in \{1,2,\dots,n\}$; the decision will be made according to the mathematical expectation associated to the the utility, expressed by

$$Eu_{ij} = \sum_{j=i}^{n} p_j u_{ij}.$$

These examples, showing how the question of utility estimation is posed, illustrate the complexity of utility measurement, a problem not fully cleared up. Nevertheless, the taking into account of utilities of various events is a compulsory condition in view of attaining a rational decision. The difficulty further increases when it comes to important economic activities, involving in most instances group decisions [14] and, considering the subjective character of utility, it is impossible to proceed to an "addition" of utilities for the members of the group.

All these difficulties have lead during recent years to important efforts directed towards an attempt to introduce more objective elements in utility definition and estimation.

There are future prospects for the endeavours in this area to result in a classification of concepts, and, in the long run, in finding methods for an objective utility estimation. In this context we have to recall the analogy made by von Neumann and Morgenstern twenty years ago when comparing utility to heat measurement; they showed that up to the discovery of the thermometer people were aware of "warmth" and "cold", but did not know how warm or how cold it was [11].

1.2. *Estimating utility according to von Neumann and Morgenstern*

Among the various definitions existing for utility, von Neumann-Morgenstern's definition is considered as being classical. Although almost twenty five years have elapsed since its presentation in the first edition of their work "Theory of Games and Economic Behavior", it is still very often used.

The definition is based on some fundamental propositions [1], [6], [11], which will be shown thereafter.

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*) In decision theory, distinct environment conditions are designated as „states of nature".

a. A decision maker comparing the worth of two alternatives*) whatever- A_i and A_j, may show one and only one of the following three behaviours:

— he prefers A_i to $A_j (A \mathcal{P} A_j)$;

— he prefers A_j to $A_i (A_j \mathcal{P} A_i)$;

— the decision maker considers both alternatives equally desirable or undesirable, or, in other words, he is indifferent towards the two alternatives $(A_i \mathcal{I} A_j)$.

(The first two behaviours are designated as preference relations, while the last one is called the indifference relation. The exclusive disjunction of the two relations $A_i \delta A_j = (A_i \mathcal{P} A_j) \veebar (A_i \mathcal{I} A_j)$, is sometimes used.)

b. The preference relation is transitive:

$$(A_i \mathcal{P} A_j) \,\&\, (A_j \mathcal{P} A_k) \Rightarrow (A_i \mathcal{P} A_k),$$

while the indifference relation is transitive and symmetrical:

$$(A_i \mathcal{I} A_j) \,\&\, (A_j \mathcal{I} A_k) \Rightarrow (A_i \mathcal{I} A_k),\ (A_i \mathcal{I} A_j) \Leftrightarrow (A_f \mathcal{I} A_i).$$

c. In addition to the set of simple alternatives $A = \{A_1, A_2, ..., A_m\}$, the decision maker may consider a particular type of alternatives, called "probabilistic gambles of two simple alternatives A_i and A_j "assuming the form $A' = [pA_i, (1 - p) A_j]$, where p is the probability of achieving alternative A_i and $1 - p$ is the probability of achieving alternative A_j.

d. For three given alternatives, A_1, A_2 and A_3 and a decision maker expressing relation $A_1 \mathcal{P} A_2 \mathcal{P} A_3$, there exists a gamble $A' = [p' A_1, (1 - p') A_3]$, so that $A' \mathcal{P} A_2$ and another gamble $A'' = [p'' A_1, (1 - p'') A_3]$, so that $A_2 \mathcal{P} A$".

e. For three given alternatives, A_1, A_2 and A_3 mutually distinct, if a decision maker expresses relation $A_1 \mathcal{P} A_2$, he will implicitly express also relation $[pA_1, (1 - p) A_3] \mathcal{P} [pA_2, (1 - p) A_3]$.

Based on these five propositions, a method for defining utility is outlined, which can also be used in decision making.

To each element belonging to the set of alternatives, an element belonging to the set of real numbers will be associated; the rule establishing the correspondence represents the utility function.

According to von Neumann and Morgenstern, utility $u(A_i)$ of alternative A_i is determined by considering as being known the utilities $u(A_1)$ and $u(A_0)$ of two alternatives A_1 and A_0 related by $A_1 \mathcal{P} A_0$.

