C. ZIDĂROIU

A Production Programming Model for Enterprises under Specific Conditions

From the practical point of view, it is more important to solve the problem than to generalize it.

One of the problems of great economic importance under the present conditions of intensifying material production is the production programming. The objective in view is to obtain with the same production capacities and the same number of workers an as great as possible volume of high quality production at as low as possible costs. The problem of production programming, taking the general case into consideration, is difficult from the mathematical point of view, as there are only rough approaching methods. This is why it seems to us more effective to try to formulate models much closer to the real character of the production in different industrial branches. In this paper we shall formulate a programming model for the use of equipment under conditions that may seem very restrictive at first sight, but which have been met in a relatively great number of types of industrial enterprises. *) More precisely, we shall examine here the problem of determining the work of a group of machines **) by means of which a greater range of goods have to be manufactured in a sequence determined according to the delivering delays provided by the contract terms. Between the production of an assortment and that of the following one the operation of the machines — has to be cut, in order to adjust them for the next assortment of goods.

In the model below we set forth the objective of making as short as possible the time-during which the running of the equipment is broken off·

*) We express here our gratitude for the suggestions made by Mr. Trattner, on conditions like the oves shown further, which are specific to enterprises dealing with food industry.

**) We use the expression "Group of machines" with the meaning of flow-sheet, and we shall further call it "equipment".

We will note the following:

m = the assortment of goods i requiered to be produced during the considered period of time;

n = the number of periods j during which the delivering of goods assortments must be carried out;

t_{ijk} = the working time assigned to assortment i, $1 \leqslant i \leqslant m$, during interval j, $1 \leqslant j \leqslant n$, in order to deliver the due production during interval k, $j \leqslant k \leqslant n$;

$t_{i,i'}$ = the set time for stopping the manufacture in order to replace assortment i by assortment i'.

Remark: We will assume that this lapse of time does not depend on assortments i and i', which is current, for instance, in the food industry.

b_{ik} = the quantity of assortment $i, 1 \leqslant i \leqslant m$, to be delivered during the period of time $k, 1 \leqslant k \leqslant n$;

T_j = the whole time that is to be assigned in order to manufacture assortment i during the period of time j, $1 \leqslant j \leqslant n$ (breaks included);

T_{ij} = the longest time which may be assigned for manufacturing assortment i during period j ($Tij = T_j$ may be considered);

T'_{ij} = the minimum time in which assortment i can be processed during the period of time j;

v_i = the manufacturing speed for assortment i, $1 \leqslant i \leqslant m$, (assortment units per hour);

s_i = the maximum stock allowed for assortment i, $1 \leqslant i \leqslant m$, (considered within the standard stock estimates of the enterprise);

y_{ij} = discrete variables assuming value 1 if assortment i is manufactured during period of time interval j, and zero if assortment i is not manufactured during interval j.

Our model is as follows:

(1) $\min \sum_{i=1}^{m} \sum_{j=1}^{n} y_{ij}$

(2) $\sum_{j=1}^{k} v_i t_{ijk} \geqslant b_{ik}, 1 \leqslant i \leqslant m, 1 \leqslant k \leqslant n$

(3) $\sum_{i=1}^{m} \sum_{k=j}^{n} t_{ijk} + \tau \sum_{i=1}^{m} y_{ij} \leqslant T_j, 1 \leqslant j \leqslant n$

(4) $\sum_{j=1}^{l} \sum_{k=j}^{n} v_i t_{ijk} - \sum_{j=1}^{l} b_{ij} \leqslant s_i, 1 \leqslant i \leqslant m; \ 1 \leqslant l \leqslant n$

(5) $\sum_{k=j}^{n} t_{ijk} - y_{ij} \ T_{ij} \leqslant 0, \ 1 \leqslant i \leqslant m, \ 1 \leqslant j \leqslant n$

(6) $\sum_{k=j}^{n} t_{ijk} - y_{ij} \ T'_{ij} \geqslant 0, \ 1 \leqslant i \leqslant m, \ 1 \leqslant j \leqslant n$

