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Simulation Convergence in a Class of Economic Systems

Systems theory and simulation are inseparable.

In order to characterize the class of economic systems referred to in this paper, we shall take a simple example, namely the production system currently described through the relationship

$$Y = f(L, K) \tag{1}$$

where:

Y = the social product (assuming that multiple entries among the branches have been removed),

L = manpower,

K = means of production.

The function f, termed the production function, has been extensively investigated and a great number of results have been published in the literature on this subject, a considerable part of these results aiming at obtaining an effective computation tool.

In this paper, however, we intend to discuss some suggestions concerning some basic concepts related to economic systems and to the simulation of a certain class of systems; therefore, starting from relation (1), we shall attempt some generalizations.

The application of relation (1) is commonly considered as having the domain in the cartesian product $L \times K$, where L and K are sets, and as taking values in Y, which is also a set. We can, however, consider the case where manpower together with the means of production can be described by class A, while the ultimate social product is described by class B. Then, a function F from A to B is a sub-class of the cartesian product $A \times B$,

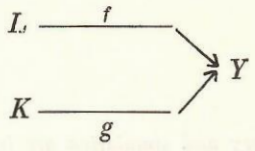
such that for each $X \in A$, there exists a small $Y \in B$, for which $(X, Y) \in F$ is true. Class A is called the domain of function F, and the class including the values function F can assume:

$$\{Y \mid Y \in B \text{ and } (X, Y) \in F \text{ for any } X \in A\}.$$

If the domain is a set, it follows that the domain of values is also a set and, as a result, we obtain the previous case.

Facts derived from actual practice show, however, that the concept of function does not incorporate the high complexity of phenomena which occur in economic systems. This is why new concepts have emerged during the last few years, such as the morphisms in the various classes of economic systems, some of them being known in the literature as ε-morphisms.

Let us consider once again relation (1), as follows:



(2)

where f and g are the morphisms f: $L \to Y$, which is to be read, f is a morphism of source L and address Y; similarly, g: $K \to Y$, i.e. g is a morphism of source K and address Y. Now, if we consider L and K as being sets belonging to a class I and Y as a set belonging to a class O, it becomes possible to represent (2) in a more generalized form, such as:

$$I \xrightarrow{\ h\ } O \tag{3}$$

where h: $I \to O$ is a morphism of source I and address O. In view of the system theory it is more suitable to term I the input and O the output, consequently, h is a morphism of input I and output O.

The question arising now is if economic systems of type (1) are suitably characterized by an input, an output and a sole morphism, i.e. $I \to O$. Actual practice shows that neither is this outline general enough, and we shall therefore consider the systems such as:

$$I \xrightarrow{\ S(I,\,O)\ } O \tag{4}$$

where $S(I, O)$ is a set, namely the set of morphisms of input I and output O.

All economic systems which can be represented by relation (4) make up a class, which we shall designate class $I\mathcal{M}O$, pointing out that morphisms $I \to O$ make up a set.

A triplet,

$$(I,\ S(I, O), O) \tag{5}$$

can be interpreted as a category, whose objects are I and O, while the morphisms make up set $S(I, O)$, on condition that identity morphisms $1_I: I \to I$ and $1_0: O \to O$ also exist. The composition of morphisms is obvious in this particular case. A triplet such as (3):

$$(I,\ h,\ O) \tag{6}$$

can hence be considered a sub-category of category (5), or, in terms of the system theory, one of the subsystems of system (4).

We can now introduce a new concept, namely a category of systems belonging to class $I\mathcal{MO}$. We consider as objects of category $\mathcal{S}$ the class of triplets

$$\langle I,\ S(I,O), O\rangle \tag{7}$$

The composition of morphisms in $\mathcal{S}$ occurs as follows: given two systems $\langle I, S(I, O), O\rangle$ and $\langle I', S(I', O'), O'\rangle$, a morphism from $\langle I, S(I, O), O\rangle$ to $\langle I', S(I', O'), O'\rangle$ will be a couple (α, β) with α: $I \to I'$ and β: $O \to O'$ and endowed with commutativity of diagram:

$$\begin{array}{ccc} I & \xrightarrow{\alpha} & I' \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f'} \\ O & \xrightarrow{\beta} & O' \end{array} \tag{8}$$

for any $f \in S(I, O)$ and $f' \in S(I', O')$. The composition of morphisms is then as follows:

$$\langle I, S(I,O), O\rangle \xrightarrow{(\alpha,\beta)} \langle I', S(I', O'), O'\rangle \xrightarrow{(\alpha',\beta')} \langle I'', S(I'', O''), O''\rangle$$

