J. AURIAN

The Use of Selection Methods for the Optimal Location of Enterprises

The laying out of investments in a given territory assumes extremely important decisions, owing to the long-term implications for economic development.

1. GENERAL CHARACTERIZATION OF THE PROBLEM

In choosing adequate locations, a great number of economic and extra-economic factors must be taken into account. This is, accordingly, an optimization problem depending on criteria which can be quantitatively expressed, as well as on several factors also exerting an influence, such as: the rational distribution and utilization of productive forces, geographical areas development, climatic variability, and specific conditions determined by the geographical environment, etc., whose effects cannot be quantified.

Modern thought considers the location of investments as a complex optimization problem, which has to be examined by taking into account the entire set of factors controlling, or being affected by the choice of a location; under present economic development conditions, the significance of macro-economic elements and the need of adopting improved procedures of decision making in this field is ever increasingly emphasized.

The solutions suggested for the location of an investment will necessarily provide the mutual adjustment of the partial branch and regional interests with the general interests of economy, in order to obtain maximal benefits in exchange of a least possible expenditure of materialized labour.

Given the multitude of available data and of potential locating alternatives, the use of mathematical optimization methods is imperative. In the socialist economy, such methods have a wide range of application, as their ability to cover large-size problems agrees with the concept of centralized management of investments.

The following questions need a closer examination, when settling the territorial location of investements:

— securing a rational agreement among demands and available resources, the balance of direct and indirect production factors (manpower, ground, available energy, water, etc., resources) which occur particularly in places of industrial concentration;

— securing co-operation relations among new and already existing enterprises;

— securing optimal production capacities for the new enterprises;

— the organization of complex industrial centres for which infrastructure investments and socio-cultural endowments are provided;

— the reduction of economic development discrepancies among geographical areas, the priority development of underdeveloped areas, by increasing the living standard of the population, through the improvement of living and working conditions;

— total expenses minimization (production costs, investment outlays, transportation costs, as well as socio-cultural expenditures).

Although a general and unitary methodology has not yet been devised for the establishment of the territorial location of enterprises, according to problems peculiarities, analytical methods considered as being most adequate, are usually applied. The primary aim was to solve location problems at the production branch level, for which the scheduling of optimal economic and territorial development is feasible, that is the setting up of the optimal network of enterprises in a given area. The most simple methods use the linear programming transportation algorithm which yields satisfactory results, particularly for homogeneous branc hes of products. Solving procedures which utilize dynamic programming, integer programming and combinatorial methods. have also been examined. They all assume the possibility of expressing the optimal criterion in the form of a function. This is why only part of the economic factors can be taken into account.

Mathematical programing methods are more convenient, as we are thus able to obtain quantitatively expressed results, with a high degree of accuracy; however, several important disadvantages are inherent to these methods, namely:

— they require a great number of primary data, assembled through extensive surveys and statistical data collections;

— the accuracy of results depends on the precision attainable in the computation of primary data and therefore the utilization of a highly refined mathematical tool is not always justified, as frequently the necessary premises are not available for obtaining accurate data;

— a great number of the most important factors cannot be included in the models, because their effect cannot be expressed in the form of quantitative relationships.

All these reasons have led to a still restricted application of accurate methods, while the optimal solutions obtained through the above-mentioned methods are accepted only after a many-sided analysis, also taking into account some other factors, besides those included in the models. On the other hand, the criteria playing a part in the selection of locations cannot be hierarchically arranged in accordance with a unique scale, having universal validity

for all cases encountered in actual practice. In chosing the optimal location of investments, it is frequently necessary to adopt a solution which takes into account several viewpoints marked by varying degrees of significance and which cannot be reduced to a single one.

2. CLASSIFICATION OF LOCATION PROBLEMS

Location problems encountered in practice are highly diversified. The study of their typology is useful for the selection of models and methods corresponding to existing requirements. The full knowledge of the whole set of location problem types involves their analysis from various standpoints, in order to make possible their combination in as homogeneous subsets (classes or categories) as possible. With this end in view, various classification criteria can be used, by taking into account:

— the number of enterprises for which the location is examined;
— the time interval considered in a specific case;
— *the character of expenditure involved in the function-objective;*
— the number of products to be manufactured;
— *the weight assigned to raw materials in the manufacturing process.*

The economic-mathematical models which can be used are also subject to a classification according to:

a. the number of optimization criteria, namely: models for which a function-objective could be defined and those involving several criteria which cannot be mutually compared (and which will from now on be designated as selection models);

b. the period provided for the location under examination:
— static models;
— dynamic models;

c. the number of potential locations:
— continuous models;
— discrete models;

d. the surface area:
— macromodels;
— micromodels.

A distinction is also marked between linear and nonlinear models, partial and global models, deterministic and stochastic models.

3. THE OPTIMAL LOCATION DETERMINED IN ACCORDANCE WITH SEVERAL CRITERIA WHICH CANNOT BE MUTUALLY COMPARED

Although this is an important category of problems, there are quite a few investigations on this subject published in the literature. The establishment of enterprise locations requires the taking into account of a great number of location factors (sometimes called location criteria). They express certain demands which must be fulfilled by available conditions; the influence exerted by such a great number of factors cannot be quantitatively estimated in a single unit of measure, hence their effects cannot be directly compared.

The effects of several political and socio-cultural factors, or of factors related to national defense, the living standard increase, a.s.f., on the choice of locations, cannot be but qualitatively expressed; by ignoring them, the practical value of solutions obtained would, however, be considerably curtailed

By using certain selection procedures, it is possible to make available for responsible factors solutions which reduce to a great extent the subjectiveness of decision making. Essentially, the selection models consist of the following steps:

— setting up a list of as many potential locations as possible;

— pointing out, for each location listed, the degree of influence exerted by each factor;

— selecting the most convenient location.

The methods can be used for macromodels, as well as for micromodels of location; only comparative, and not quantitative data are obtainable with respect to the economic effect associated to each location.

These models are particularly advisable for limiting the investigation area, for directing, concentrating and finally localizing the attention on an ever decreasing number of locations.

Among the well-known solving methods, we shall mention the following:

a. The file methods, which is a data processing method based on cards, which contain the characteristics of as many locations as possible, suitably encoded in order to make possible the elaboration of processing programs. On this basis, the needs are compared to available conditions, for each potential location, in view of selecting the most favourable location.

