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Optimization Criteria Regarding the Reliability in Operation

An investigation method for ascertaining the profitableness of investments associated to objectives with random operation in time is described. The choice among the various alternatives investigated in a design is thus possible by using a general criterion of maximum investment efficiency.

The numerical indicators enable a technical comparison of the various alternatives of a design and the choice of the optiml alternative as regards the reliability in operation. A thorough analysis of the problems concerned with reliability cannot neglect the economic side, to which they are closely related. The reliability in operation of an object is a determining factor for ascertaining the profitableness of the appropriate investment. In accordance with the mean duration of a breakdown in a given time interval (i.e., the time necessary to recover the investment) and the mean number of breakdowns in the respective time interval, the costs due to breakdowns in operation of the corresponding object can be computed. Taking into account also these costs, an economic comparison of the various alternatives of a design is possible, in agreement with the criterion of the minimal general expenditure.

However, such a criterion is improper for comparing the economic efficiency of investments. On the other hand, a mathematical model enabling the estimation of income in agreement with the running of an object with a random time operation should underlie the optimization criterion for the choice among the various alternatives of a design, according to the economic efficiency of the investments required to this end. For completing the investigation method required by a reliability survey, we shall use the so-called Markov type decision processes.

Whe shall associate an income matrix $\| r_{ij} \|$ defining the income associated to the transition from state i to state j to the transitions of a Markov process. The income is expressed in monetary units or in other physical magnitudes (production units, etc). In this way, the time evolution of the process engenders a sequence of incomes, which are random variables defining a stochastic process associated to the Markov process whose laws of probability are induced by those of the Markov process. The mean values of these random variables are time functions associated to the states in which the process is evolving.

We assume that the matrix of the transition probabilities may be chosen out of a series of such matrices, since there are several alternatives for each state of the process. The various strategies in choosing the alternatives for each state should be estimated by turning to account the process evolution and the optimal strategy will correspond to a maximal income.

Let us assume that the maintenance of the system in state i during a time interval yields an income of r_{ii} lei per time unit, and that the transition of the system from state i to state j yields an income of r_{ij} lei ($r_{ii} r \neq_{ij}$). We have to know the reliability of the system, that is the mean income cumulated at moment t, resulting from the evolution of the process under the given initial conditions.

By denoting by $v_i(t)$ the cumulated mean income associated to the system evolution at moment t, provided that it may be in state i, associated to a given strategy, we obtain the relation

$$v_i(t + \Delta t) = \left[1 - \sum_{j \neq i} a_{ij}\, \Delta t\right][r_{ii}\, \Delta t + v_i(t)] + \\ + \sum_{j \neq i} a_{ij}\, \Delta t\, [r_{ij} + v_j(t)] \qquad (1)$$

with the following significance:

In the elementary interval $(t, t + \Delta t)$ the system may remain in state i, with a probability

$$\left[1 - \sum_{j \neq i} a_{ij}\, \Delta t\right]$$

and in this case, the corresponding income is expressed by

$$[r_{ii}\, \Delta t + v_i(t)];$$

or it it may change into a state whatever $j \neq i$, with the probability $a_{ij}\, \Delta t$, and the corresponding income is expressed by $[r_{ij} + v_j(t)]$.

Taking into account the relation

$$a_{ii} = -\sum_{j \neq i} a_{ij}$$

the equation (1) is written

$$v_i(t + \Delta t) - v_i(t) = \left[r_{ii} + \sum_{j=i} a_{ij} r_{ij} + \sum_{j=1}^{N} a_{ij} v_j(t)\right] \Delta t + a_{ii} r_{ii} (\Delta t)^2. \qquad (2)$$

Assuming the derivability of function $v_i(t)$, the equation with finite differences (2) leads to the differential equation

$$v_i'(t) = r_{ii} + \sum_{j \neq i} a_{ij} r_{ij} + \sum_{j=1}^{N} a_{ij} v_j(t) \tag{3}$$

Writing

$$q_i = r_{ii} + \sum_{j \neq i} a_{ij} r_{ij}$$

we conclude that the mean cumulated incomes $v_i(t)$ satisfy the matrix differential equation

$$v'(t) = q + \mathbf{A}v(t), \quad \text{where } \mathbf{A} = \| a_{ij} \| \tag{4}$$

which defines in a single way the vector $v(t)$ in case the vector $v(t)$ is known.