(7) $t_{ijk} \geqslant 0, \ 1 \leqslant i \leqslant m, \ 1 \leqslant j \leqslant n, \ j \leqslant k \leqslant n$

(8) $y_{ij} = 1 \text{ or } 0, \ 1 \leqslant i \leqslant \text{m}, \ 1 \leqslant j \leqslant n.$

The economic significance of this model is as follows:

Let us minimize the sum (1) of time intervals during which the operation of the equipment is broken off under the conditions:

I. The production of assortment i must provide at least the quantity to be delivered during interval k (2).

II. The total time used for production during interval j plus the break-off time required by the assortment replacement must be inferior to the time available within this interval (3).

III. The stocks of each assortment have to be inferior to the maximal stock allowed for the due assortment, in each of the intervals considered (4).

The additional restrictions (5), (6), (7), (8), are equivalent to the logical conditions:

$$(7')\ y_{ij} = 1 \text{ implying } T'_{ij} \leqslant \sum_{k=j}^{n} t_{iik} \leqslant T_{ij}$$

$$(8')\ y_{ij} = 0 \text{ implying } t_{ijk} = 0$$

having outstanding significances.

If we have to do not one group of machines, but with more, the problem is to be tackled in an analogous manner.

In this respect, let us consider that we have a group of p machines, able to produce simultaneously some of the manufactured assortments, and let us note:

t^r_{ijk} = the working time assigned for assortment i, $1 \leqslant i \leqslant m$, with the group of machines r, $1 \leqslant r \leqslant p$, during interval j, $1 \leqslant j \leqslant n$, in order to be delivered during interval k, $j \leqslant k \leqslant n$;

$\tau^r_{i,i'}$ = the set time for a break-off in manufacturing with r machine group in order to replace assortment i by assortment i' (assuming $\tau^r_{i,i'} = \tau^r$);

T^r_j = the total time during which equipment r may be used during interval j, $1 \leqslant j \leqslant n$ (including the break-off);

φ^r_i = the manufacturing speed of assortment i with equipment r;

s_i = the maximum stock allowed for assortment i, $1 \leqslant i \leqslant m$ (which is included in the calculation of the circulating funds of the enterprise);

T'^r_{ij} = the maximum time that may be assigned in order to produce assortment i during interval j at equipment r ($T'^r_{ij} = T^r_j$ may be considered);

T''^r_{ij} = the minimum processing time for assortment i, during period j with equipment r (it corresponds to the processing time of a unit pertaining to assortment i);

y^r_{ij} = $\begin{cases} 1, \text{ if assortment } i \text{ is produced with equipment } r \text{ during interval } j; \\ 0, \text{ in other instances.} \end{cases}$

If assortment i cannot be produced with the group of machines r, we shall assume $y^r_{ij} = 0$ for any j, $1 \leqslant j \leqslant n$.

The corresponding mathematical model is as follows:

$$(1)\ min\left[\sum_{r=1}^{p}\sum_{i=1}^{m}\sum_{j=1}^{n}\tau^r\ y_{ij}^r\right]$$

$$(2)\ \sum_{r=1}^{p}\sum_{j=1}^{k}v_i^r\ t_{ijk}^r \geqslant b_{ik},\ 1\leqslant i\leqslant m,\ 1\leqslant k\leqslant n$$

$$(3)\ \sum_{i=1}^{m}\sum_{k=j}^{n}t_{ijk}^r+\tau^r\sum_{i=1}^{m}y_{ij}^r\leqslant T_j^r,\ 1\leqslant j\leqslant n,\ 1\leqslant r\leqslant p$$

$$(4)\ \sum_{r=1}^{p}\sum_{j=1}^{l}\sum_{k=j}^{n}t_{ijk}^r\ v_i^r-\sum_{j=1}^{l}b_{ij}\leqslant s_i;\ 1\leqslant i\leqslant m;\ 1\leqslant l\leqslant n$$

$$(5)\ \sum_{k=j}^{n}t_{ijk}^r-y_{ij}^r\ T_{ij}^r\leqslant 0,\ 1\leqslant i\leqslant m;\ 1\leqslant j\leqslant n;\ 1\leqslant r\leqslant p$$