There are several kinds of information carriers, namely:

— notched cards, for manual processing;

— punched cards, for mechanographic processing;

— punched cards, punched tape or magnetic tape, for computer processing.

b. Propositional computing methods, based on some principles of mathematical logic. The relationships between the various factors involved in the choice of locations are given in adequate forms for the application of propositional computing procedures.

c. The Electre Method (Elimination et Choix Traduisant la Réalité — Elimination and Choice Translating Reality) has been recently developed by the SEMA (Société d'Economie et de Mathématique Appliquée) and designed for the solution of problems concerning the choice of a new product to be launched on the market, out of the range of new products which could be manufactured by an industrial enterprise. This method allows the selection of an element or of an object out of a set of objects, by taking into account several viewpoints or several criteria and by considering that these criteria cannot be combined in a single one. In other words, we deal with a multidimensional problem in which the individuality of each separate criterion is maintained. It is, however, assumed that some "appraisals" or "ratings" are feasible, as regards the effect of each criterion on each object. The method is based on some properties of the graph theory, thus establishing certain priorities among the analyzed objects.

The Electre method can also be successfully applied in the selection of locations in view of investments. The problem is stated as follows:

If M objects and F factors (viewpoints or criteria) are given, it is required to set up a rating system in order to be able to select one object of the set. With this end in view, we start from the concept that a comparison of two objects — i and j — out of the set of objects M, by taking into account all possible criteria, implies the ability of making a separate comparison, according to each individual criterion. Consequently, it is assumed that for each criterion, an "appraisal" or "rating scale" is available and that a certain level or degree, on this rating scale, corresponds to each object of set M (thus expressing the effect of each criterion on each object). The ratings can be continuous or discrete, they can result from computations or from qualitative estimations.

As a result, any pair of elements, i and j, out of set M, can be compared in accordance with each separate criterion; hence, the following types of relations can be established:

— i surpasses j;
— i is surpassed by j;
— i is equivalent to j; and
— i and j are not comparable.

By applying the Electre method in choosing locations, we aim at developing a procedure allowing us to ascertain which of the above relations is met by each pair of objects i, j in view of accomplishing first a classification and subsequently a selection on this basis.

For a better understanding of the procedure employed, it is advisable to resort to the theory of graphs, by using as an illustration a schematic example.

We shall consider that set M is made up of five elements (which may represent five potential locations for an industrial investment) and that three criteria will be taken into account for the choice of the most convenient location. The "rating scale" comprises five qualitative degrees: very well, well, moderate, satisfactory and unsatisfactory. The ratings pertaining to each location are given in Table I.

Table 1

Table of Primary Data

Locations \ Criteria	1	2	3
L_1	*vw*	*s*	*m*
L_2	*w*	*vw*	*w*
L_3	*m*	*w*	*vw*
L_4	*vw*	*w*	*w*
L_5	*vw*	*s*	*u*

It is possible to associate a graph to this problem, by adopting the following conventions:

The elements of set M are considered to be vertices of the graph and the relationships prevailing among the elements of this set define the connecting

arcs between the vertices of the graph. The graph could be denoted through the set of vertices and arcs.

For instance:

$$G = (M, U)$$

where:

M is the set of vertices (locations);
U is the set of arcs.

The arcs could be defined as follows:

— if i surpasses j, an arc (arrow) is drawn from j towards i;
— if i is surpassed by j, the arc is drawn from i towards j;
— if i and j are equivalent, an arc is drawn in both directions.

By using the data contained in Table I, the relations prevailing among the elements of set M could be defined for each separate criterion and on this basis, three graphs could be drawn up (Figs. 1—3).

It can be seen that some relations are common to all graphs, hence, it is possible to draw up a new graph, representing a synthesis of the three criteria. This graph is shown in Fig.4. The arcs drawn in his graph are only those common to all graphs shown in Figs. 1—3. By denoting with U_0 the set of arcs in Fig.4, we can write the condition:

$$(i, j) \in U_f \qquad \forall f = 1, 2, ..., F \Rightarrow (i, j) \in U_0$$

The graph in Fig. 4 can be denoted:

$$G_0(M, U_0), \text{ where } U_0 = \bigcap_{f=4}^{f=3} U_f.$$

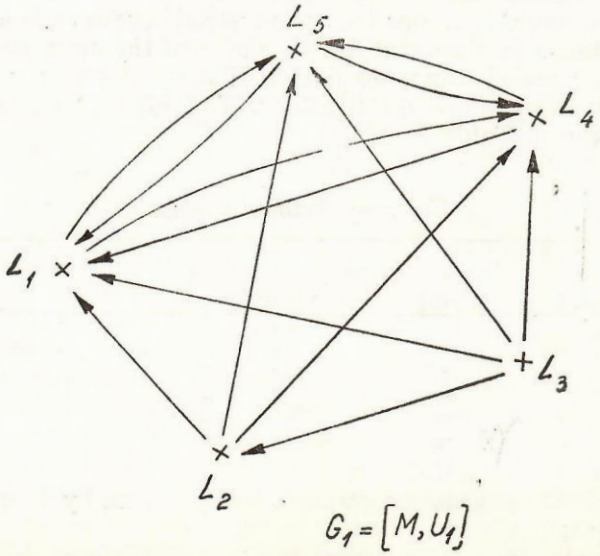


Fig. 1

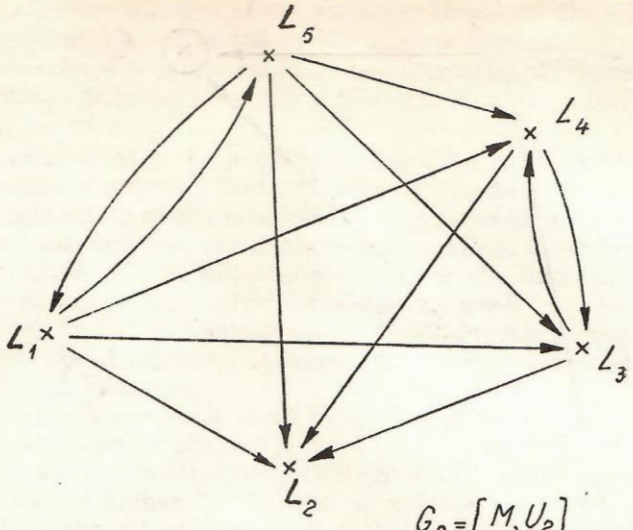


Fig. 2

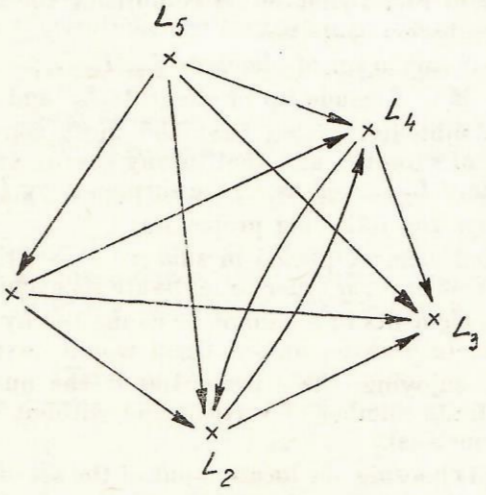


Fig. 3

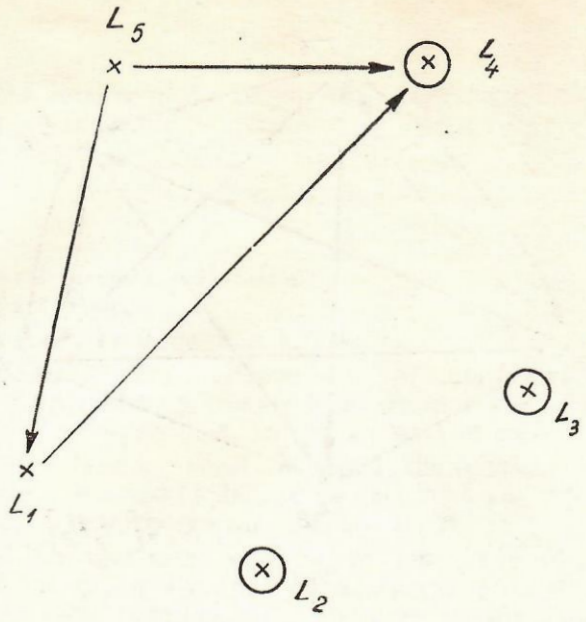


Fig. 4

It can be seen in Fig. 4 that set M comprising the five locations could be divided in two subsets, namely:

— one subset S, made up of elements L_2, L_3 L_4;

— one subset $M - S$, made up of elements L_1 and L_5.

It can be undoubtedly, stated that the most convenient location is one of the elements of subset S and that in any case it cannot be comprised in subset $M - S$, since L_1 as well as L_5 are surpassed by L_4.

Subset S enjoys the following properties:

— any excluded element placed in subset $(M - S)$ must be surpassed by at least one element of S (in order to motivate its exclusion);

— the pairs of elements in S cannot be connected by an arc, hence they cannot be compared (otherwise, one of them would have to be excluded);

— the subset enjoying these properties is the nucleus of the graph (any graph with a finite number of vertices and without circuits has always one and only one nucleus).

The problem of choosing one location out of the set of potential locations is then amounting to the determination of the nucleus of a finite graph without circuits.

Graph G_0 does not, however, contain a sufficient number of connections, allowing a more detailed analysis of the relations prevailing among locations,

as well as the effective solution of the problem; therefore, the following hypotheses and supplements are added to the Electre method:

— It is considered that criteria could be differentiated into more important and less important criteria; a weight $\pi_f (f = 0,1, 2,...,F)$ could be assigned to each criterion.

— It is considered to be possible to associate a "mark" to each degree of the rating scale of criteria effects. This mark may be a simple numbering of degrees, or an estimation of the interval existing between the degree, or it may stand for quantitative influences, liable to computation. These marks are involved in computation only as differences and their absolute value is not a limiting factor, as will be seen in the following paragraphs.

— For each pair of elements of set M, two indexes are computed, namely the concordance and discordance indexes, which are useful in establishing the priority ranking of elements.

— The concordance index is an indication of the degree with which an element i surpasses another element j. It is expressed by the ratio of the number (multiplied by the respective weight) of criteria for which the estimations in the rating scales are higher, to the sum of weights associated to all criteria.

Considering the set of criteria F as being divided into two subsets: subset $C(i, j)$ for which i surphasses j and subset $D(i, j)$ for which i is surpassed by j, the concordance index is computed according to the following formula:

$$C_{ij} = \frac{\sum\limits_{f \in C(i,j)} \pi f}{\sum\limits_{f=1}^{F} \pi f}.$$

The value of the concordance index is comprised within the limits 0 and 1.

In the above illustration, the concordance index between locations L_2 and L_3 is $C_{2,3} = \frac{2}{3} = 0.66$, as L_2 surpasses L_3 for two of the three criteria. If different weights would be considered, for instance if weight 3 would be assigned to criterion I and weights equal to unity would be assigned to the remaining criteria, then the concordance index would have the value $C_{2,3} = \frac{4}{5} = 0.8$, since L_2 surpasses L_3 for the first two criteria and the sum of weights is 3.

The values of all concordance indexes can be computed for all pairs of elements i, j. Table II shows these values for the above example.

Table II

Table of Concordance Indexes

i \ j	L_1	L_2	L_3	L_4	L_5
L_1	x	06	06	06	1
L_2	04	x	08	04	04
L_3	04	02	x	04	04
L_4	1	08	08	x	1
L_5	08	06	06	06	x

It is possible for some criteria to have particular significance and since the computation of the concordance index takes into account only the weighted number of criteria, it is necessary to introduce another index, which would express the effect of divergent criteria. In order to consider the criteria for which the previously mentioned hypothesis — that i is surpassed by j — is not satisfied, the discordance index is introduced. The computation of the discordance index requires to associate to each rating scale for criteria marks expressed by integers.

In the case of the above example, we assume the following marks to be assigned on the basis of a distinct rating scale for each separate criterion:

— for criterion 1 $vw = 10$, $w = 5$, $m = 2$
— for criterion 2 $vw = 30$, $w = 20$ $s = 5$
— for criterion 3 $vw = 5$, $w = 4$, $m = 3$ $n = 1$

Considering the weights of criteria, a table of marks (Table III) can be made up, replacing Table I for the computation of the discordance index (criterion 1 has weight 3).

Table III

Table of Marks

Criteria / Locations	1	2	3
L_1	30	5	3
L_2	15	30	4
L_3	6	20	5
L_4	30	20	4
L_5	30	5	1

The discordance index is computed as the ratio of the difference of marks assigned to elements i and j, to the highest possible difference. Thus, in the case of locations L_2 and L_3, for instance, the value of the discordance index is:

$$d_{2,3} = \frac{5-4}{25} = \frac{1}{25} = 0.04,$$

since the highest possible difference is 25.

