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Computer Program for the Estimation of Trade Matrix*

The development of knowledge and prediction requires the use of appropriate means in view of synthesizing earlier information.

1. INTRODUCTION

The export (or import) trade matrix of a particular country is a matrix in which the rows represent the commodities exported from (or imported to) the country, the columns the importing (or exporting) countries and each cell the amount in thousands of dollars of such an operation (see Annex II). Some countries report only, at first, the marginal totals of their exports or imports and eventually, later on, the internal elements of the matrix. For some economic studies and for the editing of statistical tables it is necessary to know these elements as soon as possible; therefore the problem is to estimate the trade matrix for the current year, using the previous year's trade matrix and current year marginal totals **). In a further step some of the cells of the matrix can be derived from, for instance, imports or exports of more developed countries and added to the available data to improve the estimation.

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**) The problem is often called problem of Frechet, since Professor Frechet studied it and in particular gave necessary and sufficient conditions of existence for the estimated matrix [2], [3]. An application of the problem in this context was suggested by Mr Al-Atraqchi, of the UN International Trade centre, who has also done some work on this subject.

2. NOTATION

x_{ij} = elements of the matrix of the previous year (given)
$x_{i.}$ = marginal totals of the previous year (by commodity)
$x_{.j}$ = marginal totals of the previous year (by country)
y_{ij} = element of the current year matrix to be estimated
$y_{i.}$ = marginal totals of the current year (given)
$y_{.j}$ = marginal totals of the current year (given)
$y_{ij}^{(k)}$ = kth estimation of y_{ij}
m = number of commodity (row number)
n = number of the country of destination (or provenance) (column number)

3. FIRST APPROACH: THE DIFFERENCE MINIMIZATION METHOD

3.1. *Principles*

The estimation is made in two steps. We compute a first estimate of each of the elements of the matrix as a function of the previous year's elements and the current year's marginal totals:

$$y_{ij}^{(1)} = f(x_{ij},\ y_{i.},\ y_{.j})$$

In a second step, in order that each row and each column add up to their corresponding marginal totals $y_{i.}$ and $y_{.j}$, we will compute a new estimation $y_{ij}^{(2)}$. Each element of the $m \times n$ matrix is considered as one of the $m \times n$ components of a vector X belonging to a vectorial space $E^{m \times n}$ and let $d(X, Y)$ be a function of the difference between two vectors X and Y.

Let $Y^{(1)}$ be the vector of components $y_{ij}^{(1)}$ and $Y^{(2)}$ the vector of components $y_{ij}^{(2)}$ final estimate of y_{ij}.

The problem is to minimize $d(Y^{(1)},\ Y^{(2)})$ subject to the constraints $\sum_j y_{ij}^{(2)} = y_{i.}$ and $\sum_i y_{ij}^{(2)} = y_{.j}$

3.2. *Choice of f and d*

A simple choice of f, which assumes independence between countries and commodities is:

$$f = y_{i.}\, y_{.j}$$

but we can improve on the prediction by taking into account the information given by the previous years' matrix. Usually there is no independence between i and j. From the previous year's data we can see that we have either $x_{ij} > x_{i.} \cdot x_{.j}$ or $x_{ij} < x_{i.} \cdot x_{.j}$, and if we assume that the deviation from the independence pattern is the same over a year, a possible

representation of f is *):

$$f = \frac{y_{i.}\, y_{.j}}{x_{i.}\, y_{\cdot j}}\, x_{ij}$$

The function $d\,(Y^{(1)}, Y^{(2)})$ can be chosen as the sum of the squares of the differences between the components of $Y^{(1)}$ and $Y^{(2)}$:

$$d = \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij}^{(2)} - y_{ij}^{(1)})^2$$

d is a positive definite quadratic form and hence is a convex function over the closed convex set Y in $E^{m \times n}$; therefore any relative optimum of d in Y is also the absolute optimum of d over Y and this optimum is a minimum.

The problem is to minimize the function:

$$d = \sum_{i} \sum_{j} (y_{ij}^{(2)} - y_{ij}^{(1)})^2$$

subject to $m + n - 1$ independent constraints:

$$u_i = \sum_{j} y_{ij}^{(2)} - y_{i.} \qquad i = 1.\,2, \ldots, m$$

$$v_j = \sum_{i} y_{ij}^{(2)} - y_{.j} \qquad j = 1.\,2, \ldots, n-1$$

since there is a relationship between the two sets of constraints:

$$\sum_{i} y_{i.} = \sum_{j} y_{.j}$$

Using the Lagrange's multipliers method to solve the minimization problem, we construct the following expression:

$$d + \sum_{i=1}^{m} \lambda_i u_i + \sum_{j=1}^{n-1} \mu_j v_j \tag{3.1}$$

or

$$\sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij}^{(1)} - y_{ij}^{(1)})^2 + \sum_{i=1}^{m} \lambda_i \left(\sum_{j} y_{ij}^{(2)} - y_{i.}\right) + \sum_{j=1}^{n-1} \mu_j \left(\sum_{i} y_{ij}^{(2)} - y_{.j}\right)$$

where $y_{ij}^{(2)}$ = unknown final estimate of y_{ij}
$y_{ij}^{(1)}$ = known first estimate of y_{ij}
$y_{i.}$ = given marginal totals of y_{ij}
$y_{.j}$ = given marginal totals of y_{ij}

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* Henri Theil [9] proposes this function for the prediction of international trade flow from the ith region to the jth region.

λ_i = Lagrange multipliers

μ_j = Lagrange multipliers

We then differentiate this expression with respect to $y_{ij}^{(2)}$ and set the $m \times n$ derivatives equal to zero:

$$\begin{aligned} 2y_{ij}^{(2)} - 2y_{ij}^{(1)} + \lambda_i + \mu_j &= 0; \quad i = 1, \ldots, m; \quad j = 1, \ldots, n-1 \\ 2y_{ij}^{(2)} - 2y_{ij}^{(1)} + \lambda_i \quad &= 0; \quad i = 1, \ldots, m; \quad j = n \end{aligned} \tag{3.2}$$

or

$$\begin{aligned} y_{ij}^{(2)} &= y_{ij}^{(1)}(1 + \alpha_i + \beta_j); \quad i = 1, \ldots, m; \quad j = 1, \ldots, n-1 \\ y_{ij}^{(2)} &= y_{ij}^{(1)}(1 + \alpha_i) \quad ; \quad i = 1, \ldots, m; \quad j = n \end{aligned} \tag{3.3}$$

where

$$\alpha_i = \frac{-\lambda_i}{2y_{ij}^{(1)}} \quad \text{and} \quad \beta_j = \frac{-\mu_j}{2y_{ij}^{(1)}}$$

are the new multipliers. Then we sum the $y_{ij}^{(2)}$ with respect to i and j:

$$\begin{aligned} \sum_i y_{ij}^{(2)} &= \sum_i (y_{ij}^{(1)}(1 + \alpha_i + \beta_j)); \quad j = 1, \ldots, n-1 \\ \sum_j y_{ij}^{(2)} &= \sum_j (y_{ij}^{(1)}(1 + \alpha_i + \beta_j)); \quad i = 1, \ldots, m \end{aligned} \tag{3.4}$$