With the understanding that $u(A_1) = 1$ and $u(A_0) = 0$, the following distinct cases can be outlined:

I. — $A_1 \mathcal{P} A_i \mathcal{P} A_0$, where probability p is estimated, for which $A_i \mathcal{I} [(pA_1, (i - p) A_0]$ and $u(A_i) = p$ (we shall obviously have $0 \leqslant u(A_i) \leqslant 1$);

II. — $A_i \mathcal{P} A_1 \mathcal{P} A_0$; in this case, probability q will have to be assessed, for which $A_1 \mathcal{I} [qA_i, (1 - q) A_0]$, assuming $u(A_i) = \frac{1}{q} \geqslant 0$;

*) Actually, the comparision is made between the consequences of the two alternatives. We shall assume, however a one-to-one relation existing between the set of simple alternatives and the set of consequences. Under these circumstances, it may be considered that the choice among a number of consequences is equivalent to the choice among the corresponding alternatives.

III. — $A_1 \mathcal{P} A_0 \mathcal{P} A_i$; in this case probability r is to be estimated, for which $A_0 \mathcal{I} [rA_1, (1-r)A_i]$, assuming $u(A_i) = -\frac{r}{1-r} < 0$.

The utility function defined in this way may be subject to a linear transformation, that is if $u(A_1)$ is interpreted as a utility defined in the terms above stated, then $u'(A_i) = a \times u(A_i) + b$, with a and b being two real numbers, will also be a utility function. Based on this property, cases II and III can be reduced to case I; in other words it is always possible to arrange that maximal utility be equal to 1 and minimal utility be zero.

Other important properties of the utility function are:

I. $u(A_i) > u(A_j)$ if and only if $A_i \mathcal{P} A_j$.

II. $A_i \mathcal{I} [pA_1, (1-p) A_0]\} \Rightarrow [u(A_i) = pu(A_1) + (1-p) u(A_0)]$

1.3. Operating with additive utilities.

As an illustration of the utilities operating procedure, we shall describe the following case.

In order to accomplish the serial manufacturing of an industrial product, the following four distinct technologies are possible: T_1, T_2, T_3 and T_4. The managing board of this industrial unit — where any of the four technologies may be utilized — has to choose one of them, by taking into account two criteria (objectives): profit and quality of the product. The following consequences resulting from the four alternatives can be predicted on the basis of technical and statistical data:

Table I

Alternatives	Criteria	
	C_1 amount of annual profit (thousands lei)	C_2 product quality (qualifications)
T_1	1,000	poor
T_2	800	good
T_3	600	excellent
T_4	700	good

The first part of the problem consists in estimating utilities in terms of those two criteria.

C_1. Consider $u_1(T_1) = 1$ and $u_1(T_3) = 0$; the two gambles are built up, which will define $u_1(T_2)$ and $u_1(T_4)$;

$T_2 \mathcal{I} [p_2 T_1, (1-p_2) T_3]$ and $T_4 \mathcal{I} [p_4 T_1, (1-p_n) T_3$; the subjective possibilities $p_2 = 0{,}5$ and $p_4 = 0.1$ are estimated, hence:

$$u_1(T_2) = 0.5 \text{ and } u_1(T_4) = 0.1.$$

It is to be noted, as was pointed out in the first section of this paper, that utilities are not directly related to monetary values.

This lack of proportionality may be due, in this specific case, to the fact that the profit to be achieved by the industrial unit amounts to 750,000 lei and utility decreases considerably if the real profit does not reach this predetermined level.

C_2. Take $u_2(T_3) = 1$ and $u_2(T_4) = 0$; the two gambles defining $u_2(T_2)$ and $u_2(T_4)$ are: $T_2 \mathcal{J}[0.7\ T_3; 0.3\ T_1]$ and $T_4 \mathcal{J}[0.7\ T_2; 0{,}3\ T_1]$ respectively, hence $u_2(T_2) = 0.7$ and $u_2(T_4) = 0.7$.

It is now possible to enter the values of utilities in the following:

Table II

Alternatives	Utilities	
	In terms of criterion C_1	In terms of criterion C_2
T_1	1	0
T_2	0.5	0.7
T_3	0	1
T_4	0.1	0.7

We come now to the second part of the problem, consisting of making the decision, by taking both criteria into account; we shall consider a criterion synthesis "totalling" utilities associated to both criteria keeping in mind that the two criteria considered have not the same economic importance. In view of "weighting" the relative importance of criteria, we shall associate a coefficient k_j to each criterion so that for the least important criterion we shall have $k_j = 1$. We assume $k_1 = 1$ and $k_2 = 1.5$. We shall multiply the utility values by these coefficients obtaining the values of weighted utilities (Table III). These will be added and subsequently entered in the last column of this table, containing utilities for the criterion-synthesis.

