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System Simulation, Models and Categories

From the simulation of models to a model of simulation

Thanks to the relatively recent developments in the system theory on a general background (including the biological system too), the possibility of applying certain general results to some particular instances in economy, which are a frequent objet of simulation techniques, has sprung up. To make precise the background we have in view, further details on the senses assig. ed to the working concepts will be given.

A system is defined as a collection of different elements in interaction, including men and machines, integrated in achieving a desired purpose by the manipulation and control of material, information, energy and human activity. Within the system there exist relations (in the broadest sense), and the mathematical representation of these relations is called model.

A system can be isolated and dealt with independently, and its outward connections become inputs and outputs. Part of a system may be isolated as a sub-system if its connections with the system can be represented as inputs and outputs. Consequently, the sub-system is also a system.

A model is defined as a technique of the system analysis resulting in the mathematical idealization of the system or of a part of it. The extent to which the model is full and true depends on the problems requiring a solution, on the stage of knowledge of the system and on its outward connections.

The concept of simulation is usually assumed to have two meanings:

(1) the representation of the relations in the system by a physical analog;

(2) the use of the physical and computation analogies as a means for exploring the system behaviour.

In the following we shall try to represent the system simulation by means of a model suitable for these two meanings of simulation.

Since certain elementary notions of the category theory are further used, the notions are given at the same time with their concise explanation, with the object of making easier the reading of the article.

We denote by Cs and define as a system category the concept consisting of:

a) a class of objects $ObCs$, defined as the system class;

b) for any ordered couple of objects (I, E) a set $Cs(I, E)$ is given, whose elements are denoted by $f, g, \ldots$, defined as morphisms with input I and output E. For

$$f \in Cs(I, E)$$

we shall write

$$f : I \to E$$

and we shall read: f is a morphism of input (source) I and output (address) E;

c) for two distinct couples whatever (I, E) and (I', E') we assume that $Cs(I, E) \cap Cs(I', E') = \emptyset$

d) for any ordered triplet of objects I, E, X of $ObCs$, is defined an application of sets

$$\theta(I, E, X) : Cs(I, E) \times Cs(E, X) \to C(I, X)$$

defined as the composition of morphisms, which can be written also

$$\theta(I, E, X)(f, g) = g^{\circ}f$$

for any

$$g \in Cs(E, X) \text{ and } f \in Cs(I, E)$$

A concrete example of composition of morphisms is the following: the output of a system is connected as an input to another system. The two systems connected in a cascade may be considered as a sole system, with an input, an output and the morphisms resulting from the composition:

e) The composition of morphisms is associative:

$$h^{\circ}(f^{\circ}f) = (h^{\circ}g)^{\circ}f$$

f) For any $I \in ObCs$ the set $Cs(I, I)$ contains at least one element denoted by 1_I and defined as the identical morphism, of I.

With these specifications, we shall call a system the class consisting of two objects I, E and of the set of morphisms $Cs(I, E)$.

A particular case of system is the segment system, more exactly the class made up of two objects I, E and three morphisms 1_I 1_E and $f : I \to E$.

The couple system is made up of two objects I, E and four morphisms $1_I, 1_E$ and $f, g : I \rightrightarrows E$.

It is of interest the class whose objects are the ordered triplets (I, f, E). We denote these objects by S and the respective class by MCs, called the class of C_s morphisms.

If (I, f, E), $(I', f', E') \in Ob\,MCs$, then $MCs\,[(I, f, E), (I', f', E')]$ is the set of all the couples of morphisms (α, β), $\alpha : I \to I'$, $\beta : E \to E'$, so that $f' \circ \alpha = \beta \circ f$, for which we have the commutative diagram shown in Fig. 1.

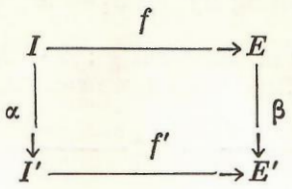

Fig. 1

SIMULATION ON THE SYSTEM

Simulation is, essentially, an experimental study. In numerous instances, the experimental study of the system behaviour, directly on the system, is not admitted for various reasons. Let us assume this to be the case of system S in the previous example. Let us assume, yet, that system S' in the same example allows a direct experimental study. In this case, simulation on the system can be conceived as an experimental study of morphism f, by the instrumentality of the commutative diagram in Fig. 1, that is the study of the composition of morphisms in the class of Cs morphisms.

If β is a reversible isomorphism, (that is simultaneously sectionable and retractible), then we have

$$f = \beta^{-1} \circ f' \circ \alpha$$

If α is reversible, we have

$$f' = \beta \circ f \circ \alpha^{-1}$$

If α and β are simultaneously isomorphisms, then the association $f \rightsquigarrow \beta \circ f \circ \alpha^{-1}$ is a canonical bijection from set $Cs\,(I, E)$ to set $Cs\,(I', E')$.

