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Two Formulas for Determining the Total Materials Stock According to the Cost Criterion

The whole economic activity, hence also the material stock management, the management of unfinished output and of finished articles, must meet contemporary efficiency requirements.

The stock management is a highly important activity field of the enterprises, where a series of principles, methods and techniques thought out by the production and labour organization and management science can be successfully applied. The present paper intends to critically present two formulas for determining the total material stock according to the cost criterion [1], [2], and an attempt is made to apply them in an enterprise, in order to grasp the practical utilization difficulties.

GENERALS

Both formulas take into account the *mean cost* depending on the stock size, expressed as follows:

— P. Massé's formula:

$$C_{(s)} = (a + b)\, s + g \int_s^{\infty} (r - s)\, \mathrm{d}F(r) - c \int_0^{\infty} (s - r)\, \mathrm{d}F(r) \qquad (1)$$

— A. Kaufmann's formula:

$$C_{(s)} = g \sum_{r=s+1}^{\infty} (r - s)\, p(r) + c \sum_{r=0}^{s} (s - r)\, p(r) \qquad (2)$$

where:

$C_{(s)}$ = mean cost connected with the stock existence (it is not clear whether it is determined per annum);

a = unit purchasing costs (including transport);

b = unit costs due to material stocking and immobilization (it is not clear whether they are per day, per order or per annum);

g = unit costs due to material absence from stock (it is also not clear whether they are considered per day or per annum);

c = unit costs due to surplus stock materials selling off — designated also as "surplus unit cost";

$r - s$ = material consumption between two consecutive deliveries;

$s - r$ = material quantity to be stocked in the interval between two consecutive deliveries;

$r - s$ = absence of materials in stock;

$s - r$ = stock surplus as against consumption, which has to be sold off at a lower price than the purchasing one;

$dF(r)$ = differential of the consumption distribution function in the interval between two deliveries;

$p(r)$ = consumtion probability density or simple probability.

Minimum of cost $C_{(s)}$ is determined by deriving with regard to s and by equalling to zero (assuming there is a minimum). The notations used by the authors were modified and unified, in order to facilitate some findings i.e.:

— in the second formula two costs are absent (a and b), which means that the formula has a more restricted applicability;

— there is a symbol (c) with a distinct significance; in the first formula it stands for the selling off price of surplus stock materials, and in the second one it means the unit loss due to surplus materials selling off;

— in the second formula, the stocking cost is added to the absence stock cost, while in the first one the cost of surplus selling off is subtracted from the other totalized costs;

— the first formula takes into account the continuous consumption variation process; it is admitted, however, that continuous processes can be dealt with as discrete processes; there results, from this form of expression, the use — in the first formula — of the consumption distribution function, and in the second — the simple probability of consumption;

— from the first formula one may deduce that the stocking and immobilization cost, bound to the first bracket ($b \cdot s$), reflects the stocking and immobilization of the whole stock, while this, as a matter of fact, takes place only for the security stock, as the current stock immobilization rate is only 1/2;

— in both cases it is not clear how unit costs bound to a stock absence are determined; in our opinion, here two cost groups can be distinguished, namely a cost to avoid slack time (the cost of urgent re-supplying or the cost of production assortment changing) and another cost, bound to the absence period caused by the slack time; these both, in their turn, have several components.

All costs can be determined per day, per the interval between two deliveries, or per annum, and from the quantitative viewpoint, per whole stock or per stock unit. It is to be noticed that P. Massé, unlike the American authors, calls one's attention to the necessity of taking into account the slack

time extent, so that the slack cost appears to be variable not only against the size of demand, but also as against the slack time duration.

* * *

To find the optimal cost with the aid of Massé's formula, one deduces the total cost from formula (1) as against stock s:

$$\frac{dC}{ds} = a + b - g\int_s^{\infty} dF(r) - c\int_0^s dF(r) = 0$$

and

$$a + b = g[F(\infty) - F(s)] + c[F(s) - F(0)],$$

hence:

$$a + b = g[1 - F(s)] + cF(s) \tag{3}$$

It is to be noticed that the first equation member represents the stocking marginal cost and the other one represents the marginal expectation of costs bound to absence and surplus. On this basis the following reasoning is made: the optimal stock takes place where both cost groups are equal.