or

$$\begin{aligned} y_{\cdot j} - \sum_i y_{ij}^{(1)} &= \sum_i \alpha_i y_{ij}^{(1)} + \beta_i \sum_i y_{ij}^{(1)}; \quad j = 1, \ldots, n-1 \\ y_{i\cdot} - \sum_j y_{ij}^{(1)} &= \alpha_i \sum_j y_{ij}^{(1)} + \sum_j \beta_j y_{ij}^{(1)}; \quad i = 1, \ldots, m \end{aligned} \tag{3.5}$$

or in matrix notation:

$$B = A \cdot Z \tag{3.6}$$

where

$$A = \begin{vmatrix} \sum_j y_{ij}^{(1)} & 0 & \ldots\ldots & 0 & y_{11}^{(1)} & y_{12}^{(1)} & \ldots\ldots & y_{1n-1}^{(1)} \\ 0 & \sum_j y_{2j}^{(1)} & \ldots\ldots & 0 & y_{21}^{(1)} & y_{22}^{(1)} & \ldots\ldots & y_{2n-1}^{(1)} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ y_{11}^{(1)} & y_{21}^{(1)} & \ldots\ldots & y_{m1}^{(1)} \sum_i y_{i1}^{(1)} & 0 & \ldots\ldots & & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ y_{1n-1}^{(1)} & y_{2n-1}^{(1)} & \ldots\ldots & y_{mn-1}^{(1)} & 0 & 0 & \ldots\ldots & \sum_i y_{in-1}^{(1)} \end{vmatrix}$$

$$Z = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ m \\ 1 \\ \vdots \\ n-1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} y_{1.} - \sum_j y_{1j}^{(1)} \\ y_{2.} - \sum_j y_{2j}^{(1)} \\ \vdots \\ y_{n-1.} - \sum_j y_{n-1j}^{(1)} \end{bmatrix}$$

We may solve for Z and get the values of α_i and β_j. Thus from equations (3.1) we compute every $y_{ij}^{(2)}$.

4. SECOND APPROACH: THE RAS METHOD

4.1. *Principles*

This method has been used by R. Stone and A. C. Brown to analyze the variation of technical coefficient of an Input-Output matrix [6].

The basic hypothesis of this method is the variation of the matrix by double proportionality, from the previous year to the current year. We assume then a relative inertia of the system where the amount of exchange varies from the previous year to the current year but without any drastic change in the distribution by country and commodity.

The method consists in having a set of matrices $Y^{(0)}$, $Y^{(1)}$, ..., $Y^{(n)}$ where the initial matrix $Y^{(0)}$ is the given matrix of the previous year. The limit of the set is the solution of the problem. W. Gorman proved [4] the convergence toward a unique solution, but P. Thionet has shown [11] that the limit matrix does not exist necessarily and that the RAS algorithm may not converge.

4.2. *The RAS algorithm*

The matrix $Y^{(1)}$ is derived from the previous year's matrix X or $Y^{(0)}$ by the following equation:

$$y_{ij}^{(2p+1)} = y_{ij}^{(2p)} \frac{y_{i.}^{(2p+1)}}{y_{i.}^{(2p)}} \tag{4.1}$$

where $p = 0$

$y_{ij}^{(0)} = x_{ij}$

$y_{i.} = x_{i.}$

Having once adjusted each element with the marginal values of each row, we thus compute the marginal total of each column $y_{.j}^{(2p+1)}$, here $p = 0$, and compare them with the given $y_{.j}$ of the current year. If any discre-

pancy arises, a new matrix $Y^{(2)}$ is set up in adjusting each element with the marginal total of each column by the following formula:

$$y_{ij}^{(2p+2)} = y_{ij}^{(2p+1)} \frac{y_{.j}^{(2p+2)}}{y_{.j}^{(2p+1)}} \tag{4.2}$$

where $p = 1$

Then the marginal totals of each row $y_{i.}^{(2)}$ are computed and compared with $y_{i.}$. If any discrepancy arises, a third matrix $Y^{(3)}$ is created using equation (4.1) with $p = 2$ and so on until the match is obtained. The last computed matrix is the solution.

5. CASE WHERE SOME OF THE CURRENT YEAR MATRIX CELLS ARE GIVEN

We have assumed, until now, that only the marginal totals of the current year matrix were known. In this section we shall assume that in addition, some of the cells of the matrix are known from different sources; for instance from international trade statistics of more developed countries than the reporting country.

The following modifications will take into account the new information to improve the estimation.

Let y_{ij} with $i = i_1, i_2, ..., i_q$ and $j = j_1, j_2, ..., j_r$ be the given data of the current year matrix ($q \leqslant m$, $r \leqslant n$).

5.1. *Difference minimization method*

The Lagrangean expression (3.1) will become:

$$d + \sum_{i=1}^{m} \lambda_i u_i + \sum_{j=1}^{n-1} \mu_j v_j + \sum_{i=i_1}^{iq} \sum_{j=j_1}^{jr} \nu_{ij} w_{ij} \tag{5.1}$$

where $w_{ij} = y_{ij}^{(2)} - y_{ij} = 0$ for $i = i_1, ..., i_q$ and $j = j_1, ..., j_r$ and ν_{ij} a third set of Lagrange multipliers.

The two systems of equations (3.3) will be replaced by four systems of equations:

$$\begin{aligned} y_{ij}^{(2)} &= y_{ij}^{(1)} \ (1 + \alpha_i + \beta_j) && \text{for } j \neq n \\ y_{ij}^{(2)} &= y_{ij}^{(1)} \ (1 + \alpha_i) && \text{for } j = n \end{aligned} \qquad \text{for } i \neq i_1, ..., i_q \text{ and } j \neq j_1, ..., j_r \tag{5.2}$$

$$\begin{aligned} y_{ij}^{(2)} &= y_{ij}^{(1)} \ (1 + \alpha_i + \beta_j + \gamma_{ij}) && \text{for } j \neq n \\ y_{ij}^{(2)} &= y_{ij}^{(1)} \ (1 + \alpha_i + \gamma_{ij}) && \text{for } j = n \end{aligned} \qquad \text{for } i = i_1, ..., i_q \text{ and } j = j_1, ..., j_r$$

The summation with respect to i and j will give, instead of (3.4):

$$\begin{aligned} y_{i.} - \sum_j y_{ij}^{(1)} &= \alpha_i \sum_j y_{ij}^{(1)} + \sum_j \beta_j y_{ij}^{(1)} + \varepsilon_i \sum_{j=j_1}^{j_r} \gamma_{ij} y_{ij}^{(1)} \\ y_{.j} - \sum_i y_{ij}^{(1)} &= \sum_i \alpha_i y_{ij}^{(1)} + \beta_j \sum_j y_{ij}^{(1)} + \varepsilon_i \sum_{i=i_1}^{i_q} \gamma_{ij} y_{ij}^{(1)} \end{aligned} \tag{5.3}$$

where $\varepsilon_i = 0$ if $i \neq i_1, \ldots, i_q$ and $\varepsilon_j = 0$ if $j \neq j_1, \ldots, j_r$

$\varepsilon_i = 1$ if $i = i_1, \ldots, i_q$ and $\varepsilon_j = 1$ if $j = j_1, \ldots, j_r$

The product $\gamma_{ij} y_{ij}^{(1)}$, when it exists in (5.3), will be replaced by its value from (5.2):

$$\gamma_{ij} y_{ij}^{(1)} = y_{ij}^{(2)} - y_{ij}^{(1)} (1 + \alpha_i + \beta_j)$$

or, more precisely, since we know the real value y_{ij}:

$$\gamma_{ij} y_{ij}^{(1)} = y_{ij}^{(2)} - y_{ij}^{(1)} (1 + \alpha_i + \beta_j) \tag{5.4}$$

The solution of the new system composed of (5.3) and (5.4) will give the values of α_i, β_j and γ_{ij} and, from (5.2), we will get the final estimation $y_{ij}^{(2)}$ of the non-given y_{ij}.