By using the Carson-Laplace transform, the differential equation (4) turns into an algebraic equation:

$$(sI - A)\, \mathcal{L}v(t) = q + sv(0) \tag{5}$$

from which

$$\mathcal{L}v(t) = (sI - \mathbf{A})^{-1}\, [q + sv(0)]$$

is inferred, or

$$\alpha v(t) = \left[\frac{1}{s}\mathbf{P} + \mathbf{R}(s)\right] q + [\mathbf{P} + s\mathbf{R}(s)]\, v(o). \tag{6}$$

where:

P is a matrix with constant elements, having identical lines;

R(s) is a matrix having as elements rational functions of argument s.

Assuming $v(0) = 0$

we obtain

$$\mathcal{L}v(t) = \frac{1}{s}\mathbf{P}q + \mathbf{R}(s)\, q$$

On applying l'Hopitâl's rule and the $\mathcal{L}$ operator's properties

$$\lim_{t \to \infty} \frac{v(t)}{t} = \lim_{t \to \infty} v'(t) = \lim_{s \to 0} \mathcal{L}v'(t) = \mathbf{P}q \tag{7}$$

is inferred.

From relation (7), it follows that, for high values of t, $v(t)$ assumes the form

$$v(t) = t\mathbf{P}q + \mathbf{R}(0)\, q \tag{8}$$

Writing

$$g = \mathbf{P}q, \; v = \mathbf{R}(0)\, q$$

equation (8) is written under the form

$$v(t) = gt + v \tag{9}$$

We remark that vector g has identical components

$$g = \begin{bmatrix} g \\ g \\ \vdots \\ g \end{bmatrix}$$

(g has the significance of a mean income in the time unit) and, consequently, equation (9) is explained as follows:

$$v_i(t) = gt + v_i \qquad (i = 1, 2, 3, ..., N)$$

Thus, the estimation of any strategy in time intervals of long standing can be carried out in the aforesaid vectors g and v. In order to find out these vectors, (9) is substituted in equation (4).

There results

$$g = q + A(tg + v)$$

and

$$g = q_i + \sum_{j=1}^{N} a_{ij} v_j \qquad (i = 1, 2, ..., N) \tag{10}$$

is inferred.

Writing

P_i = limit probability of state i of the process in equation (10), we conclude that

$$g = \sum_{i=1}^{N} q_i P_i,$$

since

$$\sum_{i=1}^{N} a_{ij} P_i = 0 \qquad (j = 1, 2, ..., N)$$

Referring again to system (10), we remark that it contains N equations and $N + 1$ unknowns (g: $v_1, v_2, ..., v_N$).

If $(\bar{g}; \bar{v}_1, \bar{v}_2, ..., \bar{v}_N)$ is a solution to the system, then $(\bar{g}; \bar{v}_1 + \alpha, \bar{v}_2 + \alpha, ..., \bar{v}_N + \alpha)$ is, irrespective of α, also a solution to the system. Consequently, only the relative values of the variables $(v_1, ..., v_N)$, but not their absolute values, are significant. Thus, an arbitrary value may be assigned to a variable v_i and the system (10) defines in a single way the other variables.

Considering $v_N = 0$, the system (10) defines both g and v_i, for a given strategy. In this, any strategy is confronted with its economic efficiency.