$$(6)\ \sum_{k=j}^{n}t_{ijk}^r-y_{ij}^r\ T_{ij}^{\prime r}\geqslant 0,\ 1\leqslant i\leqslant m;\ 1\leqslant j\leqslant n,\ 1\leqslant r\leqslant p$$

$$(7)\ t_{ijk}^r\geqslant 0,\ 1\leqslant i\leqslant m;\ 1\leqslant j\leqslant k\leqslant n;\ 1\leqslant r\leqslant p$$

$$(8)\ y_{ij}^r=1 \text{ or } 0,\ 1\leqslant i\leqslant m;\ 1\leqslant j\leqslant n;\ 1\leqslant r\leqslant p.$$

The economic significances are analogous to those shown in the previous model.

We can adopt now a different version of this model. Let us replace restriction (3) by the following one:

$$(3')\ \sum_{i=1}^{m}\sum_{k=j}^{n}t_{ijk}^r+\tau^r\sum_{i=1}^{m}y_{ij}^r+x_j^r=T_j^r,\ 1\leqslant j\leqslant n,\ 1\leqslant r\leqslant p$$

where x_j^r is a slack variable representing the time during which equipment r does not operate during interval j; we certainly assume that:

$$(7')\ x_j^r>0,\ 1\leqslant j\leqslant n;\ 1\leqslant r\leqslant p.$$

Let us also replace objective (1) by

$$(1')\ min\left[\sum_{r=1}^{p}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\tau^r\ y_{ij}^r+x_j^r\right)\right]$$

representing the minimal time during which the machines are not operating within the considered period of time.

The new model consists of restrictions (1'), (2), (3'), (4), (5), (6), (7), (7'), (8) whose economic significances are now obvious.

The models shown above are problems of mixed linear programming (with continuous and bivalent variables) for which solving methods [2], [6], etc, exist.

In our opinion, such models offer a better solution to the specific conditions of some industrial branches (e.g. food industry) than the production programming models known till to now [1], [3], [4], [5].

REFERENCES

1. FLOOR MERRILL : *The travelling salesman problem*, Op. Res, 1, 61—67, 1956.
2. GOMORY R.: *An Algorithm for integer solutions to linear programs*, Princeton, IBM M.R.P., Technical Report Nr. 1, 1958.
3. MAMNE ALAN S.: *Scheduling of petroleum refinery operations*. Harvard Univ. Press, Cambridge (Mas.) 1956.
4. LESOURNES J.: *Technique Economique et Gestion Industrielle*, Dunod, Paris, 1964.
5. SALVESON M. E.: *A computational technique for the fabrication scheduling problem*, Los Angeles, Man. Sc. Res. Rep. Un. of California, 1953.
6. SIMONNARD M. : *Programmation linéaire*, Dunod, Paris, 1962.

UN MODELE DE PROGRAMMATION DE LA PRODUCTION DES ENTREPRISES DANS DES CONDITIONS SPECIFIQUES

Le but de cet article est celui de présenter un modèle mathématique pour le problème de la détermination du fonctionnement d'un outillage qui puisse fabriquer une quantité plus grande de valeurs matérielles dans un ordre déterminé, en rapport directe avec les termes de livraison prévus par le contract.

La production d'un assortiment une fois terminée, l'outillage doit être réglé pour le suivant assortiment, ce qui implique la réduction au minimum du temps nécessaire au réglage.

Le modèle utilise de façon concomitente des variables continues et discontinues.

МОДЕЛЬ ПРОГРАММИРОВАНИЯ ПРОИЗВОДСТВА НА ПРЕДПРИЯТИЯХ В СПЕЦИФИЧЕСКИХ УСЛОВИЯХ

Целью этой статьи является представить математическую модель для задачи определения работы оборудования, на котором производится более значительное количество продукции в определенном порядке в соответствии с предусмотренными в контракте сроками поставки.

После окончания производства определенного вида продукции машину необходимо наладить для следующего вида продукции, что требует минимума времени, необходимого для наладки.

Модель использует одновременно непрерывные и прерывные переменные.