Table III

Alternatives	Weighted utilities		Utility of criterion synthesis
	For criterion C_1	For criterion C	
T_1	1	0	1
T_2	0.5	1.05	1.55
T_3	0	1.5	1.5
T_4	0.1	1.05	1.15

By applying the maximizing method to utility of criterion-synthesis, the decision maker will obviously choose alternative T_2.

In this example, utility has been estimated in agreement with von Neumann-Morgenstern's procedure. It should be mentioned that other utility estimation procedures are also known by specialists. Thus, in a recent paper, P.C. Fishburn [4] cites no less than 24 utility estimation methods. The characteristic of all these procedures consists in the fact that like von Neumann's method, the central element is the one related to subjective estimation.

2. CONCEPT ANALYSIS

As shown in the first section of this paper, in our opinion there is still much to be done as regards utility estimation in order to achieve the outline of a widely accepted point of view with practical applications.

In the following paragraphs we shall endeavour to briefly analyze the factors which in our opinion are essential for utility estimation.

2.1. *Objectives.*

When an individual, or a group of individuals pursue an objective, they try to arbitrarily isolate a part (or a facet) of reality and intend to achieve a certain level for that part of reality.

As an illustration, an economist setting as a target of an industrial unit the achievement of maximal profit over a certain time interval, is doing nothing else than isolating out of the set of facets belonging to the enterprise economic reality (production volume, production quality, prospective long-term development, commercial prestige, employees' remuneration, profits achieved, etc.) the last facet, and setting the goal of reaching the highest possible level from this point of view.

Owing to the "point of view" feature characterizing the objectives, they are sometimes called "criteria" [3], and some other times "factors" or "variables" [4].

It is to be noted that, simultaneously with isolating a certain aspect or facet of reality, in establishing the objective, a certain time interval is also isolated since, generally speaking, objectives are associated to certain terms and time intervals.

A remarkable property characterizing any objective, is the fact that it can usually be divided into constituent sub-objectives. Indeed, in the example mentioned above, the objective headed "maximal profits achieved by the enterprise during the period X" may be divided into the following sub-objectives: "maximal profit achieved by section A", "maximal profit achieved by section B", etc.

Directly related to this property, but somewhat in the opposite sense, is the second property, namely that it is always possible to group two or more objectives in order to obtain a new objective including them (objective-synthesis). Resuming the example described above, the six aspects featuring the economic situation of the industrial unit and constituting the main objectives may be combined in a sole objective (objective-synthesis) under the heading: the accomplishment of as good socio-economic results as possible. These two properties enable us to consider any objective as including a set of constituent sub-objectives: $X = \{X_1, X_2 \ldots, X_n\}$.

It is possible that two objectives X_i and X_j include common elements among their constituent sub-objectives. In this case we shall consider the objectives as being dependent. On the contrary, they will be independent if they do not contain common elements or, in other words, if they represent the disjoint sets: $X_i \cap X_j = \theta$.

Another definition for objectives (criteria) independence was suggested by P.C. Fishburn [3]. Given a decision making problem where m alternatives and n criteria appear (Table IV), the Cartesian product $X = X_1 \times X_2 \times ... \times X_n$ contains $r = m^n$ n-tuples of the type $(x_{i_1}1, x_{i_2}2, ..., x_{i_n}n)$. If we denote these n-tuples by $x^1, x^2, ..., x^r$, a set G of pairs of gambles of the form (ω_1, ω_2) by:

$$\omega_1 = (p_1 x^1, p_2 x^2, ..., p_r x^r), \qquad \sum_{j=1}^{r} pj = 1, \; x^j \in X, j \in \{1,2, ..., r\}$$

$$\omega_2 = (q_1 x^1, q_2 x^2, ..., q_r x^r) \qquad \sum_{k}^{r} q_r = 1, \; x^k = X, k \in \{1,2, ..., r\}.$$

so that $\omega_1 \neq \omega_2$, and the total probability of a $x_i \in X_i$ $(i = 1,2, ..., n)$ is the same in both gambles.