Within a range K of strategies, strategy k^* will be regarded as optimal, if:

$$g^{k^*} \geqslant g^k, \quad k \in K$$

The optimal strategy can be ascertained with the aid of an optimization iterative algorithm devised by R. Howard, which will help us to successively obtain ever better strategies from an iteration to another, in the sense:

$$g^l(\nu + 1) \geqslant g^k(\nu)$$

where ν is the order of iteration.

In the case of a strategy we can have sub-strategies to each state i, corresponding to the chosen alternative. The strategy corresponding to the iteration $(\nu + 1)$ can be found out on the basis of values $v_j(\nu)$ corresponding to the previous iteration. The sub-strategies associated to the states of the process are successively ascertained. Thus, the sub-strategy of state i of the process is found out in agreement with relation

$$g(\nu + 1) = \max_{k \in K} \left\{ q_i + \sum_{j=1}^{N} a_{ij}^k V_j(\nu) \right\}. \tag{11}$$

By examining all the states of the process, the strategy corresponding to iteration $(\nu + 1)$ results. This strategy is subsequently estimated as previously shown and the cycle is resumed. The optimization iterative process lasts as long as it leads to strategies to which successively growing incomes g correspond. If the rate does not allow the growth of income g any longer, the optimal strategy has been found out.

The algorithm described and expressed by equations (11) effectively carries out the transition from a strategy to another with a higher mean income g.

To this end, we shall consider two strategies A and B, so that strategy B may derived from strategy A through the algorithm pointed to. Consequently, the relations

$$q_i^B + \sum_{j=1}^{N} a_{ij}^B v_j^A \geqslant q_i^A + \sum_{j=1}^{N} a_{ij}^A v_j^A \qquad i = 1, 2, ..., N \tag{12}$$

subsist.

Writing

$$\gamma^i = q_i^B + \sum_{j=1}^{N} a_{ij}^B v_j^A - q_i^A - \sum_{j=1}^{N} a_{ij}^A v_j^A$$

the relations (12) are written:

$$\gamma_i \geqslant 0 \qquad i = 1, 2, ..., N$$

Taking into account (10), there follows

$$g^B = q_i^B + \sum_{j=1}^{N} a_{ij}^B v_j^B, \qquad g^A = q_i^A \sum_{j=1}^{N} a_{ij}^A v_j^A$$

and

$$g^B - g^A = \gamma_i + \sum_{j=1}^{N} a_{ij}^B (v_j^B - v_j^A), \qquad i = 1, 2, ..., N \tag{13}$$

is inferred.

Writing

$$g^B - g^A = g^\Delta, \qquad v_j^B - v_j^A = v_j^\Delta$$

equations (12) are written under the form

$$g^\Delta = \gamma_i + \sum_{j=1}^{N} a_{ij}^B v_j^A \qquad i = 1, 2, ..., N \tag{14}$$

From equations (14), we can infer, as before,

$$g^\Delta = \sum_{i=1}^{N} \gamma_i P_i^B$$

where P_i^B is the limit probability of state i associated to strategy B.

Since

$$\gamma_i \geqslant 0, \qquad P_i^B \geqslant 0,$$

there results

$$g^\Delta \geqslant 0$$

In particular

$$g^B > g^A$$

if

$$\gamma_i < 0, \qquad i = 1, 2, ..., N$$

If there exists an optimal strategy in K, then it will certainly be found out with the aid of the algorithm described. In order to prove this, we assume, on the contrary, that the previous algorithm leads to a strategy A, that cannot be improved, although there is another strategy B, in K, so that

$$g^B > g^A$$

In this case, γ_i satisfy condition

$$\gamma_i \leqslant 0 \qquad i = 1, 2, ..., N$$

which, together with condition

$$P_i^B \geqslant 0 \qquad i = 1, 2, ..., N$$

leads to the absurd condition

$$g^B \leqslant g^A$$

This signifies that the algorithm described cannot but lead to the optimal strategy.

The iterative cycle of the algorithm is illustrated by the diagram in Fig. 1.