We have the commutative diagram:

$$\begin{array}{ccccc} I & \xrightarrow{\alpha} & I' & \xrightarrow{\alpha'} & I'' \\ {\scriptstyle f}\downarrow & & {\scriptstyle f'}\downarrow & & \downarrow{\scriptstyle f''} \\ O & \xrightarrow{\beta} & O' & \xrightarrow{\beta'} & O'' \end{array} \tag{9}$$

Morphisms'composition is associative, since

$$(\alpha'', \beta'') \circ [(\alpha', \beta') \circ (\alpha, \beta)] = (\alpha'', \beta'') \circ (\alpha' \circ \alpha, \beta' \circ \beta) = (\alpha'' \circ \alpha' \circ \alpha.$$

$$\beta'' \circ \beta' \circ \beta) = (\alpha'' \circ \alpha', \beta'' \circ \beta') \circ (\alpha, \beta) = [(\alpha'', \beta'') \circ (\alpha', \beta')] \circ (\alpha, \beta)$$

The identity morphism is defined as

$$(1_I,\ 1_O)\colon \langle I,\ S(I,O),\ O\rangle \to \langle I.\ S(I.\ O).\ O\rangle$$

and we have:

$$(1_I, 1_O) \circ (\alpha, \beta) = (1_I, \circ \alpha, 1_O \circ \beta) = (\alpha, \beta)$$

$$(\alpha, \beta) \circ (1_I, 1_O) = (\alpha \circ 1_I, \beta \circ 1_O) = (\alpha, \beta)$$

For simplification purposes and if it does not lead to confusion, in the following paragraphs we shall denote the triplets by (S) and the morphism from a system to another one by:

$$(S) \xrightarrow{(\alpha, \beta)} (S') \tag{10}$$

consequently, $(S) \in Ob\mathcal{S}$.

Let (S_0) be the system to be simulated. We then assume that (S_0) is a system on which direct tests are either impossible, or unfeasible, or too expensive, etc. The simulator is another system (S_1) on which any tests can be carried out; "simulator" is meant any system allowing a derivable systematization — such as exposition, prediction and postdiction — to be effected on the specific object system. This extension of the simulator concept is necessary because a simulator can only be used with a view to obtaining the scientific explanation; since the scientific explanation is endowed with predictive potential, it follows that the simulator can also be used for this purpose. Yet, there are instances when postdiction is of interest, i.e. starting from the present condition of a system, one infers its original condition, in other words, the course of its past behaviour is traced by means of present data.

Within this concept, any system included in class $I\mathcal{MO}$, which might be viewed as a simulator, is the address of a couple of morphisms, having as a source (S_0), or:

$$(S_0) \xrightarrow{(\alpha_1, \beta_1)} (S_1). \tag{11}$$

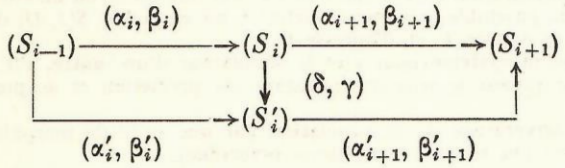
A deductive systematization is sought through investigating the system of morphisms (α_1, β_1); this is not usually possible at the first attempt, and as a result, it is necessary to proceed from (S_1) to (S_2) and so forth. A chain of morphisms is thus obtained — which are obviously finite — such as in the following diagram:

$$(S_0) \xrightarrow{(\alpha_1, \beta_1)} (S_1) \xrightarrow{(\alpha_2, \beta_2)} \ldots \xrightarrow{(\alpha_n, \beta_n)} (S_n) \tag{12}$$