If for instance the given values y_{ij} are some entire columns of the trade matrix, this will lead to the disappearance of some of the columns and rows of the working matrix to be inverted for the solution of the above system. The matrix is thus singular, its determinant equal to zero and it cannot be inverted. We therefore reduce the matrix, in order to get only non-zero rows and columns and proceed as before.

5.2. *The RAS method*

Equation (4.1) is replaced by two equations:

$$y_{ij}^{(2p+1)} = y_{ij} \qquad \text{for } i = i_1, \ldots, i_q \text{ and } j = j_1, \ldots, j_r$$

$$y_{ij}^{(2p+1)} = y_{ij}^{(2p)} \frac{y_{i.} - \sum_{j=j_1}^{j_r} y_{ij}}{y_{i.} - \sum_{j=j_1}^{j_r} y_{ij}^{(2p)}} \quad \text{for } i \neq i_1, \ldots, i_q \text{ and } j \neq j_1, \ldots j_r$$

and equation (4.2) by:

$$y_{ij}^{(2p+2)} = y_{ij} \qquad \text{for } i = i_1, \ldots, i_q \text{ and } j = j_1, \ldots, j_r$$

$$y_{ij}^{(2p+2)} = y_{ij}^{(2p+1)} \frac{y_{.j} - \sum_{i=i_1}^{i_q} y_{ij}}{y_{.j}^{(2p+1)} - \sum_{i=i_1}^{i_q} y_{ij}^{(2p+1)}} \quad \text{for } i = i_1, \ldots, i_q \text{ and } j = j_1, \ldots, j_r$$

and the algorithm is the same as before.

6. MEASURE OF INEQUALITY

To analyze the accuracy of the estimation, we have computed Theil's inequality coefficient [7], [13], [14], which is also used by the European Economic Community.

$$Q = \sqrt{\frac{\sum_i \sum_j (y_{ij}^{(k)} - y_{ij})^2}{\sum_i \sum_j y_{ij}^{(k)} 2 + \sum_i \sum_i y_{ij}^2}}$$

where $y_{ij}^{(k)}$ are the estimations and y_{ij} the corresponding actual values.

The advantage of this coefficient over some others is that, no matter whether the estimation or the actual value of one of the cells is equal to zero, the coefficient can still be computed.

The main caracteristics of Theil's coefficient are the following:

- $0 \leqslant Q \leqslant 1$
- $Q = 0$ if and only if $y_{ij}^{(k)} = y_{ij}$ whatever i and j are;
- $Q = 1$ if $y_{ij}^{(k)} = 0$ for any i and j, whatever the y_{ij}'s are;
- if $y_{ij} = 0$ for any i and j, whatever the $y_{ij}^{(k)}$'s are;
- if $y_{ij}^{(k)} = -hy_{ij}$ with $h > 0$ for any i and j.

7. PROGRAM MATRA-2

The features of the two methods, minimization of difference and the RAS method, as described in section 3 and 4, have been incorporated in the program called MATRA-2 which, in addition, used the modified equations of section 5, if the necessary information is given.

The program is written in FORTRAN IV and has been tested on the IBM 7044 of the United Nations International Computing Centre. It is composed of a main program and three sub-routines, ARRAY, MINV, and MAPRT.

The main program reads the input data and prints them. Then, depending on the user's choice, it switches to either the distance minimization method or the RAS method. If the first method is chosen, it makes a first estimation of the matrix, computes the Lagrange's multipliers for the minimization, and from them gets a second estimation, If the second method is chosen, it computes the recurrent estimations of the matrix and stops when the estimates add up to the marginal totals. Finally, the estimated matrix is printed and the results are compared with the actual values if they are given; Theil's coefficient and the matrix of the differences between estimated and actual values are printed.

The sub-routine MAPRT is a printing sub-routine for the input and output data in matrix form. It also converts data from F format to A format, so that absence of information can be printed with a special sign (here, two dots) instead of zero.

The sub-routine MINV inverts the matrix A of (3.6), using the standard Gauss-Jordan method. If the matrix is singular, the program prints out the following message "The matrix is singular" and stops.

The sub-routine ARRAY converts the matrix A from a double dimension to a single dimension array and vice versa, since the inversion subroutine operates on arrays of data in a vector fashion.

The listing of the program is given in Annex I, the format of the input data and the data themselves are given in Annex 2, the output data are given in Annex III and IV.

8. CONCLUSION

The RAS method is faster in computer time than the difference minimization method, when no information on the cells of the current year matrix is available but it tends to slow down with the addition of such information; while the Difference Minimization method is slower initially, its execution time decreases as the number of extra internal cell information increases, as we can see in Table 8.I (matrix of 16 by 14).

Execution times (in seconds) **Table 8.I.**

number of cells known	16	32	48	64	80
RAS	8	8	8	9	13
Difference minimization	18	14	14	14	13

If each method is used in a different program, the RAS method requires less computer storage. The unused core for the RAS method is of the order of 25,000 words, whereas for the other method it is of 20,000.

In terms of results, the two methods seem to give comparable solutions as we can see in comparing their respective inequality coefficients in Table 8.II.

Inequality coefficients **Table 8.II.**

number of cells known	16	32	48	64	80
RAS	0.17	0.17	0.17	0.10	0.10
Difference minimization	0.17	0.17	0.17	0.10	0.10