The values of all discordance indexes can thus be computed for the pairs of elements i and j (Table IV).

Table IV

Table of Discordance Indexes

i \ j	L_1	L_2	L_3	L_4	L_5
L_1	x	1	0.6	0.6	0
L_2	0.6	x	0.04	0.6	0.6
L_3	0.95	0.4	x	0.95	0.95
L_4	0	0.4	0.04	x	0
L_5	0.08	1	0.6	0.6	x

In view of finding out the relationships prevailing among the elements, which are necessary in order to determine the priority ranking of locations, it is agreed that i might be considered as surpassing j, if the value of c_{ij}, the concordance index, is high enough, while the value d_{ij} of the discordance index is low enough. Consequently, it will be necessary to determine two limits: a lower limit for c_{ij}, called concordance threshold and designated by p, and an upper limit for d_{ij}, called discordance threshold and denoted by q.

The most rigorous hypothesis is in agreement with the values $p = 1$ and $q = 0$. This hypothesis corresponds to the graph shown in Fig.4.

When the concordance threshold is reduced, while the discordance threshold is increased, additional connections are introduced in the graph; by computing the nuclei of the graphs for various hypotheses, indications concerning the priority ranking of elements will be obtained. If the following conditions

$$c_{ij} \geqslant p$$

$$c_{ij} \leqslant q$$

are simultaneously fulfilled, the connecting arc from j to i can be drawn. By computing the nucleus for decreasing values of p and for increasing values of q, it will contain a steadily decreasing number of elements and thus, groupings as well as priorities can be established among these elements. In the above example, if we consider the values $p = 0.8$ and $q = 0.1$, graph 0(0.8; 0.1) is obtained, which is shown in Fig. 5 (the nucleus includes only locations L_2 and L_4). If the discordance threshold is increased up to 0.4, the graph shown in Fig. 6 is obtained, whose nucleus includes only location L_4. It can be therefore stated that based on the three criteria, the five locations are grouped in the following order:

$$L_4 > L_2 > L_3 > L_1, L_5.$$

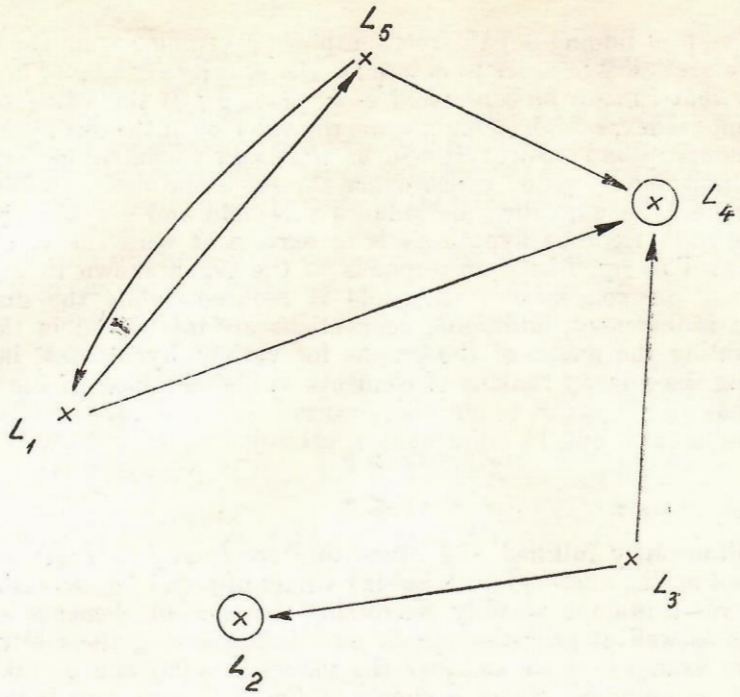
This priority ranking has been obtained by writing in the first place the location of the last graph nucleus and subsequently the locations successively excluded by the graphs made up for ever more strict thresholds p and q.

The application of the Electre method in determining the location of investement objects involves several particular aspects:

The graphs shown in figures 1—3 have two peculiar properties: they are complete and transitive. Indeed, any pair of vertices in the graphs shown in figures 1—3 is adjacent, that is there is at least one arc between any pair of vertices i, j. The graphs shown in figures 4—6 and, in general, the graphs obtained in various analytical stages for any values p and q, are not complete. Moreover, in the graphs shown in figures 1—3, each time there is a path between two vertices i and j, there is always also a unique arc uniting these vertices. For instance, in Fig.1, between vertices L_3, L_1 there is the path L_3, L_2 — L_2, L_1, as well as the direct arc L_3, L_1, which is expressed through the following property:

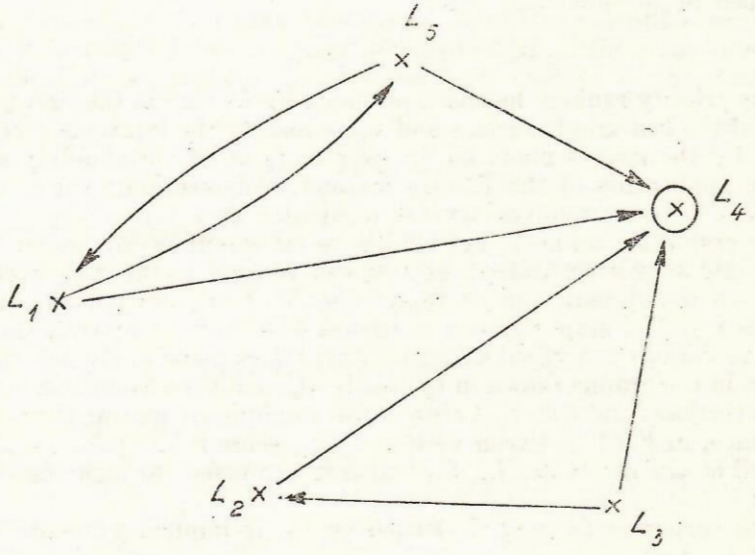
if L_2 surpasses L_3, and L_1 surpasses L_2, it implicitly results that L_1 surpasses L_3, i.e.

$$L_2 > L_3 \quad \text{and} \quad L_1 > L_2 \Rightarrow L_1 > L_3.$$



G[0,8,01]

Fig. 5



G[0,8,04]

Fig. 6

In most cases, the graphs obtained during the various analytical stages with the Electre method, do not enjoy this property.