One may give also another interpretation to formula (3), starting from the fact that $F(s)$ stands for the surplus probability, and $1 - F(s)$, for the absence probability. Thus, from relation (3) there follows:

$$a + b = g - gF(s) + cF(s)$$

hence

$$F(s) = \frac{g - (a + b)}{g - c} \tag{4}$$

and, respectively:

$$1 - F(s) = \frac{(a + b) - c}{g - c} \tag{4'}$$

Quantity $g - (a + b)$ is named "loss by absence", and $(a + b) - c$ "loss by surplus" per material unit. Carrying out the ratio between both equalities, it follows

$$\frac{F(s)}{1 - F(s)} = \frac{g - (a + b)}{(a + b) - c} \tag{5}$$

Under this form, the optimal stock appears where the ratio between surplus and absence probabilities equals the ratio between loss by absence and loss by surplus.

But P. Massé puts two conditions for the validity of his formula, namely:

$g > a + b$, as otherwise the enterprise would prefer to have systematically absence of materials;

$(a + b) > c$, as otherwise the enterprise would prefer to have continuously material excess.

In Kaufmann's formula, one starts from the assumption that *minimum cost* $C_{(s_0)}$, corresponding to an optimal stock s_0, must satisfy a double inequality:

$$C_{(s_0-1)} > C_{(s_0)} < C_{(s_0+1)} \tag{6}$$

In order to check this inequality, we compute $C_{(s_0-1)}$ and $C_{(s_0+1)}$.

$$C_{(s_0+1)} = c\sum_{r=0}^{s+1}(s+1-r)\,p(r) + g\sum_{r=s+2}^{\infty}(r-s-1)\,p(r),$$

which leads, through transformations [2], to:

$$C_{(s_0+1)} = C_{(s_0)}\,(c+g)p(r \leqslant s_0) - g \tag{7}$$

In the same manner, we compute:

$$C_{(s_0-1)} = c\sum_{r=0}^{s-1}(s-1-r)\,p(r) + g\sum_{r=s}^{\infty}(r-s+1)\,p(r) \text{ obtaining:}$$

$$C_{(s_0-1)} = C_{(s_0)} - (c+g)p(r \leqslant s_0 - 1) + g \tag{8}$$

By a series of transformations, using the assumption represented by inequality (6), the following condition results as concerns the optimal stock size:

$$p(r \leqslant s_0 - 1) < \frac{g}{g+c} < p(r \leqslant s_0). \tag{9}$$

The value of ratio $\frac{g}{g+c}$, rather inadequately called "absence coefficient" or "scarcity factor", is determined on the basis of actual data concerning the material consumption (Annex I and II). Still on the basis of these data a synthetic table containing simple and cumulative probabilities is drawn up, such as in its last column the comparison with the absence coefficient is made in order to determine the optimal stock. Of course we must previously estimate the probability law according to which the phenomenon evolves (Annex III) and must compute also the simple probability [3], [4].

In the present case, the χ^2 criterion has been used to verify the normal law evolution of the phenomenon.

APPLYING BOTH FORMULAS

Let us attempt to use both formulas for determining the optimal refined zinc stock at the Jassy Metallurgical Works on the ground of data for 1967; the enterprise data show that there were 85 deliveries and a weighted mean interval between two deliveries of 4.4 days, by a daily consumption of 8.3 tons, a mean 4-day consumption of 32.9 tons (see Annex 1 and II), and a mean effective yearly stock of 100 tons. The following costs were also determined:

— unit purchasing cost which, in a first assumption, is 7,750 lei/ton (list cost plus transport cost and other appending costs) and in another assumption is 200 lei/ton (without list cost);

— stocking-immobilization cost is 755 lei/ton; both stocking cost and immobilization cost were computed for a mean yearly stock of 100 tons (used in 1967);

— cost depending on material absence is estimated as 5,500 lei/ton-year;

— cost depending on surplus materials selling off is estimated as 193 lei per ton of stocked material; if we include in the calculation the selling off price, in P. Massé's acception, then it is 7,357 lei.