The commutative diagram in Fig. 1 may be considered a model of simulation on the system.

SIMULATION ON THE MODEL

We denote by $[S]$ the class of isomorphic systems, and by S_1 a representative of this class. The class of all the systems of Cs isomorphic with S is defined as the type of S.

The relationship between two isomorphic systems is an equivalence relationship, for if $f : S \to S'$ is an isomorphism and $f' : S' \to S''$ is an isomorphism, then also $f' \circ f : S' \to S''$ is an isomorphism.

For isomorphic segment-systems, the diagram in Fig. 1 is to be interpreted in the following way: S and S' are, in their turn, categories; hence, to assert that S and S' are isomorphic means to admit a functorial isomor-

phism $F : S \to S'$, which comes to the fact that F is a bijective and fully true functor. This leads further to the conclusion that α and β are isomorphisms.

On the basis of this reasoning, we can state that the model M of a representative S_1 of the class of isomorphic systems $[S]$ is also a model of the type of S. Indeed, on denoting by $\mu: S_1 \to M$, for $g : S_1 \to S_2$ we have $\mu \circ g^{-1}: S_2 \to M$

In particular, if M is of the form $g : D \to R$, where D stands for the initial data and R for the final results, we can draw up the diagram:

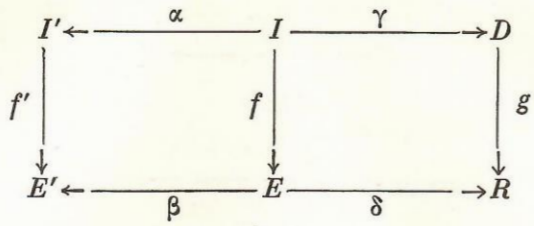

Fig. 2

from which there follows

$$\gamma \circ \alpha^{-1} : I' \to D \text{ and } \delta \circ \beta^{-1}: E' \to R$$

This means that the object of the simulation on the model is the experimental study of the morphisms of the type of a class of systems.

The diagram in Fig. 2 may be considered also a model of system simulation, in a more general case, when S and S' are distinct types and M is a model of type S.

To summarize, we may have, for instance:

— if δ is reversible, then the simulation on the model consists in the study of morphism $f = \delta^{-1} \circ g \circ \gamma$

— if β is reversible, then the simulation on the system consists in the study of morphism $f = \beta^{-1} \circ f' \circ \alpha$, avoiding in both instances the experiment on the system to be simulated.

The diagram in Fig. 2 can be reduced to the coangle

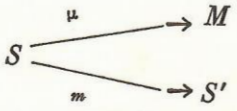

Fig. 3

which represents, in its turn, a class.

SIMULATION CONVERGENCE

From all this there follow the possibility of drawing up a class of models with $M \in Ob\ C_M$, as well as the existence of a functor $F : Cs \to C_M$.

By choosing a representative S_A for each type $[S]$, a framework Cs' of class Cs is achieved.

As a consequence, there follows also a fully true functor $F' : Cs' \to C_M$ by virtue of the fact that for any S_A, $S_B \in ObCs'$ the application $F'(S_A, S_B)$ is a bijection. The aspect under which classes Cs and C_M may be studied, from the standpoint of simulation, is the following: morphism $\mu : S \to M$ is studied by starting from an intial model M_1, knowing that $\mu_1 : S \to M$ is retractible. The succession M_1, M_2, M_3, ... is drawn up and the simulation is assumed to be a convergent process if the limit towards which this succession is tending is an isomorphism $\mu_i : S \to M_i$

In a more general manner, the simulation convergence can be expressed by the coangle in Fig. 3, where morphisms m and μ become isomorphisms at the limit

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LA SIMULATION DES SYSTEMES, DES MODELES ET DES CATEGORIES

Le système en tant qu'objet de simulation est analysé au point de vue de la théorie de la structure. Dans cet article, on signale le rôle des axiomes dans la définition du système.

Le modèle est défini comme une représentation des structures sur les ensembles échafaudés sur une échelle de base.

Le transfert au simulateur implique l'existence d'un isomorphise modèle — simulateur.

Le concept de convergence, aboutissant en feedback (original — modèle — simulateur — original) est défini sur un fondement plus général.

La méthode de traitement se fonde sur les éléments de la théorie des catégories.

СИМУЛИРОВАНИЕ СИСТЕМ, МОДЕЛИ И КАТЕГОРИИ

Система, как предмет симулирования анализируется с точки зрения теории структуры В этой статье отмечается роль аксиом в определении системы.

Модель определяется как структурное представление по множествам, развернутым по основной лестнице.

Перевод к симулятору требует наличия изоморфизма модель — симулятор.

Концепция конвергенции, исходящая из feedback (оригинал — модель — симулятор — оригинал) определяется на более общем фоне.

Метод исследования базируется на элементах теории категорий.