In comparing the estimations with the actual matrix, it must not be forgotten that they are the results of a forcast based only on one previous year, which is extremely poor from a prediction point of view. One way to improve these estimations would be to get a first estimate based on several previous years by any prediction method, (least squares or more sophisticated methods) and then adjust it on its given marginal values of the current year, using one of the two methods above.

```
4-3000-06    MATRA2                          FORTRAN SOURCE LIST
      ISN        SOURCE STATEMENT

        0 SIBFTC MATRA2  LIST,REF
          C      ***********************************************************************
          C      *MATRA2-ESTIMATION OF TRADE MATRIX                    J.AMUNATEGUI *
          C      *                                   JULY 1969                        *
          C      *                                                                    *
          C      *PROGRAM MATRA2 TO ESTIMATE THE TRADE MATRIX OF THE CURRENT YEAR *
          C      *USING THE PREVIOUS YEAR TRADE MATRIX,THE MARGINAL TOTALS OF THE *
          C      *CURRENT YEAR AND SOME OF THE ELEMENTS OF THE CURRENT YEAR IF    *
          C      *AVAILABLE.                                                        *
          C      *                                                                    *
          C      ***********************************************************************
          C      *                                                                    *
          C      *X(I,J).        =ELEMENT OF THE MATRIX OF THE PREVIOUS YEAR         *
          C      *Y(I,J)         =ELEMENT OF THE MATRIX OF THE CURRENT YEAR          *
          C      *ROWX(I)        =X-ROW TOTAL                                        *
          C      *ROWY(I)        =Y-ROW TOTAL                                        *
          C      *COLX(J)        =X-COLUMN TOTAL                                     *
          C      *COLY(J)        =Y-COLUMN TOTAL                                     *
          C      *T              =GRAND TOTAL                                        *
          C      *A              =WORKING MATRIX FOR CALCULATION OF LAGRANGE         *
          C      *                MULTIPLIERS                                        *
          C      *B              =R-H-S FOR CALCULATION OF LAGRANGE MULTIPLIERS      *
          C      *Z              =LAGRANGE MULTIPLIERS                               *
          C      *                                                                    *
          C      ***********************************************************************
        1        DIMENSION CNAM(5)
        2        DIMENSION X(16,14),Y(16,14),COLX(14),COLY(14),ROWX(16),ROWY(16)
        3        DIMENSION A(29,29),B(29),SA(29),A1(29,29),B1(29),LV(29),MV(29),S(8
                141),Z(29),Z1(29)
        4        DATA CURR/1HC/
        5        DATA RAS/3HRAS/
          C                                                   READ CHOICE OF METHOD
        6        READ(5,0) XMETH
        7      0 FORMAT(A3)
          C                                                   READ COUNTRY NAME,YEAR
          C                                                   AND SPECIFICATION OF
          C                                                   IMPORT OR EXPORT
       10        READ (5,10) (CNAM(I),I=1,5),NYEAR,PORT
       16     10 FORMAT (5A6,1X,I4,1X,A6)
          C                                                   READ MATRIX DIMENSION
       17        READ (5,20) M,N
       22     20 FORMAT (I3,I3)
          C                                                   INITIALISATION
       23        DO 30 I=1,M
       24        ROWY(I)=CURR
       25        DO 30 J=1,N
       26        COLY(J)=CURR
       27        Y(I,J)=CURR
       30     30 X(I,J)=0.
          C                                                   READ AVAILABLE DATA
          C                                                   FOR ESTIMATION
       33     40 READ (5,50) I,J,KAL,TYP
       37     50 FORMAT (I3,I3,I8,65X,A1)
       40        IF(TYP.EQ.CURR) GO TO 80
       43        IF(I.EQ.888) GO TO 90
```

```
ISN     SOURCE STATEMENT

 46       IF(I.EQ.0) GO TO 60
 51       IF(J.EQ.0) GO TO 70
 54       X(I,J)=KAL
 55       GO TO 40
 56    60 COLY(J)=KAL
 57       GO TO 40
 60    70 ROWY(I)=KAL
 61       GO TO 40
 62    80 Y(I,J)=KAL
 63       GO TO 40
 64    90 CONTINUE
       C                                                    ROW AND COLUMN TOTALS
       C                                                    FOR X
 65       DO 100 I=1,M
 66       ROWX(I)=0.
 67       DO 100 J=1,N
 70   100 ROWX(I)=ROWX(I)+X(I,J)
 73       TOTX=0.
 74       DO 110 J=1,N
 75       COLX(J)=0.
 76       DO 110 I=1,M
 77       COLX(J)=COLX(J)+X(I,J)
100   110 TOTX=TOTX+X(I,J)
       C                                                    PRINT OUT OF PREVIOUS
       C                                                    YEAR MATRIX
103       NYEAR=NYEAR-1
104       WRITE (6,120) PORT,NYEAR,(CNAM(I),I=1,5)
111   120 FORMAT (1H1,12X,A6,18H TRADE MATRIX FOR ,I4,5H FOR ,5A6/////)
112       CALL MAPRT(X,COLX,ROWX,TOTX,M,N)
       C                                                    PRINT OUT OF GIVEN
       C                                                    DATA FOR CURRENT YEAR
       C                                                    MATRIX
113       TOTY=0.0
114       DO 130 I=1,M
115   130 TOTY=ROWY(I)+TOTY
117       NYEAR=NYEAR+1
120       WRITE (6,140) PORT,NYEAR,(CNAM(I),I=1,5)
125   140 FORMAT (1H1,06X,17HREPORTED DATA OF ,A6,18H TRADE MATRIX FOR ,I4,5
          1H FOR ,5A6/////)
126       CALL MAPRT(Y,COLY,ROWY,TOTY,M,N)
       C                                                    IF CHOSEN METHOD IS =
       C                                                    RAS GO TO STAT 500
       C                                                    LAGRANGE CONTINUE
127       IF(XMETH.EQ.RAS) GO TO 500
       C     ************************************************************************
       C     *DISTANCE MINIMISATION METHOD                                           *
       C     *OPTIMISATION BY USE OF LAGRANGE MULTIPLIERS                            *
       C     ************************************************************************
       C                                                    FIRST ESTIMATE OF
       C                                                    Y(I,J) TO BE ENTERED
       C                                                    IN ARRAY X(I,J)
132       DO 200 I=1,M
133       DO 200 J=1,N
134   200 X(I,J)=(ROWY(I)*COLY(J)*X(I,J))/(ROWX(I)*COLX(J))
       C                                                    ROW AND COLUMN TOTALS
```

```
ISN        SOURCE STATEMENT

      C                                                   FOR NEW X
137         DO 210 J=1,N
140         COLX(J)=0.
141         DO 210 I=1,M
142     210 COLX(J)=COLX(J)+X(I,J)
145         DO 220 J=1,N
146         IF(COLX(J).NE.0.) GO TO 220
151         COLX(J)=0.1
152         X(M,J)=0.1
153         COLY(J)=0.1
154     220 CONTINUE
156         DO 230 I=1,M
157         ROWX(I)=0.
160         DO 230 J=1,N
161     230 ROWX(I)=ROWX(I)+X(I,J)
164         DO 240 I=1,M
165         IF(ROWX(I).NE.0.) GO TO 240
170         ROWX(I)=0.1
171         X(I,N)=0.1
172         ROWY(I)=0.1
173     240 CONTINUE
      C                                                   FILLING UP OF WORKIN
      C                                                   MATRIX NEEDED TO
      C                                                   COMPUTE LAGRANGE
      C                                                   MULTIPLIERS
175         LAG=M+N-1
176         DO 250 I=1,LAG
177         B(I)=0.
200         DO 250 J=1,LAG
201     250 A(I,J)=0.
204         DO 270 I=1,M
205         DO 260 J=1,N
206         IF(Y(I,J).EQ.CURR) GO TO 260
211         A(I,I)=A(I,I)-X(I,J)
212         B(I)=B(I)-Y(I,J)+X(I,J)
213     260 CONTINUE
215         A(I,I)=A(I,I)+ROWX(I)
216         B(I)=B(I)+ROWY(I)-ROWX(I)
217     270 CONTINUE
221         N1=N-1
222         DO 290 J=1,N1
223         J1=J+M
224         DO 280 I=1,M
225         IF(Y(I,J).EQ.CURR) GO TO 280
230         A(J1,J1)=A(J1,J1)-X(I,J)
231         B(J1)=B(J1)-Y(I,J)+X(I,J)
232     280 CONTINUE
234         A(J1,J1)=COLX(J)+A(J1,J1)
235         B(J1)=B(J1)+COLY(J)-COLX(J)
236     290 CONTINUE
240         DO 300 I=1,M
241         DO 300 J=1,N1
242         IF(Y(I,J).NE.CURR) GO TO 300
245         J1=M+J