With a view to applying Massé's formula, one has first to verify if both its using conditions are satisfied. In the first variant, which assumes the inclusion of the material list cost, condition $g > a + b$ is not complied with, because $5{,}500 < 7{,}750 + 755$, but the second condition $a + b > c$ is complied with, as $7{,}750 + 755 > 7{,}357$. In the second variant, i.e. without list cost, both conditions $g > a + b$ and $a + b > c$ are complied with, as $(5{,}500 > > 200 + 755)$ and $(200 + 755 > 193)$, respectively. It is to be remarked that only the second variant of the formula can be applied.

In order to find the optimal value, we use relations (4) and (4'), then the optimum condition, expressed by relation (3). Thus, the surplus probability will be:

$$F(s) = \frac{g - (a + b)}{g - c} = \frac{5{,}500 - (200 + 755)}{5{,}500 - 193} = 0.85641$$

and the absence probability:

$$1 - F(s) = \frac{(a + b) - c}{g - c} = \frac{(200 + 755) - 193}{5{,}500 - 193} = 0.14359$$

The optimal stock should be where both cost groups are equal, according to relation (3). Applying this equation to the above costs, valid for 100-ton stock, it would follow that precisely this stock is the optimal one, as the equation is exactly verified:

$$200 + 755 = 5{,}500 \times 0.14359 + 193 \times 0.85641$$

Under these terms, one can also verify relation (5), which settles the optimal value of the same 100-ton stock:

$$\frac{0.85641}{0.14359} = \frac{5{,}500 - (200 + 755)}{(200 + 755) - 193}.$$

Here arises the problem if this stock is actually optimal, as the optimum conditions are also satisfied by other stock sizes. Then, which is the optimal stock? This is the deadlock of Massé's formula.

If *Kaufmann's formula is used*, the optimal stock is found by a simpler method, i.e. by comparing the absence coefficient to the values in the cumulative probability column, the latter being settled with the aid of empirical data of the methodology in Annex III.

The value of the absence coefficient is $\frac{g}{c + g} = \frac{5{,}500}{5{,}500 + 193} = 0.9661$. By comparing the absence coefficient with the cumulative probability values, we notice that the optimal stock lies between 50 and 70 tons (the last interval is not 10, but 20 tons, this being prescribed by the requirement of probability estimate calculation). In order to increase the stock size

accuracy in this interval, we perform an interpolation (assuming proportional increases), namely:

50 tons	0.8729	20 tons	0.1216
70 tons	0.9945 hence	x tons	0.0932

$$x = \frac{20 \times 0.0932}{0.1216} = 15.3 \text{ tons}$$

Hence the optimal material stock, according to Kaufmann's formula, is

$$50 + 15 = 65 \text{ tons (zinc).}$$

According to Kaufmann, the same result may be analytically obtained *by digital computation* in settling the total cost for each stock size (10; 20; 30; 40; 50; 70). The optimal stock will correspond to a minimum total cost, as follows:

$$C_{(s=10)} = 0 + 157{,}042 = 157{,}042 \text{ lei.}$$

$$C_{(s=20)} = 124 + 103{,}142 = 103{,}266 \text{ lei.}$$

$$C_{(s=30)} = 500 + 61{,}942 = 62{,}424 \text{ lei.}$$

$$\cup_{=40)} = 1{,}320 + 30{,}596 = 31{,}996 \text{ lei.}$$

$$C_{(s=50)} = 2{,}634 + 8{,}354 = 10{,}987 \text{ lei.}$$

$$C_{(s=70)} = 5{,}998 + 0 = 5{,}998 \text{ lei.}$$

The result obtained by analytical computation is hence quite close to the result deduced by simplified calculation, placing itself at the upper limit of range 50 — 70 tons.

Nevertheless, using these two formulas, very dissimilar results have been obtained, the first one being totally unconvincing.