If the accepted concordance threshold is reduced, it is possible to be placed in such circumstances that object j surpases object i, while there is simultaneously another object k surpassing j, which does not mean that object k also surpasses i. These circumstances are arrived at owing to the hypotheses made for the definition of the concept of surpassing and derive from the fact that the surpassing thus defined is a result of taking into account several criteria, having frequently contradictory effects. This is why, in order to reach accurate conclusions, on applying the Electre method, it is advisable to compute several variants for various pairs of values p and q and to analyze the results obtained.

In the applications of the Electre method, circuits among the various elements are frequently obtained; in Fig. 3, for instance, there is a circuit between locations L_2 and L_4. In order to be able to compute the nucleus in this specific case, we resort to the so-called "circuit reduction". The procedure is as follows: if one keeps in mind the hypotheses introduced, it is natural to consider that the elements placed on the same circuit are mutually equivalent. In this case, it will be reasonable to replace the subset of elements from one circuit by one of these elements.

Denoting with M the set of all ellements, with φ the subset of elements in the circuit, with $M - \varphi$ the subset of elements not included in the circuit, and with t one element selected from the circuit, the reduction of circuits is based on the following relationships:

— t surpasses $i \in M - \varphi$, if there is an element j, surpassing element i;

— t is surpassed by $j \in M - \varphi$, if there is an element i which is surpassed by element j.

This method does not always lead to the finding of a single element surpassing all other elements. For sufficiently high values p and very low values q only one subset of elements is obtained within which it is certain that the searched element is also to be found. This means that for the accepted thresholds, one cannot obtain a synthesis sufficiently rich in pairs of comparable locations. In such instances an additional choice is required, a more thorough analysis of the characters belonging to each element, either by computing a greater number of variants for different values of parameters p and q, or by using other methods.

In problems involving a great number of criteria, it is possible to ignore the effect of criteria giving rise to the greatest discordance. The negative differences between the marks associated to elements i and j are accordingly arranged in the decreasing order of their absolute values and the rank of these values is designated with the numbers $1, 2, \ldots, s$. New variants are thus computed, where in addition to the differing values of parameters p and q, an entirely new parameter is introduced, with the following signification:

— if $s = 1$, the calculation will be effected as previously described;

— if $s = 2,3,\ldots$ the discordance coefficients are computed by using the difference ranked 2,3,... in the above mentioned sequence.

A series of graphs $G(p, q, s)$ is thus obtained, contributing to the achievement of a much more thorough analysis.

It can be argued that the assignement of marks in this method introduces subjective elements. In fact, the marks appear only in the form of differences and have no other purpose than to specify a priority order among each pair of elements. The practical application of this procedure has shown that if the marks are altered, without changing the ranking of the various pairs of elements, the results obtained are identical, or at least very close to each other.

In order to check this property, the example previously described has been again computed, after new marks have been assigned, as shown in Table V, in the columns headed "hypothesis No. 2".

Table V

Comparative Table

Ratings \ Marks	Criterion 1		Criterion 2		Criterion 3	
	Hyp. No. 1	Hyp. No. 2	Hyp. No. 1	Hyp. No. 2	Hyp. No. 1	Hyp. No. 2
vw	10	3	30	30	5	10
w	5	2	20	20	4	8
m	2	1			3	6
s			5	5		
n					1	1

After the computation of coefficients c_{ij} and d_{ij} and then of graphs G(1 ; 0), G(0.8 ; 0.1) and G(0.8 ; 0.4), the resuls are identical to those derived from the first hypothesis.

It is noteworthy that the method which consists of computing the sum of weighted marks, leads to different results. Indeed, by adding the marks multiplied by the weight of criteria in both hypotheses, the following totals are obtained (Table VI).

Table VI

Hypotheses \ Location	L_1	L_2	L_3	L_4	L_5
Hypothesis 1	38	49	31	54	36
Hypothesis 2	20	44	33	37	16

It can be remarked that:

— The weighted sum of the marks leads to divergent results in the first and second hypotheses, since in the second hypothesis the most favourable location would be L_2, which is not a satisfactory solution, as can be seen from an inspection of Table I ; for both hypotheses the Electre method leads to the selection of location L_4.

— The weighted sum of marks suggests that the most unfavourable elements in the first hypothesis are L_3 and L_5, while in the second hypothesis the respective elements are L_1 and L_3. In an early stage, the Electre method had ruled out locations L_1 and L_5 from both hypotheses, which seems to be more reasonable.

4. AN EXPERIMENTAL APPLICATION OF THE METHOD

In order to point out the practicability of the Electre method as a means for the determination of optimum location of investments, we shall now present an example.

We consider ten potential locations, denoted L_1, L_2..., L_{10} (which may be districts or regions, where the investment could eventually be located). It is assumed that the preliminary works, as well as the territorial systematization or zone systematization survey are already completed and, as a result, it is advisable to take into account a total of eight criteria for the choice of the locations, namely:

— four functional criteria, i.e. Cr 1 materials, Cr 2 fuel and energy supplies, Cr 3 outward transports and Cr 4 manpower;

— three socio-economic criteria, i.e. Cr 5 the effect of connected investments, Cr 6 the rational utilization of manpower and Cr 7 co-operations;

— one special criterion — Cr 8 natural reservation protection.

A rating scale is developed for each criterion, depending on the degree to which preliminary studies have exhausted the subject; it is assumed that the method of cost increments could be applied for the functional criteria, i.e. it is possible to make an estimate of the effect on the specific cost, which can be expressed in value form; a number of rating levels or degrees is adopted for the remaining criteria. Moreover, this is also the stage when weights are assigned to criteria. The following rating scales are considered:

1. Criterion 1: range of the rating scale 50—70, weight 1
2. Criterion 2: range of the rating scale 3—7, weight 1
3. Criterion 3: range of the rating scale 10—20, weight 1
4. Criterion 4: range of the rating scale 20—30, weight 1
5. Criterion 5: range of the rating scale with three degrees 8—4 weight 5
6. Criterion 6: rating scale with 4 degrees 8—11, weight 3
7. Criterion 7: rating scale with 4 degrees 6—12, weight 2
8. Criterion 8: rating scale with 2 degrees, 1—8, weight 1.