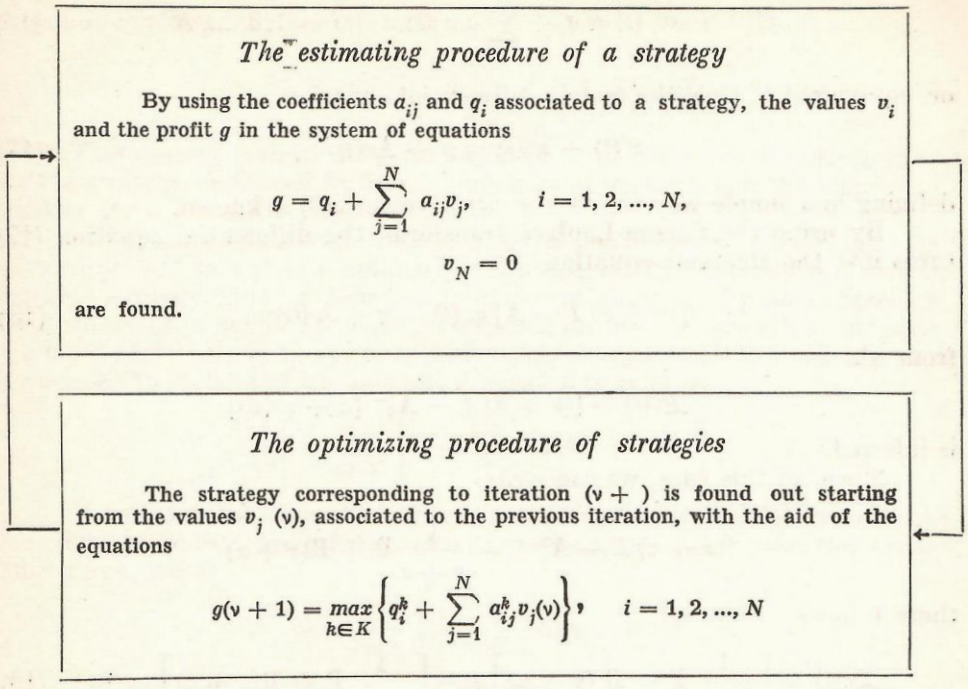


Fig. 1

We see that, on choosing initially $v_i = 0 (i = 1,2, ..., N)$, the first strategy provided by the algorithm is the one leading to q_i maximum in each state i of the process $(i = 1, 2, ..., N)$.

In the model described, the interest corresponding to the investment is not taken into account when establishing the mean incomes $v_i(t)$ (fund updating). In order to consider this interest too, we shall make use of a value $\alpha < 0$ defined as follows:

$l - \alpha \Delta t$ monetary units, after an elementary time interval Δt, are worth l units.

(This definition corresponds to the case in which the updating is continuous).

Assuming the reference to be effected at the end of the time interval Δt the equation (1) associated to this process is written:

$$v_i(t + \Delta t) = (1 - \alpha \Delta t) \left\{ \left[1 - \sum_{j \neq i} a_{ij} \Delta t \right] [r_{ii} \Delta t + v_i(t)] + \right.$$

$$\left. + \sum_{j \neq 1} a_{ij} \Delta t [r_{ij} + v_j(t)] \right\} \qquad (15)$$

Following the reasoning described before, we conclude that functions $v_i(t)$ satisfy the system of differential equations.

$$v_i'(t) + \alpha v_i(t) = q_i + \sum_{j=1}^{N} a_{ij} v_j(t), \qquad i = 1, 2, \ldots, N \tag{16}$$

or, equivalent to this, the matrix differential equation

$$v'(t) + \alpha v(t) = q + \mathbf{A}v(t) \tag{17}$$

defining in a simple way the vector $v(t)$ if vector $v(0)$ is known.

By using the Carson-Laplace transform, the differential equation (17) turns into the algebraic equation

$$[(s + \alpha) I - \mathbf{A}]\, \alpha v(t) = q + sv(0) \tag{18}$$

from which

$$\mathcal{L}v(t) = [(s + \alpha) I - \mathbf{A}]^{-1}[q + sv(0)]$$

is inferred.