246         A(I,J1)=X(I,J)
```

```
4-3000-06     MATRA2                          FORTRAN SOURCE LIST MATRA2
      ISN         SOURCE STATEMENT

      247         A(J1,I)=X(I,J)
      250     300 CONTINUE
      253         DO 310 I=1,LAG
      254     310 SA(I)=0.0
      256         DO 320 I=1,LAG
      257         DO 320 J=1,LAG
      260     320 SA(I)=SA(I)+A(I,J)
              C                                                  ELIMINATION OF ROWS
              C                                                  AND COLUMNS OF MATRIX
              C                                                  WITH ZERO ELEMENTS
      263         K=0
      264         DO 330 I=1,LAG
      265         L=0
      266         IF(SA(I).EQ.0.0) GO TO 330
      271         K=K+1
      272         B1(K)=B(I)
      273         DO 330 J=1,LAG
      274         IF(SA(J).EQ.0.0) GO TO 330
      277         L=L+1
      300         A1(K,L)=A(I,J)
      301     330 CONTINUE
      304         LAG1=K
      305         DO 340 I=1,LAG1
      306         DO 340 J=1,LAG1
      307     340 A1(I,J)=A1(I,J)/10.**3
              C                                                  INVERSION OF MATRIX A1
      312         CALL ARRAY(2,LAG1,LAG1,29,29,S,A1)
      313         CALL MINV(S,LAG1,D,LV,MV)
      314         CALL ARRAY(1,LAG1,LAG1,29,29,S,A1)
      315         IF(D.NE.0.) GO TO 360
      320         WRITE (6,350)
      321     350 FORMAT (1HO,22HTHE MATRIX IS SINGULAR)
      322         STOP
      323     360 CONTINUE
      324         DO 370 I=1,LAG1
      325         DO 370 J=1,LAG1
      326     370 A1(I,J)=A1(I,J)/10.**3
              C                                                  CALCULATION OF
              C                                                  LAGRANGE MULTIPLIERS
              C                                                  BY MULTIPLYING MATRIX
              C                                                  A1 BY VECTOR B1
      331         KA=0
      332         DO 380 I=1,LAG1
      333         KA=KA+1
      334         Z1(KA)=0.0
      335         DO 380 J=1,LAG1
      336     380 Z1(KA)=Z1(KA)+A1(I,J)*B1(J)
      341         K=0
      342         DO 400 I=1,LAG
      343         IF(SA(I).EQ.0.0) GO TO 390
      346         K=K+1
      347         Z(I)=Z1(K)
      350         GO TO 400
      351     390 Z(I)=0.0
      352     400 CONTINUE
```

```
ISN     SOURCE STATEMENT

      C                                                     ESTIMATION OF MATRIX
  354         DO 420 I=1,M
  355         N1=N-1
  356         DO 410 J=1,N1
  357         MJ=M+J
  360         X(I,J)=X(I,J)*(1.+Z(I)+Z(MJ))
  361         IF(Y(I,J).NE.CURR) X(I,J)=Y(I,J)
  364     410 CONTINUE
  366         X(I,N)=X(I,N)*(1.+Z(I))
  367         IF(Y(I,N).NE.CURR) X(I,N)=Y(I,N)
  372     420 CONTINUE
  374         GO TO 800
      C       ************************************************************************
      C       *RAS METHOD
      C       *COMPUTATION OF A SET OF MATRICES THE LIMIT OF WHICH IS THE
      C       *ESTIMATE OF THE TRADE MATRIX
      C       ************************************************************************
      C                                                     ADJUSTMENT TO COLUMN
      C                                                     MARGINAL TOTALS
  375     500 DO 540 I=1,M
  376         DENO=0.0
  377         XNUM=0.0
  400         DO 520 J=1,N
  401         IF(Y(I,J).EQ.CURR) GO TO 520
  404         XNUM=XNUM+Y(I,J)
  405         DENO=DENO+X(I,J)
  406     520 CONTINUE
  410         DO 540 J=1,N
  411         IF(Y(I,J).NE.CURR) GO TO 530
  414         X(I,J)=X(I,J)*((ROWY(I)-XNUM)/(ROWX(I)-DENO))
  415         GO TO 540
  416     530 X(I,J)=Y(I,J)
  417     540 CONTINUE
  422         DO 550 J=1,N
  423         COLX(J)=0.0
  424         DO 550 I=1,M
  425     550 COLX(J)=COLX(J)+X(I,J)
      C                                                     CONVERGENCE TEST
  430         DO 560 J=1,N
  431         D=COLX(J)-COLY(J)
  432         T=ABS(D)
  433         IF(T.GE.0.05) GO TO 570
  436     560 CONTINUE
  440         GO TO 800
      C                                                     ADJUSTMENT TO ROW
      C                                                     MARGINAL TOTALS
  441     570 DO 600 J=1,N
  442         XNUM=0.0
  443         DENO=0.0
  444         DO 580 I=1,M
  445         IF(Y(I,J).EQ.CURR) GO TO 580
  450         XNUM=XNUM+Y(I,J)
  451         DENO=DENO+X(I,J)
  452     580 CONTINUE
  454         DO 600 I=1,M
```

```
-3000-06    MATRA2                          FORTRAN SOURCE LIST MATRA2
     ISN        SOURCE STATEMENT

     455        IF(Y(I,J).NE.CURR) GO TO 590
     460        X(I,J)=X(I,J)*((COLY(J)-XNUM)/(COLX(J)-DENO))
     461        GO TO 600
     462    590 X(I,J)=Y(I,J)
     463    600 CONTINUE
     466        DO 610 I=1,M
     467        ROWX(I)=0.0
     470        DO 610 J=1,N
     471    610 ROWX(I)=ROWX(I)+X(I,J)
          C                                                  CONVERGENCE TEST
     474        DO 620 I=1,M
     475        D=ROWX(I)-ROWY(I)
     476        T=ABS(D)
     477        IF(T.GE.0.05) GO TO 500
     502    620 CONTINUE
          C                                                  CALCULATION OF GRAND
          C                                                  TOTAL AND MARGINAL
          C                                                  TOTALS OF X
     504    800 TOTX=0.0
     505        DO 830 I=1,M
     506        ROWX(I)=0.0
     507        DO 830 J=1,N
     510        IF(X(I,J).LE.0.0) X(I,J)=0.0
     513        ROWX(I)=ROWX(I)+X(I,J)
     514    830 TOTX=TOTX+X(I,J)
     517        DO 840 J=1,N
     520        COLX(J)=0.0
     521        DO 840 I=1,M
     522    840 COLX(J)=COLX(J)+X(I,J)
          C                                                  PRINT OUT OF ESTIMATED
          C                                                  MATRIX FOR CURRENT
          C                                                  YEAR
     525        WRITE(6,850) PORT,NYEAR,(CNAM(I),I=1,5)
     532    850 FORMAT(1H1,06X,14HESTIMATION OF ,A6,18H TRADE MATRIX FOR ,I4,5H FO
                1R ,5A6/////)
     533        CALL MAPRT(X,COLX,ROWX,TOTX,M,N)
          C                                                  READ REAL CURRENT YEAR
          C                                                  DATA FOR COMPARAISON
          C                                                  WITH ESTIMATION OF X
     534        INDX=0
     535        DO 860 I=1,M
     536        DO 860 J=1,N
     537    860 Y(I,J)=0.0
     542    870 READ(5,880) I,J,KAL
     546    880 FORMAT(I3,I3,I8)
     547        INDX=INDX+1
     550        IF(I.EQ.999) GO TO 890
     553        Y(I,J)=KAL
     554        GO TO 870
     555    890 IF(INDX.EQ.1) STOP
          C                                                  PRINT OUT OF REAL
          C                                                  CURRENT YEAR MATRIX
     560        WRITE(6,900) PORT,NYEAR,(CNAM(I),I=1,5)
     565    900 FORMAT(1H1,10X,5HREAL ,A6,18H TRADE MATRIX FOR ,I4,5H FOR ,5A6////
                1/)
```

```
4-3000-06     MATRA2                                FORTRAN SOURCE LIST MATRA2
      ISN          SOURCE STATEMENT

      566          CALL MAPRT(Y,COLY,ROWY,TOTY,M,N)
                   C                                                    CALCULATION OF Q INDE
                   C                                                    OF THEIL
      567          QN=0.0
      570          QD=0.0
      571          DO 910 I=1,M
      572          ROWX(I)=ROWY(I)-ROWX(I)
      573          DO 910 J=1,N
      574          QN=QN+(Y(I,J)-X(I,J))**2
      575          QD=QD+Y(I,J)**2+X(I,J)**2
      576          X(I,J)=Y(I,J)-X(I,J)
      577      910 CONTINUE
      602          DO 920 J=1,N
      603      920 COLX(J)=COLY(J)-COLX(J)
      605          TOTX=TOTY-TOTX
      606          Q=SQRT(QN)/SQRT(QD)
                   C                                                    PRINT OUT OF Q AND
                   C                                                    MATRIX OF DIFFERENCE
                   C                                                    BETWEEN PREDICTION AN
                   C                                                    REALISATION
      607          WRITE(6,930) Q
      610      930 FORMAT(1H1,02X,67HINDEX Q AND MATRIX OF DIFFERENCE BETWEEN PREDIC
                  1ION AND REALISATION//40X,10HINDEX Q = ,F6.2///)
      611          CALL MAPRT(X,COLX,ROWX,TOTX,M,N)
      612          STOP
      613          END
```