On this basis, the table of primary data is made out, where locations are successively recorded according to the rank they hold for each criterion. It is considered that rank 1 is the least valuable on the rating scale. Moreover, it has been assumed that for functional criteria, it is possible to calculate the impact of each criterion on total or unit production costs and that this impact has been expressed in percent form.

In order to bring into uniformity these ratings, they are transformed in indexes, by assigning index 100 to the lowest rating degree. The table of indexes is thus obtained.

Since the Electre method involves only rating differences among elements, the table of indexes could be simplified by consistently reducing all indexes by 100. Tentative marks are thus obtained, which when multiplied by the weights assigned to criteria, result in the ultimate marks. Finally, the table of marks (Table VII) is made up, (which supplies the primary data used in the application of the Electre method).

Subsequently, for each location pair L_i, L_j, the number of criteria is computed for which the marks assigned to location i are higher than those pertaining to location j and — otherwise — the difference between marks.

The coefficients c_{ij} are then computed as the ratio of the weighted sum of the criteria number for i and j, to the sum of criteria weights ($\Sigma\pi_i = 13$). The coefficients d_{ij} are computed as the ratio of the maximal difference pertaining to location pairs i, j, to the maximal range of the difference (in this particular case, it amounts to 300). Coefficients c_{ij} and d_{ij} have been computed only for the value of parameter $s = 1$, i.e. it was thought as being unadvisable to disregard the effect of any criterion. The computations are systematized in tables. The values of concordance and discordance indexes are shown in a condensed form in Tables VIII and IX.

Table VII

Table of Weighted Marks

Locations \ Criteria	Cr 1	Cr 2	Cr 3	Cr 4	Cr 5	Cr 6	Cr 7	Cr 8	Sum of weighted marks
L_1	10	100	0	35	0	36	200	0	381
L_2	30	30	60	10	0	108	160	100	398
L_3	40	30	100	20	0	75	0	0	265
L_4	36	30	80	20	0	75	0	100	341
L_5	0	60	40	50	300	36	120	0	606
L_6	20	100	10	0	0	36	200	100	466
L_7	8	0	50	45	150	108	60	100	521
L_8	36	130	30	25	0	0	120	100	441
L_9	20	0	50	0	0	108	60	100	338
L_{10}	30	130	10	10	0	0	120	0	300

Table VIII

Table of c_{ij} Coefficients

i \ j	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
L_1	—	054	063	054	063	077	031	069	054	077
L_2	069	—	077	077	046	077	069	061	100	077
L_3	069	054	—	092	046	069	023	061	054	077
L_4	069	063	084	—	046	077	031	069	063	077
L_5	069	054	063	054	—	063	054	077	054	084
L_6	092	034	054	054	063	—	038	069	069	077
L_7	069	077	077	077	046	069	—	069	092	069
L_8	054	069	061	069	038	063	038	—	069	100
L_9	069	069	069	069	046	077	069	063	—	063
L_{10}	054	061	054	046	038	054	031	069	061	—

Table IX

Table of d_{ij} Coefficients

i \ j	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
L_1	—	033	033	033	100	033	050	033	033	010
L_2	046	—	013	007	100	046	050	033	000	033
L_3	066	033	—	033	100	066	050	040	033	040
L_4	066	020	007	—	100	066	050	040	020	040
L_5	027	033	020	033	—	033	033	033	033	023
L_6	012	024	030	023	100	—	050	010	024	010
L_7	047	010	017	010	030	047	—	043	004	043
L_8	027	036	025	025	100	027	050	—	036	000
L_9	047	010	017	010	100	047	050	043	—	043
L_{10}	027	036	033	033	100	033	050	033	036	—

In accordance with the procedure described in this paper, graphs are drawn for various pairs of values of the concordance and discordance thresholds and subsequently the nucleus is computed, which makes possible the selection of locations and the establishment of priorities. The pairs of values p, q are chosen in such a manner that if we start from the extreme values ($p = 1$ and $q = 0$) the concordance threshold should gradually diminish, simultaneously with the increase of the discordance threshold. In order to ascertain these values the results obtained during each stage are analyzed, as well as the potential changes of indexes c_{ij} and d_{ij}.

For each pair of values p, q, graph

$$\mathrm{G}(p\,;\,q) = (M,\ U_{p,q})$$

is drawn; it is made up of the set M of locations $L_1, L_2, \ldots, L_{10}$ and the set of arcs i, j, which fulfil the condition

$$(i, j \in U(p, q) \Rightarrow c_{ij} \geqslant p \quad \text{and} \quad d_{ij} \leqslant q$$

that is, the arc uniting location i with location j exists only if the relations determining the limits of indexes c_{ij} and d_{ij} are satisfied for the location pair i, j.

With a view to selecting the most favourable location, about thirty graphs have been developed for the above mentioned example, the values p and q being included within the limits

$$1 \geqslant p \geqslant 0.05$$

$$0 \leqslant q \leqslant 0.33.$$

In the following paragraphs we shall describe some of the most characteristic graphs thus obtained.

a. Graph $G(1.00\,;\,0.00)$. This graph contains only the arcs uniting those pairs of locations for which the marks pertaining to one of them are higher

than the marks of the other, for all eight criteria. In Tables VII and VIII t can be seen that there are only two arcs simultaneously fulfilling conditions

$$c_{ij} = 1 \text{ and } d_{ij} = 0,$$

namely the arcs $L_2 - L_9$ and $L_2 - L_{10}$. The graph is shown in Fig. 7 and its nucleus contains eight locations, L_1, L_2, L_3, L_4, L_5, L_6, L_7 and L_8; locations L_9