FINDING THE OPTIMUM, ACCORDING TO MASSE'S FORMULA, USING KAUFMANN'S METHOD

1. *Presentation of the simplified calculation.* Taking into account the more analytic character of Massé's formula, we have attempted to use Kaufmann's simplified method, in order to obtain a new optimum condition in Massé's formula. Massé's formula was previously transposed in the situation of a discrete character consumption, as follows:

$$C_{(s)} = (a + b)s + g \sum_{r=s+1}^{\infty} (r - s)\, p(r) - c \sum_{r=0}^{s} (s - r)\, p(r) \qquad (10)$$

Just like with Kaufmann's method, for the optimum the following double inequality will be satisfied:

$$C_{(s_0-1)} > C_{(s_0)} < C_{(s_0+1)}$$

Since transformations lead, for the optimum, to a simplified relationship between costs, differing from Kaufmann's absence coefficient, we shall present the whole calculation of the relationship.

We begin by computing

$$C_{(s_0+1)} \text{ and } C_{(s_0-1)}:$$

$$C_{(s_0+1)} = (a+b)(s+1) + g\sum_{r=s+2}^{\infty}(r-s-1)\,p(r) -$$

$$-c\sum_{r=0}^{s+1}(s+1-r)\,p(r) = (a+b)(s+1) + g\sum_{r=s+1}^{\infty}(r-s-1)\,p(r) -$$

$$-g(s+1-s-1)p(s+1) - c\sum_{r=0}^{s}(s+1-r)\,p(r) - c(s+1-s-$$

$$-1)\,p(s+1) = (a+b)\,s + a + b + g\sum_{r=s+1}^{\infty}(r-s)\;p(r) - g\sum_{r=s+1}^{\infty}p(r) -$$

$$-c\sum_{r=0}^{s}(s-r)\,p(r) - c\sum_{r=0}^{s}p(r).$$

This leads to:

$$C_{(s_0+1)} = C_{s_0} + (g-c)\,p(r \leqslant s) + (a+b) - g \qquad (11)$$

The same transformations take place also for finding $C_{(s_0-1)}$. Thus:

$$C_{(s_0-1)} = (a+b)(s-1) + g\sum_{r=s}^{\infty}(r-s-1)\,p(r) -$$

$$-c\sum_{r=0}^{s-1}(s-1-r)\,p(r) = (a+b)(s-1) + g\sum_{r=s+1}^{\infty}(r-s+1)\,p(r) + g(s-$$

$$-s+1)\,p(s) - c\sum_{r=0}^{s}(s-1-r)\,p(r) + c(s-1-s)\,p(s) =$$

$$(a+b)s - (a+b) + g\sum_{r=s+1}^{\infty}(r-s)\,p(r) + g\sum_{r=s+1}^{\infty}p(r) + gp(s) -$$

$$-c\sum_{r=0}^{s}(s-r)\,p(r) + c\sum_{r=0}^{s}p(r) - cp(s) = C_{(s_0)} - (a+b) +$$

$$+g\left[1-\sum_{r=0}^{s}p(r)\right] + gp(s) + c\sum_{r=0}^{s}p(r) - cp(s) = C_{(s_0)} -$$

$$-(a+b) + g - g\sum_{r=0}^{s}p(r) + gp(s) + c\sum_{r=0}^{s+1}p(r) = C_{(s_0)} -$$

$$-g\sum_{r=0}^{s-1}p(r) + c\sum_{r=0}^{s-1}p(r) - (a+b) + g.$$

Hence:

$$C_{(s_0-1)} = C_{(s_0)} - (a+b) + g - (g-c)\, p(r \leqslant s-1). \tag{12}$$

As

$$C_{(s_0+1)} - C_{(s_0)} > 0,$$

it means that

$$(g-c)\, p(r \leqslant s) + (a+b) - g > 0,$$

hence

$$p(r \leqslant s) > \frac{g-(a+b)}{g-c}. \tag{13}$$

Equally as

$$C_{(s_0-1)} - C_{(s_0)} > 0,$$

it follows that

$$-p(r \leqslant s-1)(g-c) - (a+b) + g > 0$$

or

$$p(r \leqslant s-1) < \frac{g-(a+b)}{g-c}. \tag{14}$$

Relations (13) and (14) can also be written thus:

$$p(r \leqslant s-1) < \frac{g-(a+b)}{g-c} < p(r \leqslant s) \tag{15}$$

This double inequality allows to find the optimal stock under the conditions of the new solving. Here, the comparison relationship does not express any more the absence probability or the so-called absence coefficient $\frac{g}{g+c}$, but a surplus coefficient or the surplus probability $\frac{g-(a+b)}{g-c}$; this occurs when using the simplified calculation to solve Massé's formula.