A 2.1 *Input format*

The input data are set in the following order:

— First card: col. 1—3 DFM if Difference Minimisation method is chosen or RAS if RAS method is chosen

— Second card: col. 1—30 name of country under consideration
col. 32—35 current year
col. 37—42 export or import

— Third card: col. 1—3 number of rows
col. 4—6 number of columns

Then three different sets of data with one value on each card are ordered:

— previous year matrix element: col. 1—3 row number
col. 4—6 column number
col. 7—14 value

— current year marginal totals: same format but col. 14—6 blank for row marginal total and col. 1—3 blank for column marginal totals

— current year matrix element: same format with "C" in column 80

— control card with 888 in col. 1—3 to indicate the end of the necessary data for the estimation

Finally, if available, the actual values of the current year matrix in the same format as the previous year matrix and a control card with a 999 in col. 1—3 to indicate the end of the input data.

A 2.2 *Description of matrix headings*

Column headings (countries):

1 = U.S.A.
2 = Canada
3 = Federal Republic of Germany
4 = Other EEC country
5 = U.K.
6 = Other EFTA countries
7 = Other Western European countries
8 = U.S.S.R.
9 = Other Eastern European countries
10 = Middle East
11 = Australia, New Zealand, South Africa
12 = Japan
13 = Other Asian countries (except China)
14 = Rest of the world

Row heading (commodities — S.I.T.C. classification)

1 = 05 (Fruits and Vegetables)
2 = rest of 0 plus 1 (rest of Food and Live Animal plus Beverage and Tobacco)
3 = 21 (Hides, Skins and fur skins, undressed)
4 = 26 (Textiles fibres and their waste)
5 = 27 (Crude fertilizers and crude minerals)
6 = 28 (Metalliferous ores and metal scrap)
7 = rest of 2 (rest of crude materials, inedible, except fuels)
8 = 3 (Mineral fuels, lubricants and related materials)
9 = 4 (Animal and vegetable oils and fats)
10 = 5 (Chemicals)
11 = 65 (Textile yarn, fabrics, made-up articles and related products)
12 = 67 (Iron and steel)
13 = 68 (Non-ferrous metals)
14 = Rest of 6 (manufactured goods classified chiefly by material)
15 = 7 (Machinery and transport equipment)
16 = 8 plus rest (Footwear plus rest)

A 2.3 *Source of data*

The data of the 1965 and 1966 Trade Matrix are taken from Commodity Trade Statistics 1965 and 1966 — Series D — Statistical Office — United Nations — New York

Export trade matrix for 1965 for Turkey

	1	2	3	4	5	6	7	8
1	7938.0	360.0	45127.0	18483.0	11480.0	5529.0	1245.0	8540.0
2	58309.0	27.0	11082.0	9591.0	211.0	16340.0	3438.0	4338.0
3	1117.0	0.0	117.0	1514.0	0.0	491.0	1127.0	218.0
4	395.0	0.0	9395.0	25513.0	22854.0	15024.0	9529.0	4227.0
5	359.0	0.0	735.0	4121.0	541.0	449.0	141.0	0.0
6	2724.0	0.0	540.0	2005.0	277.0	863.0	1207.0	0.0
7	2601.0	0.0	1434.0	3044.0	713.0	1188.0	1824.0	19.0
8	0.0	0.0	191.0	1163.0	3605.0	0.0	647.0	0.0
9	884.0	0.0	0.0	6934.0	0.0	284.0	0.0	1331.0
10	150.0	0.0	222.0	1114.0	540.0	70.0	223.0	0.0
11	116.0	0.0	2355.0	189.0	421.0	793.0	0.0	0.0
12	217.0	0.0	0.0	763.0	410.0	247.0	0.0	0.0
13	6082.0	0.0	0.0	9415.0	0.0	0.0	1451.0	0.0
14	659.0	0.0	16.0	42.0	74.0	78.0	110.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	201.0	0.0	118.0	15.0	0.0	0.0	0.0	0.0
T	81752.0	387.0	71332.0	83906.0	41126.0	41356.0	20942.0	18673.0

