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On a Queueing Model for a Time-Sharing Computation System

A number of parameters, such as the time of reply, the length of the queue, etc. can be estimated with the aid of the queueing models for computation systems.

By setting out from a model of Wei-Chang for time-shared computation systems, we shall evaluate the generating function of distribution of the Markov chain associated with the model.

In a time-sharing computation system, each program is being processed by the computer in very short time intervals called "quantums", quantums which generally are random variables. If the given quantum is smaller than the time necessary for a service, the processing is interrupted and the program is again placed in the queue (in this case, the programme awaits the continuation of processing).

In the time-shared computation system, the processing of programmes is carried out according to an algorithm which takes into consideration the priority of the programme as well. Several classes of priority are being considered.

The service discipline is of two types: with the interruption (Ist type) or without the interruption (IInd type) of the given quantum.

In case of the first type, all programs with greater priorities accumulated in the meantime are answered and only afterwards, the rest of the quantum is given to the program whose processing quantum had been interrupted.

The time-shared computation systems are functioning, as a rule, according to the IInd type of discipline in which the given quantum is entirely used even if in the meantime a request has occured from another program which had greater priority than that of the program being carried out.

During each processing, a quantum is given for the computation time of the respective programme. The following events may appear:

— E_k consisting in the fact that the execution of the programme ends at the expiration of the quantum.

— $\bar{E}_k$ consisting in the fact that the execution of the programme can not end in the given quantum.

We associate with these complementary events the probabilities:

$$P(E_k) = q_k, \quad P(\bar{E}_k) = p_k, \quad p_k + q_k = 1, \qquad k = 1, 2, \ldots n.$$

In case of the event which is carried out with probability p_k, the programme is sent to the queueing line and will await the giving of the next quantum by observing the service discipline (the IInd type of discipline).

Corresponding to this type of problems are queuineg models in which a single class of priorities is considered and the random input with Poisson's distribution and the service time with an exponential distribution are implied.

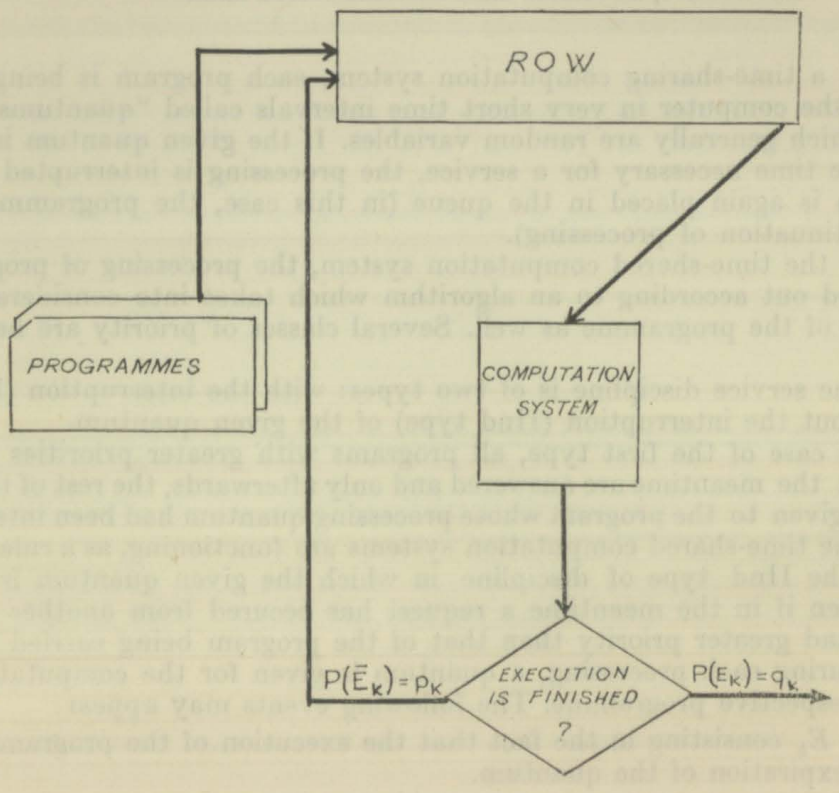
The most important elements occuring in the analysis of the queueing model of a computation system are:

— the expectation time of a program representing the time of the programme queueing in the row before its processing is started; the total length of a programme's remaining in the system is equal to the queueing time plus the service time.

— the occupation period, that is the time interval during which the computation system is occupied.

— the number of programmes in the system which is given by the number of programmes existent in the queue and of those in an execution stage at a given moment $t(t > 0)$.

In his model, Wei-Chang considers the input flow a Poisson process, the service discipline of the IInd type and a single class of priority.