Since, in this case, we can write

$$[(s + \alpha) I - \mathbf{A}]^{-1} = \frac{1}{s + \alpha} \mathbf{P} + \mathbf{R}(s + \alpha)$$

there follows

$$\mathcal{L}v(t) = \left[\frac{1}{s + \alpha} \mathbf{P} + \mathbf{R}(s + \alpha)\right] q + \left[\frac{1}{s + \alpha} \mathbf{P} + \mathbf{R}(s + \alpha)\right] sv(0) \tag{19}$$

Taking into account the properties of the operator α, from equation (19) we infer

$$\lim_{t \to \infty} v(t) = \left[\frac{1}{\alpha} \mathbf{P} + \mathbf{R}(\alpha)\right] q$$

Thus, the behaviour of the process for high values of t (asymptotic behaviour of the process) is expressed by the vector

$$v = \left[\frac{1}{\alpha} \mathbf{P} + \mathbf{R}(\alpha)\right] q. \tag{20}$$

The equation (20) is equivalent, according to relation (6), to the equation

$$v = (\alpha I - \mathbf{A})^{-1} q \tag{21}$$

The components of vector v stand for updated incomes, corresponding to certain moments quite distant from the initial moment, which are associated to states i of the process.

Equation (21) can be written under the form

$$\alpha I v = \mathbf{A} v + q$$

which is explained by the system

$$\alpha v_i = q_i + \sum_{j=1}^{N} a_{ij} v_j \quad i = 1, 2, ..., N \tag{22}$$

This system defines the limit incomes v_i of the process, corresponding to the strategy expressed by the components of vector q and the elements of matrix A.

Thus, the estimation of any strategy depends on the parameter α and is materialized in vector v defined by the system (22). We can find out the optimal strategy starting from the iterative optimization algorithm described.

In order to achieve a maximal speed of income $v_i(t)$ growth at moment t we must previously achieve an optimization, as regards all the sub-strategies associated to states i of the process, according to relation

$$v_i'(t) = \max_{k \in K} \left\{ q_i^k + \sum_{j=1}^{N} a_{ij}^k v_j(t) - \alpha v_i(t) \right\}.$$

If we have to know the behaviour of the process for high values of t, the limit incomes v_i are substitutes for incomes $v_i(t)$. In this case, the optimization relation is

$$v_i' = \max_{k \in K} \left\{ q_i^k + \sum_{j=1}^{N} a_{ij}^k v_j - \alpha v_i \right\}.$$

The strategy corresponding to iteration $(\nu + 1)$ is ascertained in agreement with limit incomes $v_i(\nu)$, associated to the previous iteration, according to relations

$$v_i'(\nu + 1) = \left\{ \max q_i^k + \sum_{j=1}^{N} a_{ij}^k v_j(\nu) - \alpha v_i(\nu) \right\} \quad i = 1, 2, ..., N$$

Since the relations above $v_i(\nu)$ are independent of k, it follows that these relations are equivalent to

$$v_i'(\nu + 1) = \max_{k \in K} \left\{ q_i^k + \sum_{j=1}^{N} a_{ij}^k v_j(\nu) \right\}, \quad i = 1, 2, ..., N \tag{23}$$

The iterative optimization process will last as long as relation

$$v'(\nu + 1) > v'(\nu)$$

subsists between two successive iterations, which leads to improved strategies.

If

$$v'(\nu + 1) = v'(\nu)$$

it follows that the strategy corresponding to iteration (ν) is the optimal strategy.

This algorithm actually carries out the changing from a strategy to another at a speed of incomes growth v'.

To this end, two strategies A and B are considered, so that strategy B should be derived from strategy A through the algorithm described.