	9	10	11	12	13	14	TOTAL
1	8068.0	3749.0	208.0	118.0	0.0	351.0	111196.0
2	15420.0	24246.0	16.0	3827.0	721.0	201.0	147767.0
3	1689.0	978.0	0.0	0.0	0.0	11.0	7262.0
4	14858.0	5540.0	0.0	237.0	3366.0	2354.0	113292.0
5	662.0	0.0	0.0	247.0	0.0	34.0	7289.0
6	2567.0	0.0	0.0	230.0	0.0	0.0	10413.0
7	1726.0	3016.0	0.0	490.0	10.0	65.0	16130.0
8	0.0	0.0	0.0	0.0	0.0	40.0	5646.0
9	2269.0	0.0	0.0	0.0	0.0	117.0	11819.0
10	325.0	123.0	0.0	0.0	263.0	1.0	3031.0
11	311.0	0.0	0.0	0.0	0.0	121.0	4306.0
12	0.0	0.0	0.0	0.0	0.0	123.0	1760.0
13	0.0	235.0	0.0	0.0	0.0	1.0	17184.0
14	136.0	147.0	0.0	0.0	0.0	0.0	1262.0
15	0.0	0.0	0.0	0.0	0.0	119.0	119.0
16	0.0	112.0	0.0	0.0	0.0	0.0	446.0
T	48031.0	38146.0	224.0	5149.0	4360.0	3538.0	458922.0

Reported data of export trade matrix for 1966 for Turkey

	1	2	3	4	5	6	7	8
1	5069.0	461.0	30412.0	• •	12293.0	• •	• •	• •
2	64101.0	180.0	18658.0	• •	1020.0	• •	• •	• •
3	1429.0	0.0	185.0	• •	0.0	• •	• •	• •
4	455.0	196.0	16063.0	• •	26943.0	• •	• •	• •
5	824.0	0.0	314.0	• •	426.0	• •	• •	• •
6	3539.0	0.0	1003.0	• •	394.0	• •	• •	• •
7	1930.0	44.0	3049.0	• •	621.0	• •	• •	• •
8	308.0	0.0	0.0	• •	2583.0	• •	• •	• •
9	0.0	0.0	0.0	• •	0.0	• •	• •	• •
10	349.0	0.0	245.0	• •	431.0	• •	• •	• •
11	0.0	0.0	447.0	• •	101.0	• •	• •	• •
12	0.0	0.0	0.0	• •	353.0	• •	• •	• •
13	1680.0	0.0	5768.0	• •	1456.0	• •	• •	• •
14	247.0	0.0	66.0	• •	93.0	• •	• •	• •
15	0.0	0.0	0.0	• •	0.0	• •	• •	• •
16	279.0	0.0	202.0	• •	0.0	• •	• •	• •
T	80210.0	881.0	76412.0	94987.0	46714.0	45312.0	22089.0	18474.0

J.	9	10	11	12	13	14	TOTAL
1	• •	• •	• •	• •	• •	• •	105260.0
2	• •	• •	• •	• •	• •	• •	157773.0
3	• •	• •	• •	• •	• •	• •	7909.0
4	• •	• •	• •	• •	• •	• •	139152.0
5	• •	• •	• •	• •	• •	• •	9157.0
6	• •	• •	• •	• •	• •	• •	12790.0
7	• •	• •	• •	• •	• •	• •	16977.0
8	• •	• •	• •	• •	• •	• •	4380.0
9	• •	• •	• •	• •	• •	• •	2508.0
10	• •	• •	• •	• •	• •	• •	3464.0
11	• •	• •	• •	• •	• •	• •	2373.0
12	• •	• •	• •	• •	• •	• •	2211.0
13	• •	• •	• •	• •	• •	• •	24758.0
14	• •	• •	• •	• •	• •	• •	797.0
15	• •	• •	• •	• •	• •	• •	171.0
16	• •	• •	• •	• •	• •	• •	828.0
T	56050.0	36067.0	229.0	8129.0	3407.0	1547.0	490508.0

Real export trade matrix for 1966 for Turkey

	1	2	3	4	5	6	7	8
1	5069.0	461.0	30412.0	17974.0	12293.0	5498.0	1487.0	11849.0
2	64101.0	180.0	18658.0	11714.0	1020.0	17444.0	2591.0	3573.0
3	1429.0	0.0	185.0	1659.0	0.0	408.0	404.0	166.0
4	455.0	196.0	16063.0	38781.0	26943.0	16025.0	12755.0	2868.0
5	824.0	0.0	314.0	4414.0	426.0	1610.0	164.0	0.0
6	3539.0	0.0	1003.0	2583.0	394.0	1532.0	248.0	0.0
7	1930.0	44.0	3049.0	3713.0	621.0	1528.0	1580.0	18.0
8	308.0	0.0	0.0	348.0	2583.0	291.0	850.0	0.0
9	0.0	0.0	0.0	2200.0	0.0	0.0	0.0	0.0
10	349.0	0.0	245.0	1269.0	431.0	91.0	0.0	0.0
11	0.0	0.0	447.0	252.0	101.0	815.0	0.0	0.0
12	0.0	0.0	0.0	368.0	353.0	47.0	0.0	0.0
13	1680.0	0.0	5768.0	9596.0	1456.0	0.0	1976.0	0.0
14	247.0	0.0	66.0	39.0	93.0	23.0	34.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	279.0	0.0	202.0	77.0	0.0	0.0	0.0	0.0
T	80210.0	881.0	76412.0	94987.0	46714.0	45312.0	22089.0	18474.0

	9	10	11	12	13	14	TOTAL
1	13015.0	6390.0	209.0	165.0	0.0	438.0	105260.0
2	16911.0	18792.0	20.0	2303.0	291.0	175.0	157773.0
3	2062.0	1594.0	0.0	0.0	0.0	2.0	7909.0
4	14231.0	6886.0	0.0	1185.0	2576.0	188.0	139152.0
5	836.0	252.0	0.0	278.0	0.0	39.0	9157.0
6	3435.0	0.0	0.0	0.0	0.0	56.0	12790.0
7	2415.0	1623.0	0.0	353.0	45.0	58.0	16977.0
8	0.0	0.0	0.0	0.0	0.0	0.0	4380.0
9	113.0	115.0	0.0	0.0	0.0	80.0	2508.0
10	450.0	0.0	0.0	0.0	495.0	134.0	3464.0
11	570.0	0.0	0.0	0.0	0.0	188.0	2373.0
12	1443.0	0.0	0.0	0.0	0.0	0.0	2211.0
13	437.0	0.0	0.0	3845.0	0.0	0.0	24758.0
14	132.0	163.0	0.0	0.0	0.0	0.0	797.0
15	0.0	0.0	0.0	0.0	0.0	171.0	171.0
16	0.0	252.0	0.0	0.0	0.0	18.0	828.0
T	56050.0	36067.0	229.0	8129.0	3407.0	1547.0	490508.0

