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A Mathematical Model for the Ascertainment of an Optimal Foreign Currency Policy

A scientific foreign currency policy can be achieved by using such methods as those made available by modern mathematics.

The routine (short term) management of the foreign currency fund of a depository (such as the national bank, or others) sets forth a major economic problem: ascertaining and maintaining such a ratio among the various freely convertible currencies as to provide the maximal value of the foreign currency fund, considered as being associated to a low predetermined risk. This risk is engendered by the daily fluctuations exhibited by the rates of exchange of the various currencies on foreign markets; it is assumed that the necessary data in order to determine the common random character and distribution law are well known by the bodies entitled to make decisions concerning the foreign currency fund structure. In actual practice, classical methods are used for solving this problem, namely the qualitative analysis of current information and management skill; it is clear that such solutions are not — nor could they be but incidentally — the optimal ones.

A scientific currency policy can be achieved by using such methods as those made available by modern mathematics.

The following notations are used in order to make possible the statement of the foreign currency fund management problem in accurate terms:

n = the number of freely convertible foreign currencies taken into account;

m = the number of markets considered;

ξ_{ij} = the profit achieved on market i, $1 \leqslant i \leqslant m$, assuming that 1 US$ is deposited in the form of foreign currency i, $1 \leqslant j \leqslant n$ and that a time unit is maintained. These values are random variables whose common distribution law can be obtained by using available

data (the daily rate of exchange of foreign currencies on various markets and the rates of interest offered by various banks) *);

x_{ij} = the percentage of the given foreign currency funds, which will be deposited on market i, $1 \leqslant i \leqslant m$ in the form of foreign currency j, $1 \leqslant j \leqslant n$.

The following mathematical model can be formulated by using these notations:

$$max\ v \tag{1}$$

$$P\left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \xi_{ij} x_{ij} \geqslant v \right\} \geqslant \alpha \tag{2}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = 1 \tag{3}$$

$$x_{ij} \geqslant 0,\ 1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n \tag{4}$$

where $1 - \alpha$ is the (predetermined) risk that the value of the fund deposited at a given time be lower that its maximal value.

This model takes into account the major features required for a currency fund management policy, namely flexibility, reliability and efficiency.

Relationship (2) expresses the reliability condition, through establishing the accepted risk $1 - \alpha$ at sufficiently low values and it simultaneously contains, along with (1), the efficiency condition, namely the achievement of a maximal value for the foreign currency fund, with a very high probability P. The flexibility condition is not explicitly stated in the model; however, it is implied by the fact that the problem is limited to the *short term* foreign currency fund management case.

The mathematical model (1) — (4) can be supplemented with definite constraints, which may show up in various specific situations. It is thus possible to enjoin the condition according to which on certain banking markets minimal quantities from various foreign currencies be provided:

$$\sum_{j=1}^{n} x_{ij} \geqslant a_i, \qquad 1 \leqslant i \leqslant m, \tag{5}$$

where a_i, $1 \leqslant i \leqslant m$, are (minimal) percentages out of the whole fund on markets i, $1 \leqslant i \leqslant m$.

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*) If we denote by η_{ij} the rate of exchange of foreign currency j, $1 \leqslant j \leqslant n$, on market i, $1 \leqslant i \leqslant m$ (in US $) and by d_{ij} the rate of interest (in $) offered on market i, $1 \leqslant i \leqslant m$, for a foreign currency unit j, $1 \leqslant j \leqslant n$, kept in foreign currency j, $1 \leqslant j \leqslant n$, during a time unit, we have then

$$\xi_{ij} = \eta_{ij} + d_{ij}$$

where η_{ij} is a random variable whose achievements are to be found in foreign currency rates of exchange tables and d_{ij} is a known constant.

Another condition can be expressed as the requirement to provide minimal quantities of specific foreign currencies, useful for various current, or other, payments:

$$\sum_{i=1}^{m} x_{ij} \geqslant b_j \qquad 1 \leqslant j \leqslant n, \tag{6}$$

where b_j, $1 \leqslant j \leqslant n$, are the necessary (minimal) percentages out of the whole fund of foreign currencies j, $1 \leqslant j \leqslant n$.