Consequently, the relations

$$\gamma_i \geqslant 0, \qquad i = 1, 2, ..., N$$

subsist, where γ_i have the aforesaid expressions.

Taking into account (22), it follows

$$\alpha v_i^A = q_i^A + \sum_{j=1}^{N} a_{ij}^A v_j^A, \quad \alpha v_i^B = q_i^B + \sum_{j=1}^{N} a_{ij}^B v_j^B,$$
$$i = 1, 2, ..., N$$

and

$$\alpha v_i^\Delta = \gamma_i + \sum_{j=1}^{N} a_{ij}^B v_j^\Delta, \qquad i = 1, 2, ..., N \tag{24}$$

is inferred, where

$$v_i^\Delta = v_i^B - v_i^A$$

The equations (24) are equivalent, as proved through the equation

$$v^\Delta = (\alpha I - \mathbf{A})^{-1}\gamma \tag{25}$$

where

$$\gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma N \end{bmatrix}$$

Since the elements of matrix $(\alpha I - \mathbf{A})^{-1}$ and γ are non-negative, there follows

$$v^\Delta \geqslant 0$$

In particular, if

$$\gamma_i \geqslant 0, \qquad i = 1, 2, ..., N$$

then, for at least one index i, the relation

$$v_i^B > v_i^A$$

subsists.

Of course, if there exists an optimal strategy in K, then it will certainly be ascertained with the aid of the previous algorithm. In order to prove the truthfulness of the assertion we assume the contrary, that is, that the previous

algorithm leads to a streategy A that cannot be improved although there exists in K another strategy B, so that for one index i at least the relation

$$v_i^B > v_i^A$$

In this case, γ_i satisfy condition

$$\gamma_i \leqslant 0, \qquad i = 1, 2, ..., N$$

that leads us to the absurd conclusion

$$v_i^B \leqslant v_i^A \qquad i = 1, 2, ..., N$$

It follows that the algorithm described leads to an optimal strategy.

The iterative cycle of the algorithm can be plotted on a diagram similar to the one drawn in Fig.1.

We note that, by choosing $v = 0 (i = 1, 2, ..., N)$ originally, the first strategy provided by the algorithm is the one leading to q_i maximum in each state i of the process $(i = 1, 2, ..., N)$.

We shall illustrate the way of applying the systems dwelt upon with the aid of five common examples.

Case I. A single element

$$\mathbf{A} = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} r_0 & 0 \\ -r_1' & -r_1 \end{bmatrix};$$

$$q_0 = r_0, \quad q_1 = -(r_1 + \mu r_1'),$$

$$\alpha_0'(t) = q_0 - \lambda v_0(t) + \lambda v_1(t); \qquad v_0(0) = 0;$$

$$v_1'(t) = q_1 + \mu v_0(t) - \mu v_1(t); \qquad v_1(0) = 0;$$

$$\mathcal{L}v_0(t) = \frac{q_0 s + \lambda q_1 \mu + q_0}{s(s + \lambda + \mu)s} \times \mathcal{L}v_1(t) = \frac{q_1 s + \lambda q_1 + \mu q_0}{s(s + \lambda + \mu)};$$

$$v_0(t) = \frac{\lambda}{\lambda + \mu}(q_0 - q_1)[1 - e^{-(\lambda+\mu)t}] + \left[\frac{\mu}{\lambda + \mu} q_0 + \frac{\lambda}{\lambda + \mu} q_1\right] t;$$

$$v_1(t) = -\frac{\mu}{\lambda + \mu}(q_0 - q_1)[1 - e^{-(\lambda+\mu)t}] + \left[\frac{\mu}{\lambda + \mu} q_0 + \frac{\lambda}{\lambda + \mu} q_1\right] t;$$

$$\lim_{t\to\infty} \frac{v_0(t)}{t} \lim_{t\to\infty} \frac{v_1(t)}{t} = \frac{\mu}{\lambda + \mu} q_0 + \frac{\lambda}{\lambda + \mu} q_1$$

$$g = \frac{\mu}{\lambda + \mu} q_0 + \frac{\lambda}{\lambda + \mu} q_1 = q_0 P_0 + q_1 P_1$$