Practical application. On using the same synthetical table containing zinc consumption probabilities and computing the surplus coefficient $\frac{g-(a+b)}{g-c} = \frac{5{,}500-(200-755)}{5{,}500-193} = 0.85641$, we can compare this coefficient with the consumption cumulative probability. There results an optimal stock ranging from 40 to 50 tons. Performing an interpolation, we get:

40 tons ..... 0.6808 10 tons 0.1921

50 tons ..... 0.8729 hence x tons 0.1756

$$x = \frac{1.756}{0.1921} = 9.1 \text{ tons.}$$

The optimal stock will be $40 + 9.1 = 49.1$ tons zinc.

We notice that by using an analytical (numerical) computation a minimum total cost is obtained, also in the assumption of a stock of about 50 tons—very close to the above optimum, namely:

$$C_{(s=10)} = (755 + 200)\ 10 - 193\ [(10 - 0)0 + (10 - 10)0.0643] +$$
$$+ 5{,}500[(20 - 10)\ 0.1306 + (30 - 10)0.2298 + (40 - 10)0.2561 +$$
$$+ (50 - 10)\ 0.1921 + (70 - 10)\ 0.1214 = 166{,}592 \text{ lei.}$$

$$C_{(s=20)} = (755 + 200)20 - 193\ [(20 - 10)0.0643] + 5{,}500[(30 - 20)$$
$$0.2298 + (40 - 20)0.2561 + (50 - 20)0.1921 + (70 - 20)0.1214 =$$
$$= 122{,}118 \text{ lei.}$$

$$C_{(s=30)} = (755 + 200)30 - 193[(30 - 10)0.0643 + (30 - 20)0.1306] +$$
$$+ 5{,}500\ [(40 - 30)0.2561 + (50 - 30)0.1921 + (70 - 30)0.1214 =$$
$$= 90{,}074$$

$$C_{(s=40)} = (755 + 200)40 - 193[(40 - 10)0.643 + (40 - 20)0.1306 +$$
$$+ (40 - 30)0.2298] + 5{,}500\ [(50 - 40)0.1921 + (70 - 40)0.1214] =$$
$$= 67{,}476$$

$$C_{(s=50)} = (755 + 200)\ 50 - 193[(50 - 10)0.0643 + (50 - 20)0.1306 +$$
$$+ (50 - 30)0.2298 + (50 - 40)0.2561] + 5{,}500[(70 - 50)0.1214] =$$
$$= 53{,}470 \text{ lei}$$

$$C_{(s=70)} = (755 + 200)70 - 193[(70 - 10)0.0643 + (70 - 20)0.1306 +$$
$$+ (70 - 30)0.2298 + (70 - 40)0.2561 + (70 - 50)0.1921 =$$
$$= 60{,}852 \text{ lei}$$

A 50-ton stock (near to 49) appears to be an optimal zinc stock as it assumes the lowest cost (53,470 lei).

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Daily zinc consumption in 1967