Table IV

Alternatives	Criteria			
	X_1	X_2	...	X_n
A_1	x_{11}	x_{12}	...	x_{1n}
A_2	x_{21}	x_{22}	...	x_{2n}
A_m	x_{m1}	x_{m2}	...	x_{mn}

According to Fishburn, given n criteria $X_1, X_2, ... X_n$, they are mutually independent in agreement with the utility theory if, and only if, $\omega_1 \mathcal{J} \omega_2$ for any $(\omega_1, \omega_2) \in G$.

Fishburn's definition is hardly applicable to the practical cases in which the strategies and criteria are quite numerous.

The importance of the notion of objectives (criteria) independence in agreement with the utility theory lies in the fact that, according to some authors utilities are additive only for independent objectives.

In our opinion, the problems related to criteria independence and to utility additivity have not been fully cleared up and they require further discussion.

Each separate facet of reality, which is isolated when considering an objective, has a number of potential levels more or less approximating the objective set forth. For instance, if we consider the objective "optimal" quality", the following levels may be accomplished: "exceptional", "very" good", "good", "moderate" "poor", "very poor". In this case, the respective objective level has a discrete variation. There are obviously other cases, where level variation is continuous, for instance, when considering the objective "maximal profit".

When the levels can be expressed by numbers, the objective is viewed as quantitative, while in case this quantification is not possible, the objective is qualitative.

An important observation is that potential levels of an objective can each represent an outcome of a certain alternative.

In establishing objectives, it must be remembered that in the decision making process, the decision makers very often take into account, either consciously or not, obscure and indefinite objectives. These might completely alter the decisions. For this reason, the unambiguous statement of all actual objectives by decision makers, is a first rank concern.

2.2. *The states of nature.*

The achievement of a certain level for a given objective, depends on a number of conditions, constituting a state of nature.

In order to illustrate the influence exerted by the states of nature, the best examples are to be found in decision making in agricultural activities. In this area, for any objectives, such as highest production per hectare, there is a level corresponding to each combination of alternatives, (such as various sorts of cereals which may be sown) with the states of nature (such as the situations: high humidity level, normal weather, drought).

Essentially, the states of nature represent realities assumed to prevail at the time that the effects of some alternatives are expected to become manifest namely those realities exerting an influence over the respective effect. It is obvious that these realities cannot be expected to appear simultaneously, and as a result, if we are considering them from the probabilistic point of view, they form a complete system of events (if we denote, in the above mentioned example, high humidity with N_1, normal weather with N_2 and drought with N_3, we shall obtain: $N_1 \cap N_2 \cap N_3 = \emptyset$, $p_1(N_1) + p_2(N_2) + p_3(N_3) = 1$.

Once the states of nature are well defined, the decision maker is faced with the problem of estimating their corresponding probabilities of accomplishment. Two situations are possible:

a) the decision maker is able to estimate these probabilities ;

b) the decision maker is not able to estimate them.

In both situations, diagrams of decision making are available, which will be discussed in the final part of this article.

2.3 *The individuals estimating utility.*

We have shown that in various definition and estimation procedures, utility is viewed as a subjective unit, depending on the estimator.

In our opinion, the subjective character of utility is a result of the following aspects:

1) the individual differences in appraising the objectives set forth and the relative significance bestowed upon these objectives;

2) the individual differences in appraising the potential states of nature and associated probabilities;

3) various assessments of consequences (levels) belonging to some given objectives and states of nature;

4) different psychological responses towards the same consequence, even if the objectives and states of nature are well determined;

5) the lack of a generally accepted unit of measure and of an estimation method for utility.

As to the first aspect, its effects may be considerably reduced if an organized concern exists for the assessment of objectives ranked differently and of

their relative significance. It is undeniable that this operation will always be endowed with a subjective appraisal character; however, if achieved by expert staff, based, on manysided analyses and consultations, it may lead, in time, to the elaboration of "codes of objectives" similar to the codes of laws. These "codes of objectives" together with the respective significance coefficients might be argued upon and subjected to public approval, constituting true guides in decision making.

The following two aspects are a result of insufficient available forecasting means. We think that, as these forecasting means will improve, considerable advances will be achieved in the description and listing of the potential states of nature as well as in the description and listing of the potential levels for given objectives and states of nature. Moreover, it is our opinion that important advances will be made in estimating probabilities associated to the various states of nature, the subjective probabilities being thus progressively replaced by objective probabilities.