Case II. A system of series connected elements.

$$\mathbf{A} = \begin{bmatrix} -\lambda_0 & \lambda_1 & \lambda_2 & \dots & \lambda_n \\ \mu_1 & -\mu_1 & 0 & \dots & 0 \\ \mu_2 & 0 & -\mu_2 & \dots & 0 \\ \vdots & & & & \vdots \\ \mu_n & 0 & 0 & \dots & -\mu_n \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} -r_0 & 0 & & 0 \\ -r_1' & -r_1 & 0 \dots & 0 \\ -r_2' & 0 & -r_2 & \\ \vdots & & & \\ -r_n' & 0 \dots & 0 & -r_n \end{bmatrix}$$

$$q_0 = r_0, \quad q_1 = -(r_1 + \mu_1 r_1'), \quad q_2 = -(r_2 + \mu_2 r_2'),$$

$$q_3 = -(r_3 + \mu_3 r_3'), \dots, q_n = -(r_n + \mu_n r_n');$$

$$P_0 = \left[1 + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}\right]^{-1} \qquad P_i = \frac{\lambda_i}{\mu_i} P_0 \qquad i = 1, 2, \dots, n$$

$$g = \frac{r_0 - \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}(r_i + \mu_i r_i')}{1 + \sum_{i=i}^{n} \frac{\lambda_i}{\mu_i}}$$

Case III. A system of identical elements operating in parallel.

$$r_{ii} = (n - i)\, r - ir', \quad r_{i,\,i+1} = 0, \quad r_{i,\,i-1} = -r'';$$

$$q_i = [(n - i)\, r - ir'] - i\mu r'', \quad i = 0, 1, 2, \dots, n;$$

$$g = \sum_{i=0}^{n} q_i P_i;$$

$$P_i = C_n^i \left(\frac{\mu}{\lambda + \mu}\right)^{n-i} \left(\frac{\lambda}{\lambda + \mu}\right)^{i}, \quad i = 0, 1, 2, \dots, n.$$

Case IV. The choice between two types of equipment. The technical and economic characteristics related to the operation and repair of such equipment is illustrated by the following table.

State	Strategy	Matrix **A**	Vector q ($q_0 > 0, q_1 < 0$)	Unit income g
0	1	$\begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$	$\begin{bmatrix} q_0 \\ q_1 \end{bmatrix}$	$q_0\left(\frac{\mu}{\lambda+\mu}\right) + q_1\left(\frac{\lambda}{\lambda+\mu}\right)$
	2	$\begin{bmatrix} -\lambda & \lambda \\ l\mu & -l\mu \end{bmatrix}$	$\begin{bmatrix} q_0 \\ l' q_1 \end{bmatrix}$	$q_0\left(\frac{l\mu}{\lambda+l\mu}\right) + l' q_1\left(\frac{\lambda}{\lambda+l\mu}\right)$
1	3	$\begin{bmatrix} -k\lambda & k\lambda \\ \mu & -\mu \end{bmatrix}$	$\begin{bmatrix} k' q_0 \\ q_1 \end{bmatrix}$	$k' q_0\left(\frac{\mu}{k\lambda+\mu}\right) + q_1\left(\frac{k\lambda}{k\lambda+\mu}\right)$
	4	$\begin{bmatrix} -k\lambda & k\lambda \\ l\mu & -l\mu \end{bmatrix}$	$\begin{bmatrix} k' q_0 \\ l' q_1 \end{bmatrix}$	$k' q_0\left(\frac{l\mu}{k\lambda+l\mu}\right) + l' q_1\left(\frac{k\lambda}{k\lambda+l\mu}\right)$

The choice of the optimal strategy depends on the size ordering of coefficients k, k', l, l'. For instance, if the relations

$$k < k' < l < l' \leqslant 1$$

subsist, it follows that the optimal strategy is the strategy 4.