The problem we are facing now is to specify (S_n), the simulator which completes the simulation process. It is to be inferred that this is the last simulator, since it allows the required deductive systematization to be made. This happens only if (S_n) is equivalent to an object system (S_0). Through the equivalence of systems, $(S_0) \sim (S_n)$, it is meant that there exists a couple of isomorphisms $(\alpha_{n+1}, \beta_{n+1})$ with the signification that the two isomorphisms are reversible — i.e., simultaneously sectionable and rectractable — such that

$$(\alpha_{n+1} \circ \alpha_{n+1}^{-1}, \quad \beta_{n+1} \circ \beta_{n+1}^{-1}) = (1_{I_0}, 1_{O_0}) : (S_0) \to (S_0)$$

In general, any of the systems (S_i) could be isomorphic with other systems. We shall assume, for instance, that $(S_i) \sim (S'_i)$. In this case we obtain the commutative diagram:



It is derived from this diagram that:

$$(\alpha'_{i+1} \circ \delta \circ \alpha_i, \beta'_{i+1} \circ \gamma \circ \beta_i) = (\alpha_{i+1} \circ \delta^{-1} \circ \alpha'_i, \beta_{i+1} \circ \gamma^{-1} \circ \beta'_i).$$

We also have, however:

$$(\delta \circ \alpha_i, \gamma \circ \beta_i) = (\alpha'_i, \beta'_i).$$

Furthermore,

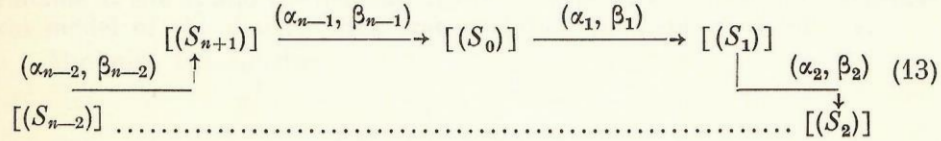
$$(\delta^{-1} \circ \alpha'_i, \gamma^{-1} \circ \beta'_i) = (\alpha_i, \beta_i).$$

We finally derive from the above relation:

$$(\alpha'_{i+1} \circ \alpha'_i, \beta'_{i+1} \circ \beta'_i) = (\alpha_{i+1} \circ \alpha_i, \beta_{i+1} \circ \beta_i).$$

Consequently, it is necessary that no system isomorphic with any other exists in chain (12), a conclusion which leads to the selection of systems (S_i) as the corresponding representatives of the sub-classes (S_i) of isomorphic systems. We also can resort to another way for expressing the same requirement, namely that chain (12) is built up on a frame of category $\mathcal{S}$.

Since we have isomorphism $(S_n) \sim (S_0)$, it follows that the chain is closed in a circuit by means of the isomorphisms $(\alpha_{n+1}, \beta_{n+1})$, that is in the sub-class made up of systems isomorphic with (S_0), and we get the diagram:



(13)

To conclude, we can state that under these circumstances, the simulation process is convergent if the circuit of morphisms illustrated in diagram (13) starts from the given object system and is closed in the sub-class of systems isomorphic with the same object system. This is the convergence condition for systems belonging to class $I\mathcal{MO}$.

LA CONVERGENCE DE LA SIMULATION D'UNE CLASSE DE SYSTEMES

On prend en considération les systèmes décrits par une classe ou un ensemble I, nommé entrée, une classe ou un ensemble O, nommé sortie et un ensemble $S(I, O)$ dont les éléments sont des morphismes de source I et d'adresse O.

On démontre qu'un système peut être le simulateur d'un autre, s'il permet une systématisation déductive (ayant le sens d'explication, de prédiction et de postdiction) sur un système-objet donné.

On définit la convergence de la simulation par une suite de morphismes qui aboutissent à un isomorphisme (au sens de morphisme reversible).

СХОДИМОСТЬ СИМУЛИРОВАНИЯ КЛАССА ЭКОНОМИЧЕСКИХ СИСТЕМ

Принимаются в расчет системы, описываемые классом или множеством I, называемым входом, классом или множеством, называемым выходом, и множеством $S(I, O)$, элементы которого являются морфизмами источника I и адреса O.

Указывается, что одна система может быть симулятором другой, если она делает возможной дедуктивную систематизацию (в отношении объяснения, предвидения и констатации) данной системы-объекта.

Определяется сходимость симулирования при помощи ряда морфизмов, ведущих, в конечном счете, к изоформизму (в смысле обратимого морфизма).