The scheme of Wei-Chang's model for a time-shared computation system is the following:

It is considered that the service type of a programme is an independent random variable with the distribution function:

$$H(x) = \begin{cases} 1 - e^{-x} & x \geqslant 0 \\ 0 & x < 0. \end{cases} \tag{1}$$

Noting with $\psi(x)$ Laplace's transformed of $H(x)$, we have:

$$\psi(s) = \int_0^\infty e^{-st}\, dH(x). \tag{2}$$

For this model, there is the following ratio between $H(x)$ and $Q(x)$

$$H(x) = q \sum_{n=1}^{\infty} p^{n-1} Q_n(x) \tag{3}$$

where $Q_n(x)$ is the convolution product of $Q(x)$ with itself, $Q(x)$ being the quantum distribution.

Wei Chang estimates the row after the completion of quantum n with the aid of the generating function given by the following ratio:

$$U(z) = \frac{U(0)[q(1-z) + pz(1-z)]\Phi[\lambda(1-z)]}{(q + pz)\Phi(1-z)} \tag{4}$$

A Markov chain can be associated with the queueing model (of M/G/1 type) for time-shared computation systems. Consider $\xi(t)$ the number of clients who are served or await to be served at moment t. We define the Markov chain $\xi(t)$ associated with the service system, where $\xi(t)$ is the number of clients left after the departure of a client served.

In case of the queueing model for a time-shared computation system, $\xi(t)$ represents the number of programmes at moment t left after the giving of the last quantum. This quantum is not always that which is necessary for processing the programme as a whole.

The possibilities of transition of the associated Markov chain are:

$$P_{ij} = P(\xi_{n+1} = j / \xi_n = i) \qquad i, j, \in N. \tag{5}$$

In that case

$$P_{0j} = P_j, \quad j \in N, \quad P_{ij} = P_{j-i+1}, \quad i\ N, \quad j_{n-1}$$

where

$$P_k = \frac{1}{k!} \int_0^\infty e^{-\frac{x}{a}} \left(\frac{x}{a}\right)^k dQ(x), \quad P_k, > 0 \text{ and } \sum_{k \in N} P_k = 1. \tag{6}$$

The conditions of the Markov chain $\xi(t)$ form a single essential class of period 1.

Noting with J the set of conditions of the associated chain, we say that condition i is essential, if ratio $i \to j$ takes place, $j \to i$ $(i, j \in J)$ is also taking place.

It is known from the theory of the finite Markov chains that the conditions are positive for the traffic intensity $\rho < 1$, where $\rho = \sum_{k \in N} p_k Z^k$.

For a Markov chain associated with a service system $M/G/1$, the theorem is proved:

If $\rho < 1$, in that case the stationary distribution Π_k, $k \in N$ of the Markov chain associated with the service system $M/G/1$ has the generating function

$$\Pi(z) = \frac{(1-q)\,p(z)\,(1-z)}{p(z)-z}. \tag{7}$$

The Markov chain $\xi(t)$ associated with the model for the time-shared computation system satisfies the conditions of the above enunciated theorem. The stationary distribution of chain $\xi(t)$ thus has the generating function $\Pi(z)$ with the aid of which the associated Markov chain $\xi(t)$ can be described.

For a given moment, the number of programmes in the queue awaiting to be processed by the computation system can be evaluated.

Considering that (7) can be applied in the above presented queueing model with the aid of the associated chain $\xi(t)$, the impact of the quantum on the chain and on the time of reply can be examined.

Considering the queueing model for time-shared computation systems with several priority classes, the generating function of the stationary distribution of chain $\zeta(t)$ can be estimated according to (7) for each class of priority.

The aim of the work was thus to show that by constructing the Markov chain associated with the queueing model, by considering the quantum a random variable and a single class of priority, the queueing row at a given moment can be estimated with the aid of the generating function.

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D'UN MODELE D'ATTENTE POUR UN SYSTEME DE CALCUL A TEMPS DIVISES

On peut estimer, à l'aide des modèles d'attente pour les systèmes de calcul, une série de paramètres tels que le délai de réponse, la durée d'attente, etc.

Prenant comme point de départ un modèle de Wei-Chang pour les systèmes de calcul à temps divisés, on présente l'évaluation de la fonction génératrice de la répartition de la chaîne Markov associée au modèle.

О МОДЕЛИ ОЖИДАНИЯ ДЛЯ СИСТЕМЫ РАСЧЕТА С НАПРАВЛЕННЫМ ВРЕМЕНЕМ

При помощи моделей ожидания для систем расчета можно получить оценку ряда параметров как, например, время ответа, длина хвоста и т.д.

Исходя из модели Вей-Чанга для систем расчета с разделенным временем делается оценка генерирующей функции распределения цепи Маркова, ассоциированной с моделью.