As to the possible numerical solution of model (1) — (4) (possibly (5) (6) as well) it can be seen that in actual practice, the random variables ξ_{ij}, $1 \leqslant i \leqslant m$, $1 \leqslant j \leqslant n$ are normal, though generally not independent. If ξ and $\mathbf{x}$ denote the column vectors with components ξ_{ij} and $\mathbf{x}_{ij}$ (ranked in the same order), the model (1)—(4) can also be written as:

$$max\, v \tag{7}$$

$$P\{\xi' \mathbf{x} \geqslant v\} \geqslant \alpha \tag{8}$$

$$\mathbf{x}\, \mathbf{e}' = 1 \tag{9}$$

$$\mathbf{x} \geqslant 0, \tag{10}$$

where $\mathbf{e}$ is the column vector having all components equal to one.

Let μ be the vector having as constituents the average values of the components of vector ξ. We assume that the respective multidimensional distribution is normal with the covariant matrix Σ. In this case, the random variable

$$\frac{\xi'\mathbf{x} - \mu'\mathbf{x}}{(\mathbf{x}'\Sigma\mathbf{x})^{\frac{1}{2}}} \tag{11}$$

is normal, with the average equal to zero and the dispersion equal to one.

Constraint (8) can also be written:

$$P\left\{\frac{\xi'\mathbf{x} - \mu'\mathbf{x}}{(\mathbf{x}'\Sigma\mathbf{x})^{\frac{1}{2}}} \geqslant \frac{v - \mu'\mathbf{x}}{(\mathbf{x}'\Sigma\mathbf{x})^{\frac{1}{2}}}\right\} \geqslant \alpha \tag{12}$$

or

$$1 - \Phi\left[\frac{v - \mu'\mathbf{x}}{(\mathbf{x}'\Sigma\mathbf{x})^{\frac{1}{2}}}\right] \geqslant \alpha \tag{13}$$

where Φ (*) is the Laplace function.

Relationship (13) can also be written as:

$$\frac{v - \mu'\mathbf{x}}{(\mathbf{x}'\Sigma\mathbf{x})^{\frac{1}{2}}} \leqslant \Phi^{-1}(1 - \alpha) \tag{14}$$

*) The accent indicates the transposition.

By squaring relationship (14) and by taking into account that the value $\Phi^{-1}(1-\alpha)$ is negative, a quadratic constraint is obtained, having the following expression:

$$[\Phi^{-1}(1-\alpha)]^2 \mathbf{x}'\Sigma\mathbf{x} - (\mu'\mathbf{x} - v)^2 \leqslant 0. \tag{15}$$

Model (1)—(4), (possibly (5), (6) as well) becomes then a program, where the function objective is linear, and one of the constraints is quadratic, while the others are linear. A specific algorithm for the solution of this problems is given in [1]. A similar approach to an agricultural problem is given in [2].

The model described in the preceding paragraphs allows the computation of vector μ and of matrix Σ, by using the data concerning values ξ_{ij} for a previous period; the quadratic relationship (15) can be explicitly written by using these data. Subsequently, the algorithm suggested in [1] is applied and the optimal ratio $\hat{x}$ (among various foreign currencies) is determined for the maintenance of the foreign currency fund. The periodic recalculation of μ and Σ (based on recent achievements of the random variables ξ_{ij}) makes possible the updating of the optimal ratio $\hat{x}$.

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UN MODELE MATHEMATIQUE POUR LA DETERMINATION D'UNE POLITIQUE OPTIMALE DES DEVISES ETRANGERES

Dans l'article on considère le problème de la détermination de la façon optime d'administrer la quantité de devises étrangères disponible, tenant compte des fluctuations qui apparaissent pour les cours des devises étrangères, fluctuations qui ont un prononcé caractère aléatoire.

L'optimalité est prise ici au sens de maximisation de la valeur du dépôt total à un moment donné, tout en acceptant un risque (petit) pré-établi.

Le modèle mathématique correspondant est une question de programmation stochastique ayant une seule restriction probabilistique.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДЛЯ ОПРЕДЕЛЕНИЯ ОПТИМАЛЬНОЙ ВАЛЮТНОЙ ПОЛИТИКИ

В статье исследуется проблема определения оптимального метода, по которому нужно администрировать количество наличной валюты, с учетом изменений, появляющихся в курсах иностранных валют, изменений имеющих явно случайный характер.

Оптимальность понимается в смысле максимизации стоимости общих запасов в данный момент, учитывая заранее предусмотренный риск (небольшой).

Соответствующая математическая модель является задачей стохастического программирования (с одним единственным вероятностным ограничением).