The last two aspects reflect the true subjective character of utility. In our opinion, utility viewed as an objective magnitude has to be conceived as a "sum" of subjective utilities, hence as a social utility.

Consequently, the need of estimating subjective utility remains a fundamental problem, to which the problem of subjective utilities summation in view of obtaining social utility must be added.

The first of these last two problems is for the time being a question of psychometrics, all estimation methods suggested being, in the last analysis, reduced to measuring some psychical responses. Simultaneously with the progress of the measuring techniques of these responses, the possibility may exist for the quantification of psychologic correlates belonging to the respective responses. This will, of course, represent a sensible "objectification" of the measurement.

The second problem, namely the "combination" of subjective utilities, is discussed by the group decision theory [14]. In this theory it is considered that utilities cannot be interpersonally added and that it is, however, possible to make group decisions by handling the preference and indiference relations. An objectification of response measurement, through eventually using the physiological correlates, may remove the restraint concerning the interpersonal utility addition.

The subjective character of utility estimation may be, if not reduced, at least transferred into the area of social consensus, if a method is used for "encoding" the levels corresponding to the various objectives and states of nature. At the same time with the encoding operation, the expert groups to whom this task has been assigned would also estimate the values of the respective utilities, which would thus become "social utilities". Undoubtedly, the task is immense, since there is an infinite number of levels for the various objectives and states of nature. We must not forget, however, that in this operation computers can be used and that at the beginning only important objectives will be chosen, also selecting an acceptable number of potential levels.

3. OUTLINE OF A COMPLEX MODEL OF DECISION MAKING PROCESS

The decision making problems which may appear in economic practice are so different and intricate that it is necessary to consider the most complex possible one. Further, we shall present a complex model of decision making process.

Consider a finite set of compatible *objectives* (criteria): $X = \{X_1, X_2,..,X_n\}$, each objective X_j $(1j \in \{1,2,...,n\})$ being associated to an *importance coefficient* c_j so that $\sum_{j=1}^{n} c_j = 1$.

There is a set of *states of nature*: $N = \{N_1\, N_2,...,N_s\}$, each state of nature $N_k (k \in \{1,2,...,s\})$ being associated to an *achievement probability* $p_k \left(\sum_{p_k=1}^{s} p_k = 1 \right)$

Set Y is defined as the Cartesian product $X \times N = \{Y_{jk}\}$, $(j = 1,2,...,n;$ $k = 1,2,...,s)$, where element Y_{jk} represents objective X_j in the state of nature N_k. Each successive element Y_{jk} of set Y is also a set of levels: $Y_{jk} = \{x_{jk1}, x_{jk2}, \ldots, x_{jkr}\}$

There is a *set of individuals* $H = \{H_1, H_2, \ldots, H_s\}$ to whom the task of *estimating utilities* has been assigned, an authority coefficient $a_h \left(\sum_{h=1}^{t} a_h = 1 \right)$ being associated to each individual.

Consider further a *set**) *of alternatives* $A = \{A_1, A_2, \ldots, A_m\}$, each distinct alternative A_i being also a set with $n \times s$ elements called consequences: $A_i = \{x_{jki},\}$, $(i = 1,2,...,m;\ j = 1,2,...,n;\ k = 1,2,...,s)$. Each consequence x_{jki} is an element of one of the sets Y_{jk}, namely $x_{jki} \in Y_{jk}$.

For the individual $H_h (h \in \{1,2,...,t\})$, the *conventional* (subjective) *utility* associated to level x_{jkg} $(g \in \{1,2,...,r_{jk}\})$, and hence to the objective x_j and to the state of nature N_k, will be a number varying between 0 and 1, denoted by U_{hjkg}**). The same meaning belongs to the concept of subjective utility associated to consequence $x_{jki} (i \in \{1,2,...,m\})$, denoted by U_{hjki}. The *group utility*, or *social utility* or, in short, utility, associated to level x_{jkg}, will be considered as the weighted sum-total of conventional utilities of the group members H:

$$U^H_{jkg} = \sum_{h=1}^{n} a_h\, U_{hjkg}\text{***}).$$

The *utility for the state of nature* N_k, *alternative* A_i and *objective-synthesis* X, built up by component objectives $X_1, X_2,...,X_n$ is $U^{HX}_{ki} = \sum_{j=1}^{n} c_j\, U^H_{jki}$.