Estimation of export trade matrix for 1966 for Turkey

	1	2	3	4	5	6	7	8
1	5069.0	461.0	30412.0	23293.5	12293.0	6877.1	1449.4	9629.5
2	64101.0	180.0	18658.0	8940.2	1020.0	15185.2	2946.9	3662.0
3	1429.0	0.0	185.0	1592.1	0.0	513.3	1091.2	206.9
4	455.0	196.0	16063.0	31353.6	26943.0	18339.1	10786.5	4683.4
5	824.0	0.0	314.0	5378.4	426.0	581.5	169.6	0.0
6	3539.0	0.0	1003.0	2198.6	394.0	942.9	1217.4	0.0
7	1930.0	44.0	3049.0	3025.4	621.0	1174.6	1668.7	17.1
8	308.0	0.0	0.0	976.7	2583.0	0.0	500.9	0.0
9	0.0	0.0	0.0	1593.1	0.0	64.7	0.0	275.1
10	349.0	0.0	245.0	1338.4	431.0	83.4	246.9	0.0
11	0.0	0.0	447.0	258.0	101.0	1058.8	0.0	0.0
12	0.0	0.0	0.0	1350.9	353.0	431.2	0.0	0.0
13	1680.0	0.0	5768.0	13609.8	1456.0	0.0	1932.1	0.0
14	247.0	0.0	66.0	32.8	93.0	60.3	79.4	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	279.0	0.0	202.0	45.6	0.0	0.0	0.0	0.0
T	80210.0	881.0	76412.0	94987.0	46714.0	45312.0	22089.0	18474.0

	9	10	11	12	13	14	TOTAL
1	10937.3	4239.3	217.2	228.5	0.0	153.3	105260.0
2	15799.8	21014.8	11.8	5748.8	443.7	61.0	157773.0
3	1940.7	947.0	0.0	0.0	0.0	3.8	7909.0
4	19911.9	6248.4	0.0	461.3	2746.6	964.2	139152.0
5	940.3	0.0	0.0	508.3	0.0	14.9	9157.0
6	3089.9	0.0	0.0	405.2	0.0	0.0	12790.0
7	1876.8	2765.8	0.0	776.8	6.6	21.3	16977.0
8	0.0	0.0	0.0	0.0	0.0	11.3	4380.0
9	566.1	0.0	0.0	0.0	0.0	9.1	2508.0
10	424.7	135.0	0.0	0.0	210.2	0.4	3464.0
11	447.9	0.0	0.0	0.0	0.0	60.3	2373.0
12	0.0	0.0	0.0	0.0	0.0	75.9	2211.0
13	0.0	311.7	0.0	0.0	0.0	0.5	24758.0
14	114.8	103.7	0.0	0.0	0.0	0.0	797.0
15	0.0	0.0	0.0	0.0	0.0	171.0	171.0
16	0.0	301.4	0.0	0.0	0.0	0.0	828.0
T	56050.0	36067.0	229.0	8129.0	3407.0	1547.0	490507.9

Estimation of export trade matrix for 1966 for Turkey

	1	2	3	4	5	6	7	8
1	5069.0	461.0	30412.0	23169.3	12293.0	6868.7	1434.7	9630.3
2	64101.0	180.0	18658.0	9012.0	1020.0	15215.8	2969.7	3666.8
3	1429.0	0.0	185.0	1595.7	0.0	512.9	1091.9	206.7
4	455.0	196.0	16063.0	31388.4	26943.0	18318.1	10777.1	4678.2
5	824.0	0.0	314.0	5375.4	426.0	580.4	169.1	0.0
6	3539.0	0.0	1003.0	2208.7	394.0	942.1	1222.3	0.0
7	1930.0	44.0	3049.0	3033.2	621.0	1173.2	1670.8	17.0
8	308.0	0.0	0.0	977.7	2583.0	0.0	500.0	0.0
9	0.0	0.0	0.0	1592.4	0.0	64.6	0.0	275.0
10	349.0	0.0	245.0	1338.8	431.0	83.4	246.4	0.0
11	0.0	0.0	447.0	254.7	101.0	1059.3	0.0	0.0
12	0.0	0.0	0.0	1351.2	353.0	433.5	0.0	0.0
13	1680.0	0.0	5768.0	13612.8	1456.0	0.0	1928.6	0.0
14	247.0	0.0	66.0	32.6	93.0	60.0	78.5	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	279.0	0.0	202.0	44.2	0.0	0.0	0.0	0.0
T	80210.0	881.0	76412.0	94987.0	46714.0	45312.0	22089.0	18474.0

	9	10	11	12	13	14	TOTAL
1	11005.0	4316.8	216.5	235.8	0.0	148.0	105260.0
2	15766.1	20926.6	12.5	5732.5	448.6	63.5	157773.0
3	1937.1	946.8	0.0	0.0	0.0	3.9	7909.0
4	19890.8	6260.7	0.0	464.8	2742.4	974.4	139152.0
5	939.6	0.0	0.0	513.6	0.0	14.9	9157.0
6	3077.0	0.0	0.0	403.9	0.0	0.0	12790.0
7	1871.5	2760.5	0.0	778.4	6.6	21.8	16977.0
8	0.0	0.0	0.0	0.0	0.0	11.3	4380.0
9	567.0	0.0	0.0	0.0	0.0	9.0	2508.0
10	425.0	135.8	0.0	0.0	209.3	0.4	3464.0
11	456.1	0.0	0.0	0.0	0.0	54.9	2373.0
12	0.0	0.0	0.0	0.0	0.0	73.3	2211.0
13	0.0	312.1	0.0	0.0	0.0	0.5	24758.0
14	114.9	104.9	0.0	0.0	0.0	0.0	797.0
15	0.0	0.0	0.0	0.0	0.0	171.0	171.0
16	0.0	302.8	0.0	0.0	0.0	0.0	828.0
T	56050.0	36067.0	229.0	8129.0	3407.0	1547.0	490507.9

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UN PROGRAMME POUR ORDINATEUR EN VUE D'ESTIMER UNE MATRICE DE COMMERCE INTERNATIONAL

Cet article présente un programme qui donne la possibilité d'utiliser deux méthodes différentes pour estimer une matrice de commerce international d'un pays pour une année donnée, connaissant ses totaux marginaux et la matrice de l'année précédente.

La première méthode consiste à calculer d'abord une première estimation en fonction des données, puis à obtenir une deuxième estimation en minimisant une fonction de la différence entre la première et la deuxième estimation, compte tenu des contraintes marginales. La deuxième méthode, RAS, par un procédé de récurrence, calcule une série de matrices dont la limite est l'estimation recherchée. Enfin, les deux méthodes sont modifiées pour tenir compte de la connaissance supplémentaire de certains éléments intérieurs de la matrice de l'année étudiée et permettre ainsi d'améliorer l'estimation.

ПРОГРАММА ДЛЯ КАЛЬКУЛЯТОРОВ В ЦЕЛЯХ ОЦЕНКИ ТОРГОВОЙ МАТРИЦЫ

Автор приводит программу, позволяющую применять два различных метода оценки матрицы международной торговли какой-либо страны на данный год, если известны ее конечные итоги и матрица предыдущего года.

Первый метод заключается, прежде всего, в расчете первой оценки в зависимости от данных, а затем в получении второй оценки путем минимизации функции разницы между первой и второй оценками с учетом конечных ограничений. Второй метод RAS рассчитывает путем рекуренции ряд матриц, предел которых является искомой оценкой. Наконец, оба метода изменяются для учета дополнительного знания определенных внутренних элементов матрицы рассматриваемого года, что позволяет улучшить оценку