ANNEX I

Day \ Month	January	February	March	April	May	June	July	August	September	October	November	December
1	0	5.6	0	9.9	0	8.6	0	0.4	17.0	0	20.9	7.8
2	3.1	5.8	0.3	4.0	4.0	1.1	0	3.0	20.6	9.2	12.8	16.8
3	0.5	5.6	0	9.5	1.0	0.4	31.6	4.8	15.8	9.1	8.9	6.1
4	3.7	7.8	0	7.8	1.8	0.9	2.8	12.7	9.3	7.9	8.6	12.2
5	4.7	6.7	4.6	5.0	0	10.6	6.9	17.2	13.1	6.5	9.5	5.1
6	6.6	8.2	9.5	6.7	6.5	7.2	4.9	7.7	12.5	11.1	17.7	10.6
7	11.0	5.9	3.3	9.8	6.0	6.2	6.0	8.1	8.6	23.2	18.0	4.2
8	4.5	6.9	3.0	8.0	6.7	7.2	9.9	19.1	5.1	3.9	3.8	6.8
9	14.4	3.3	1.3	6.7	8.4	4.3	5.3	13.2	9.5	22.4	9.3	0
10	2.9	9.1	4.1	12.5	8.3	3.1	17.5	16.5	9.2	17.5	12.2	24.9
11	8.0	6.9	11.5	7.6	0.7	5.7	13.5	5.5	9.0	5.3	6.1	14.0
12	3.9	5.4	13.2	9.9	0	14.4	4.6	9.7	5.1	3.0	5.6	10.0
13	3.7	7.4	13.7	11.1	7.3	13.9	4.8	7.1	8.6	10.6	7.1	22.8
14	2.9	20.1	4.7	13.9	5.7	2.9	5.3	1.7	2.2	11.8	25.6	10.7
15	0.1	4.1	11.0	12.2	1.8	0.7	7.9	10.5	5.6	4.4	4.5	9.2
16	1.9	9.5	6.3	5.2	3.0	4.9	6.5	11.0	9.9	32.3	0	21.1
17	0.8	10.7	6.4	11.7	5.9	7.2	0.7	5.4	13.9	22.0	0	0
18	2.6	11.5	8.5	0	19.9	5.2	13.3	3.2	19.3	10.1	0	9.8
19	6.6	9.2	5.6	5.2	27.3	0	13.6	3.1	20.7	3.5	5.5	11.3
20	5.9	4.3	6.3	12.6	5.4	3.2	1.0	7.4	14.1	7.3	5.8	8.9
21	6.2	2.1	4.2	14.2	0	1.6	5.1	6.0	13.7	0	9.9	3.6
22	0.6	0	8.5	9.7	0	0.7	9.2	14.5	3.9	0	16.2	2.7
23	0	5.8	8.0	1.9	0	5.4	7.8	0	5.7	10.5	6.1	10.2
24	3.0	16.9	8.0	10.1	10.5	18.0	8.1	3.9	6.4	7.6	8.3	11.7
25	18.2	22.1	9.7	2.9	6.7	7.6	8.8	11.1	4.8	5.3	7.1	17.8
26	11.1	18.3	0.7	6.0	12.7	7.9	12.4	12.3	8.8	2.9	5.7	2.5
27	24.3	9.1	1.6	0	12.4	6.9	9.7	11.0	8.3	5.0	20.8	0
28	15.8	2.4	8.2	8.8	10.1	29.2	9.5	4.3	6.8	8.6	14.3	3.0
29	22.5	—	17.7	16.3	12.8	14.2	4.4	15.2	5.5	0	0	1.7
30	15.6	—	6.5	13.1	22.6	25.7	12.9	10.3	6.3	14.7	1.9	0
31	12.5	—	33.0	—	13.6	—	22.3	10.7	—	0	—	0

ANNEX II

Zinc consumption per equal successive periods (4 days)

Month / No. of Period	January	February	March	April	May	June	July	August	September	October	November	December
1	7.3	25.9	4.9	29.0	6.8	13.0	57.3	26.9	58.8	24.6	51.2	26.2
2	26.8	24.3	17.1	37.0	19.2	23.9	20.6	52.1	35.7	48.7	49.0	34.0
3	29.2	28.8	42.3	42.5	17.4	37.0	46.2	44.9	30.9	49.1	33.2	35.9
4	8.6	44.4	28.4	29.1	17.8	21.7	22.6	30.3	31.0	29.8	37.2	57.5
5	15.9	27.1	24.6	41.7	58.5	10.0	34.1	19.1	47.8	67.9	11.3	40.1
6	10.8	44.8	34.2	20.9	10.5	31.7	23.1	24.4	20.8	17.8	40.5	26.5
7	69.4	30.8	28.2	39.2	41.9	58.2	39.0	38.6	29.4	20.8	47.9	42.2
8	56.2	—	53.4	—	57.6	—	49.1	53.2	24.6	23.3	—	4.7