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*) Sets Y_{jk} and A may sometimes be infinite sets. For instance, they may represent possible solutions to a system of relations, such as a problem of mathematical programming.

**) When the decision making process takes place under certainty conditions, conventional utilities estimation may be made independently for each criterion. However when various states of nature are possible (risk or uncertainty conditions), conventional utilities estimation is performed by simultaneously taking into account all consequences appearing in the decision matrix.

***) In the model, the interpersonal addition of utilities is considered possible.

The expected value of utility for alternative i, is given by the expression

$$\bar{U}_i^{HX} = \sum_{k=1}^{s} p_k \, U_{ki}^{HX}$$

There are two possible cases:

a. When probabilities p_k are known, the decision making process will consist of choosing the alternative associated to a mathematical expectation of maximum utility.

b. When probabilities p_k are not known, the alternative for which the expression min U_{ki}^{HX} is maximum*) $(1 \leqslant k \leqslant s)$ will be chosen.

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CERTAINES REMARQUES SUR LE CONCEPT D'UTILITE ET SUR SON ROLE DANS LA DECISION

L'étude est consacrée à l'analyse des principaux facteurs qui interviennent dans l'appréciation de l'utilité. L'auteur examine la question des buts et de leur indépendence dans le sens de Fishburn, les états de la nature, ainsi que l'élément subjectif dans l'estimation de l'utilité. Il donne aussi la solution d'un modèle complexe du processus de décision.

On considère un ensemble fini d'*objectifs* indépendents $\{X = X_1, X_2,...,X_n\}$ ainsi qu'un ensemble *d'états de la nature* $N = \{N_1, N_2,...,N_s\}$. On définit le produit cartésien $Y = X \times N = \{Y_{jk}\}$. Chaque élément $\{Y_{jk}\}$ de l'ensemble Y est à son tour un ensemble de *niveaux*: $Y_{jk} =$

*) We have supposed the pessimistic criterion to be used; the expression will undergo the required changes when another criterion is used.

$\{x_{jk_1}, x_{jk_2},...,x_{jk_{rjk}}\}$. Il y a un ensemble $H=\{H_1, H_2,...,H_t\}$ d'*individus* qui estiment les utilités et un ensemble d'*alternatives* $A = \{A_1, A_2,...,A_m\}$, chaque alternative A_i étant á son tour un ensemble ayant n.s. *conséquences*: $A_i = x_{jki}$ avec $x_{jki} \in Y_{jk}$.

On définit *l'utilité conventionnelle* U_{hjkg} pour l'individu H_h et on associe au niveau x_{jkg} en tant que nombre compris entre 0 et 1, *l'utilité de groupe* et l'utilité pour des *objectifs synthèse*, et l'on indique le mode de leur utilisation dans le processus de décision.

НЕКОТОРЫЕ ЗАМЕЧАНИЯ О ПОНЯТИИ ПОЛЕЗНОСТИ И ЕЕ РОЛЬ В ПРОЦЕССЕ ПРИНЯТИЯ РЕШЕНИЙ

Исследование посвящено анализу главных факторов, влияющих на оценку полезности. Экзаменуется проблема показателей и их независимость в смысле Фишбурна, состояние природы, а также субъективный элемент при оценке полезности. Одновременно дается решение сложной модели процесса принятия решения.

Принимается за законченное множество независимых *показателей* $\{X = X_1, X_2,...,X_n\}$ и за множество *природных состояний* $N=\{N_1, N_2, \dots, N_s\}$. Определяется произведение Декарта $Y=X \times N = \{Y_{jk}\}$. Каждый элемент $\{Y_{jk}\}$ множества Y является, в свою очередь, множеством *уровней*: $Y_{jk} = \{x_{jk_1}, x_{jk_2}, \dots x_{jk_{rjk}}\}$

Существует множество $H=\{H_1, H_2...H_t\}$ *индивидов*, оценивающих полезности, и множество *альтернатив* $A = \{A_1, A_2, \dots, A_m\}$, где каждая альтернатива является, в свою очередь, множеством, имеющим n. s *последствия:* $A_i = \{x_{jki}\}$ *с* $x_{jki} \in Y_{jk}$.

Определяется *субъективная полезность* U_{hjkg} для индивида H_h приобщенная к уровню x_{jkg} как число, находящееся между 0 и 1.

В продолжение определяются: *групповая полезность* и полезность для синтетизирующих показателей и указывается способ применения в процессе принятия решений.