Case V. Let us consider the case of a system consisting of two identical elements, a basic one and a spare one. Assuming that the operation of the system is performed without penalties as long as at least one of the system elements is operating, the question arises whether the active reserve or the passive reserve is preferable?

Active reserve

$$\mathbf{A} = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -(\lambda+\mu) & \lambda \\ 0 & 2\mu & -2\mu \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} r & 0 & 0 \\ -r'' & r-r' & 0 \\ 0 & -r'' & -2r' \end{bmatrix}$$

$$q_0 = r, \quad q_1 = (r-r') - \mu r'', \quad q_2 = -2r' - 2\mu r''$$

$$P_0 = \frac{\mu^2}{(\lambda+\mu)^2}, \quad P_1 = \frac{2\lambda\mu}{(\lambda+\mu)^2}, \quad P_2 = \frac{\lambda^2}{(\lambda+\mu)^2}$$

$$g = \frac{q_0\mu^2 + 2q_1\lambda\mu + q_2\lambda^2}{(\lambda+\mu)^2}$$

Passive reserve

$$\mathbf{A} = \begin{bmatrix} -\lambda & \lambda & 0 \\ \mu & -(\lambda+\mu) & \lambda \\ 0 & 2\mu & -2\mu \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} r & 0 & 0 \\ -r'' & r-r' & 0 \\ 0 & -r'' & -2r' \end{bmatrix}$$

$$q_0 = r, \quad q_1 = (r-r') - \mu r'', \quad q_2 = -2r' - 2\mu r''$$

$$P_0 = \frac{2\mu^2}{(\lambda+\mu)^2+\mu^2}, \quad P_1 = \frac{2\lambda\mu}{(\lambda+\mu)^2+\mu^2}, \quad P_2 = \frac{\lambda^2}{(\lambda+\mu)^2+\mu^2}$$

$$g^* = \frac{2q_0\mu^2 + 2q_1\lambda\mu + q_2\lambda^2}{(\lambda+\mu)^2+\mu^2}$$

If $\frac{g}{g^*} < 1$, it follows that, from the standpoint of the operation proper, the active reserve is preferable, whereas from the standpoint of the reliability proper, the passive reserve is advantageous, since the latter leads to a full length of breakdown less than the active reserve.

Here are a few conclusions we can draw from the topic under discussion:

1. The investigation method surveyed enables finding out the profitableness of investments associated to objectives having a random operation in time.

2. It is possible to use a general optimization criterion as to the choice of the alternatives of a design, that is the maximum efficiency criterion of the corresponding investment.

3. The computation methodology described is similar to the one used in the investigation of problems related to reliability. The computation is liable to be fully carried out on digital or analog computers.

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CRITERES D'OPTIMISATION DU POINT DE VUE DE LA SURETE DU FONCTIONNEMENT

L'article présente certains critères d'optimisation qui conjuguent la sureté du fonctionnement à des considérations économiques. Du point de vue des mathématiques, ces critères conjuguent le thème des processus « stochastiques » du type Markov à la programmation dynamique. Ces critères sont d'une large utilité. Ils permettent de fonder la solution des problèmes tels que l'efficacité économique de la réserve, ou le choix de l'équipement.

Certaines applications simples dans le domaine de l'énergétique sont également présentées.

КРИТЕРИИ ОПТИМИЗАЦИИ НАДЕЖНОСТИ В ФУНКЦИОНИРОВАНИИ

В статье приводится целая серия критериев оптимизации, сочетающих надежность в функционировании с экономическими расчетами. С математической точки зрения они сочетают тему стохастических процессов типа Маркова с динамическим программированием.

Эти критерии широко применяются. Ими можно обосновать решение проблемы экономической эффективности резерва, выбора оборудования и т.п.

Представляются некоторые простые примеры применения в энергетике.