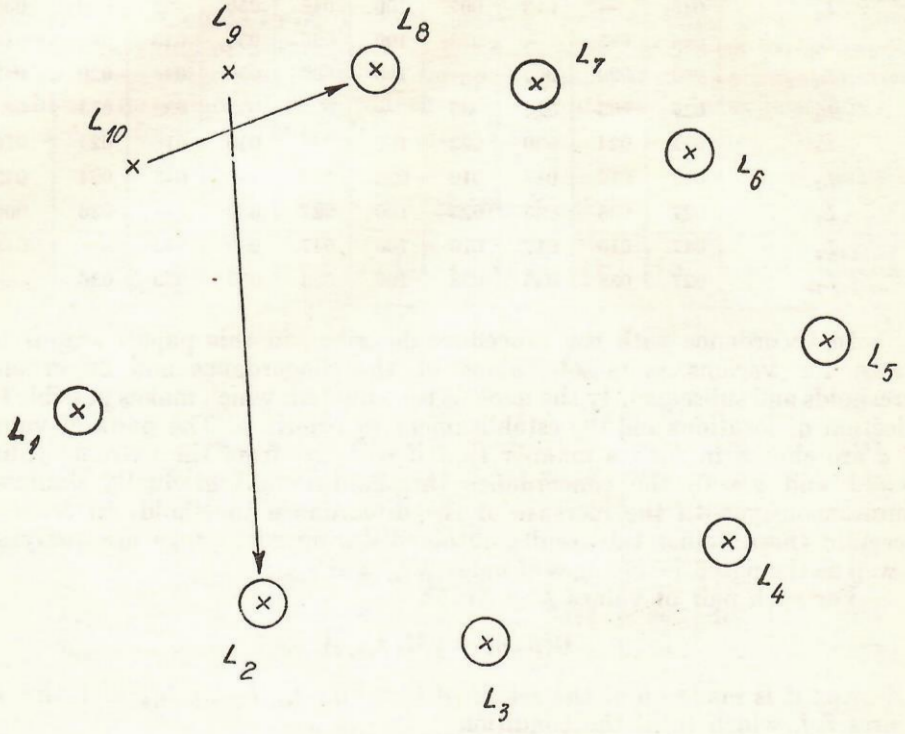


Fig. 7

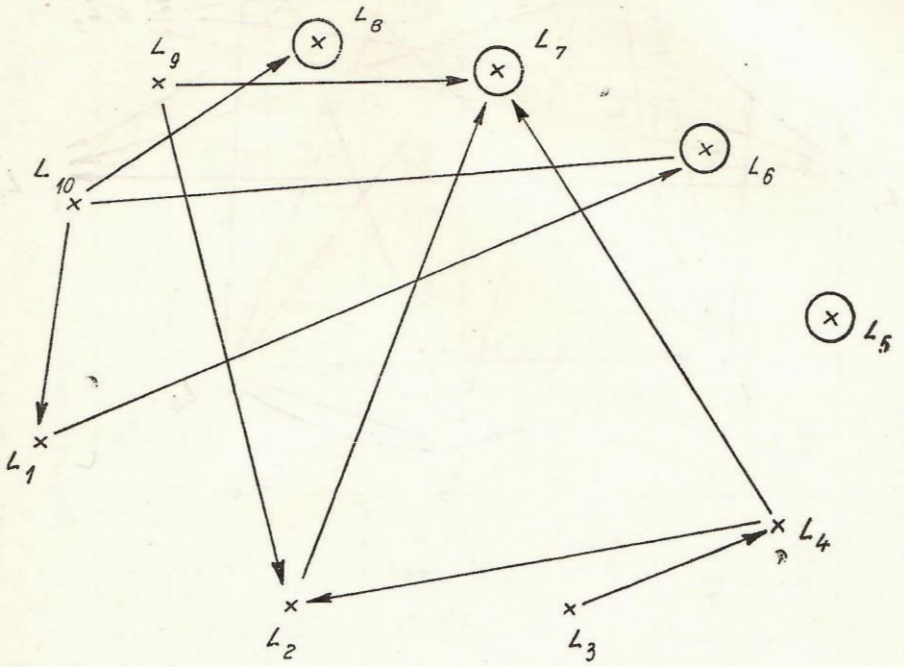
and L_{10} are excluded, as they are surpassed by L_2 and L_8. Consequently, in these very strict circumstances, it can be stated that locations L_9 and L_{10} are not involved in the choice of the most convenient location. An inspection of Table VII shows that this conclusion has not resulted from the simple comparison of the weighted sums of the marks; actually, the lowest weighted sum corresponds to another location, L_3, which is not excluded in case it is stipulated that any mark must be lower than the mark of any other location, for any of the eight criteria.

b. Graph G (0.9; 0.12) is suited to comparatively severe conditions. It contains arcs $L_2 - L_9$; $L_8 - L_{10}$; $L_7 - L_9$; $L_6 - L_1$. It can be seen that in addition to locations L_9 and L_{10}, location L_1 is also excluded. We may men-

tion that the weighted sum of the marks pertaining to L_1 is higher than the sum pertaining to locations L_3 and L_4 (which must not, however, be yet excluded).

c. Graph G (0.75; 0.12) contains the following arcs:

$$L_8 - L_{10};\ L_2 - L_9;\ L_6 - L_1;\ L_7 - L_9;\ L_7 - L_2;\ L_4 - L_3;$$
$$L_2 - L_4;\ L_7 - L_4;\ L_1 - L_{10};\ L_6 - {}_{10}.$$



G [075,012]

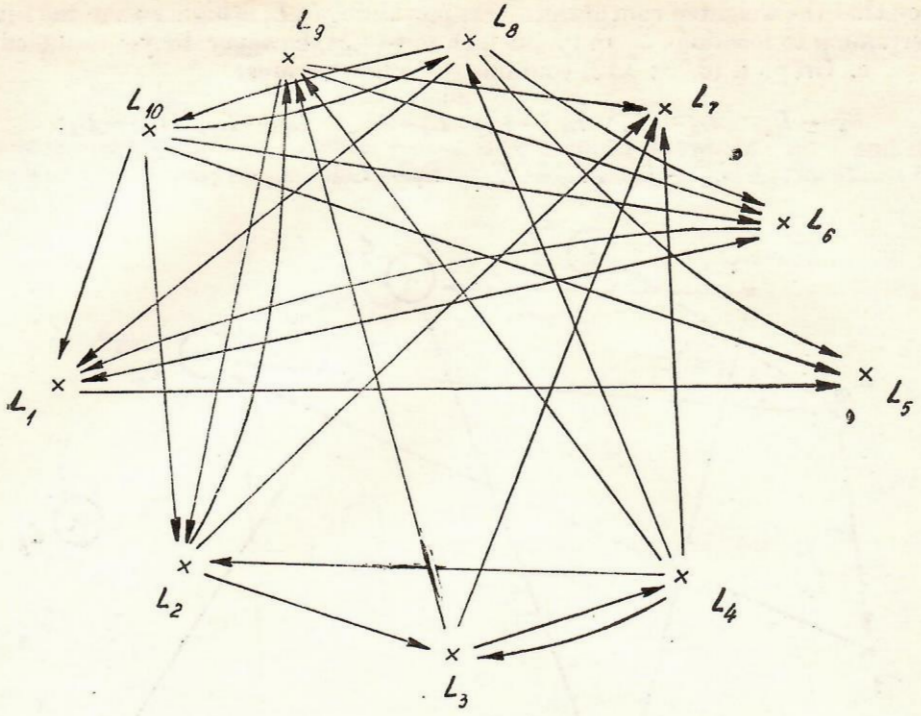
Fig. 8

Fig. 8 shows that the nucleus included only locations L_5, L_6, L_7, L_8, consequently, the ten locations could be divided into two groups:

— group L_5, L_6, L_7, L_8, including the most convenient location;

— group L_1, L_2, L_3, L_4, L_9, L_{10}, containing the locations surpassed by at least one location from the previous group.

d. Graph G(0.65; 0.35). In order to mark a distinction among the most convenient locations it is necessary to continue the decrease of the concordance threshold, while increasing the discordance threshold. The corresponding graph is shown in Fig. 9; it can be seen that several circuits have appeared, which must previously be reduced. The graph shown in Fig. 9 is thus obtained; its nucleus is made up of two elements, L_5 and L_7.



G(065,033)

Fig. 9

The nuclei obtained for various values p, q are recorded in the Table of Solutions (Table X).

By using this table, the following priority ranking can be developed

$$L_7 > L_5 > L_6 > L_8 > L_1 > L_2, L_4 > L_3 > L_9, L_{10}.$$

Considering the qualitative appraisals which lay at the basis of computations, it is sometimes advisable to restrict the procedure only to the determination of groups of locations, while more accurate methods are used for the final selection. In this case, the following groups can be derived:

$$L_5, L_6, L_7, L_8 > L_1, L_2, L_3, L_4 > L_9, L_{10}.$$

It can be stated that he first group surpasses all other groups, in the following conditions (simultaneously fulfilled):

Table X

*Table of solutions**

Discordance threshold									
Discordance threshold	p \ q	C	010	012	015	020	025	030	033
	1.00	1.2.3.4. 5.6.7.8.							
	0.90		1.2.3.4. 5.6.7.8.	2.3.4. 5.6.7.8.		2.3.4. 5.6.7.8.	2.3.4. 5.6.7.8.	2.3.4. 5.6.7.8.	2.4. 5.6.7.8.
	0.80		1.2.4. 5.6.7.8.	2.4. 5.6.7.8.		2.4. 5.6.7.8.	2.4. 5.6.7.8.	2.4. 5.6.7.8.	2. 6.7.8.
	0.75		1 5.6.7.8.	5.6.7.8.	5.6.7.8.		5.6.7.8.	5.6.7.8.	5.7.
	0.70		1. 5.6.7.8.	5.6.7.8.			5.6.7.8.	5.6.7.8.	5.7.
	0.65		1. 5.6.7.	5.6.7.		5.6.7.	5.6.7.	5.6.7.	5.7.
	0.60		1. 5.6.7.	5.6.7.	5.6.7.		5.6.7.	5.7.	5.7.
	0.50								5.

* Only the indexes of locations included in the nucleus have been recorded in this table

— $p \geqslant 0.75$, i.e. the ratio of the number, multiplied by the corresponding weights, of criteria on whose bases these locations surpass the others, to the sum of weights, is at least equal to 75 percent;

— $q \leqslant 0.12$, i.e. the ratio of the highest excess to the maximal possible range of excesses, is at most equal to 12 percent,

We can therefore consider that there are excesses for three quarters of the criteria, and for those criteria where this stipulation is not fulfilled, the discordance amounts to 12 percent of the highest possible discordance.

In order to select a single location only, these limits would have to be reduced to $p = 0.5$ and $q = 0.35$.

REFERENCES

1. STANCU-MINASIAN, I. M. and BOLDUR, G. *Metoda Electre.* Studii şi cercetări de calcul economic şi cibernetică economică, No. 2, 1969.
2. CSAS-ISCAS: *Alegerea amplasamentelor investiţiilor industriale pe baza calculelor de eficienţă economică* (metodologie) Tema III, 3, 1968.
3. INSTITUTUL DE CERCETĂRI ECONOMICE AL ACADEMIEI RSR: *Eficienţa economică a investiţiilor.* Cercetări teoretice metodologice, Bucureşti, 1967.
4. COMISIA ECONOMICĂ ONU PENTRU EUROPA: *Criterii pentru amplasarea întreprinderilor industriale*
5. BUNET, P., GREMY, J. P., MARC, M., SUSSMANN, B.: *Peut-on choisir en tenant compte de critères multiples? Une méthode (Electre) et trois applications. Metra, Tome VI* No. 2, juin 1967.
6. ROY, B.: *Classement et choix en présence de points de vue multiples (La méthode Electre).* Revue française d'informatique et de recherche opérationnelle. No.8 mars-avril, 1968.
7. INCERC: *Studiul alegerii amplasamentelor industriale pe baza calculelor de eficienţă economică. Posibilităţi şi metode de calcul matematic şi modul de comparaţie între variantele studiate.*

L'EMPLACEMENT OPTIMUM DES ENTREPRISES PAR LA METHODE DE LA SELECTION

L'auteur présente brièvement les aspects généraux et les implications économiques de l'emplacement des entreprises de production; il essaye, sur cette base, d'établir une typologie des problèmes d'emplacement territorial et d'analyser les méthodes économiques et mathématiques.

Lorsque pour choisir l'emplacement nous prenons en considération plusieurs critères non-comparables entre eux, nous devons étudier le mode d'application de la méthode « Electre » élaboré par la SEMA (FRANCE).

Cette méthode permet d'orienter et de délimiter l'aire des investigations en fixant l'ordre de priorité des différents emplacements.

L'auteur analyse, à partir de certaines applications d'essais, les possibilités offertes par la méthode « Electre », sa sensibilité à la variation des paramètres qui interviennent et ses limites d'applicabilité.

ОПТИМАЛЬНОЕ РАСПОЛОЖЕНИЕ ПРЕДПРИЯТИЙ ПО МЕТОДУ СЕЛЕКЦИОНИРОВАНИЯ

Автор кратко излагает общие замечания и экономические последствия расположения производственных предприятий, и на этой основе пытается установить типологию проблем территориального распределения, анализирует экономико-математические методы.

Когда при выборе расположения предприятий учитывается много критериев, несравнимых между собой, изучается способ применения метода Electre разработанного SEMA (Франция).

Этот метод дает возможность ориентировки и разграничения поверхности исследования, установления преимущественного порядка различных расположений.