$n = 92$ periods, $r = 32.9$ tons and $\sigma = 15$ tons

ANNEX III

Computation of zinc consumption probability, assuming equal periods and verifying the probability law

Number of interval	Consumption intervals, according to sizes $r_{min} \rightarrow r_{max}$	Group empirical frequency n	Upper limit normed value $z = \frac{r_{max} - r}{\sigma}$	Distribution function value (consumption cumulative probability) $P = p(r \leqslant s)$	Consumption (demand) simple probability $P_{(r)} = P_{(n+1)} - P_{(n)}$	Elements for verifying the probability law	
						Theoretical frequency $n_t = p_{(r)} \times \Sigma n$	Weighted deviation $\frac{(n - n_t)^2}{n_t}$
1	0—10	5	—1.5266	0.0643	0.0643	5.9	0.14
2	10—20	12	—0.8606	0.1949	0.1306	12.0	0
3	20—30	28	—0.1933	0.4247	0.2298	21.1	2.25
4	30—40	17	0.4737	0.6808	0.2561	23.1	1.61
5	40—50	17	1.1407	0.8729	0.1921	17.7	0.27
6	50—60	11	2.5417	0.9945	0.1216	11.2	0.29
7	60—70	2					
—	—	92	—	—	—	—	4.56

Mean 32.9; mean quadratic deviation 15.0 (exactly 14.99)

$$\chi^2_v = \begin{cases} 9.49 \rightarrow 0.05 \\ 13.3 \rightarrow 0.01 \end{cases}$$

DEUX FORMULES POUR DETERMINER LE STOCK TOTAL DE MATERIAUX SELON LE CRITERE DES COUTS

Les auteurs examinent de façon critique deux formules pour déterminer le stock total de matériaux utilisant le critère des coûts et essaient leur application dans une entreprise afin de saisir les difficultés pratiques d'utilisation.

Tant la formule de Massé que celle de Kaufmann, considèrent le coût moyen rapporté à la grandeur du stock. Tous les coûts peuvent être journellement déterminés, par an ou pour l'interval entre deux livraisons, et au point de vue de la quantité, pour tout le stock ou pour une seule unité de celui-ci.

Par l'emploi des deux formules à la détermination du stock optimum de zinc raffiné à l'Usine Métalurgique de Jassy, utilisant les données de 1967, on a obtenu toutefois des résultats très différents. Les auteurs s'efforcent de déterminer le niveau optime selon la formule de Massé et selon le procédé simplifié utilisé par Kaufmann. Au préalable, la formule de Massé a été transposée dans la situation d'une utilisation (consommation) à caractère discret. On a obtenu une nouvelle condition d'optimum que les auteurs considèrent satisfaisante.

О ДВУХ ФОРМУЛАХ ОПРЕДЕЛЕНИЯ ОБЩЕГО ЗАПАСА МАТЕРИАЛОВ ПО КРИТЕРИЮ СЕБЕСТОИМОСТИ

Авторы критически рассматривают две формулы определения общего запаса материалов на основе критерия себестоимости и пытаются применить их на предприятии для выявления практических трудностей при применении.

Как формула Массэ, так и формула Кауфманна имеют в виду среднюю себестоимость в связи с величиной запаса. Любая себестоимость может быть определена за день, за промежуток времени между двумя поставками или за год, а в количественном отношении — по всему запасу или на единицу запаса.

В результате применения обеих формул при определении оптимального запаса рафинированного цинка на Ясском металлургическом заводе на основе данных за 1967 г. были все же получены весьма различные результаты. Авторы пытаются определить оптимальный уровень по формуле Массэ на основе упрощенного способа, применяемого Кауфманом. Предварительно формула Массэ была приспособлена к положению потребления дискретного характера. Было получено новое условие оптимума, которое авторы считают удовлетворительным.