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Models for Investment Optimization in Chemical Fertilizers Industry

..."Researches in the field of applied economy must aim at promoting advanced planning methods, formed upon modern computation means, ... upon economic cybernetics and economic information processing"...

N. Ceauşescu

Up to the present moment, the applied mathematics provided the specialists with quite sufficient model classes, but when the modelling of concrete phenomena of reality is concerned, there appear difficulties whose removal represents every time a useful experience for researchers in the field of applied mathematics. Difficulties are greater in the field of economic phenomena, as these phenomena have peculiarities which can integrate themselves only with difficulty with the general model forms. Moreover, when modelling economic phenomena, there appears a subjective aspect of the phenomenon (corresponding to the requirements of economic policy), which makes that a model is different from another to such an extent, that any attempt of generalizing them is an ambitious scheme. This is why, — and with the hope of being useful to the experts in the field of modelling economic phenomena —, we present here a model of determination of the optimal developing model in the sector of chemical fertilizers, a model which resulted from the works of a team of experts from the Computation Centre of the Economic Cybernetics Department in the Academy of Economic Studies, and from the Designing Institute for Organic Chemistry and Fertilizers. Of course, some aspects appearing in this model can be assigned also to other industrial branches, but since we aim at presenting a model as concrete as possible, we have insisted upon this branch, which offers a sufficient number of difficult problems, as concerns the quantizing of economic phenomena.

The problem essentially consisted in the optimal distribution of an investment fund, intended for a longer time period, to the sector of chemical fertilizers. We had in view, at the same time, that the achieved objectives ensure a production, which satisfies the domestic demands of fertilizers (agriculture), and use the exportation possibilities at the utmost. It is worth noticing that we differently dealt with the domestic demand and with the exportation one. Thus, while the demand for fertilizers of our own agriculture is expressed in amounts of fertilizing substances (nitrogen, phosphorus and potassium), the demand on the international market is done in certain fertilizer types (which can possibly contain all three types of fertilizing substances), according to importers' preferences. Due to this aspect, when working out the models, we were obliged to take into account the fertilizer types (ammonium nitrate, urea, lime superphosphate, a.s.o.), their composition and their economic characteristics (costs, specific investment, a.s.o.). In these conditions, the mathematical model was composed of three submodels, linked between them by the fact that the results of a model are input data for another one. The three above mentioned submodels aim at answering the main requirements of the modelling of the here treated phenomenon.

The economic correspondent of the three submodels consists in:

1. the prognosis of the evolution of the fertilizing substances demand by the agriculture;
2. establishing the optimal profile in the dynamics of the chemical fertilizer branch (establishing the types and amount of chemical fertilizers, to be produced at any moment);
3. establishing the optimal location of the fertilizer plants, correlated with the optimal version of supplying fertilizers to beneficiaries.

Answers to these problems are provided by the submodels subsequently presented.

I. THE PROGNOSIS OF THE EVOLUTION OF THE CHEMICAL FERTILIZER CONSUMPTION

As a result of an economic and econometric analysis, carried out with the help of statistic data concerning the chemical fertilizer consumption (fertilizing substances), in our country and in other 80 countries, upon a time period of 17—20 years, we reached the conclusion that the prognosis of the evolution of the chemical fertilizer consumption per hectare can be effected with the help of a logistic curve. Both the analysis of the innermost structure of the chemical fertilizer consumption phenomenon, and the mathematical properties of the logistic model, advocate this conclusion. We remind the reader that the logistic function has the expression:

$$L(t) = \frac{k}{1 + be^{-at}}$$

Constants a, b, and k are the curve parameters. It can be seen from the above formula that straight line $L(t) = k$ is the horizontal asymptote

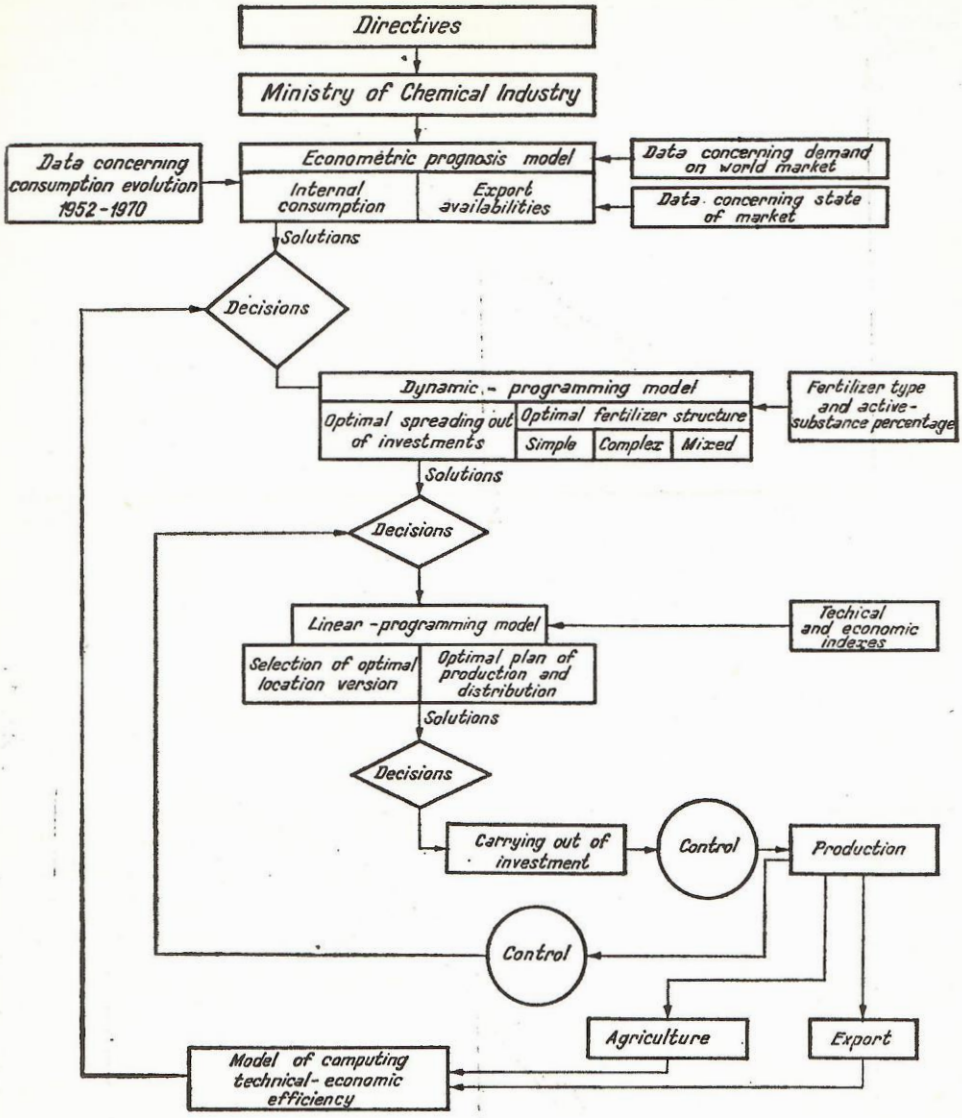


Fig. 1

of the logistic function. It follows that magnitude k may be considered as „saturation level" for the variables in the left member.

There generally are two elements which limit the consumption of fertilizers per hectare, namely the fact that, beginning with a certain consumption level, most of it cannot be economically justified (the production increase does not compensate any more the expenses connected with the fertili-

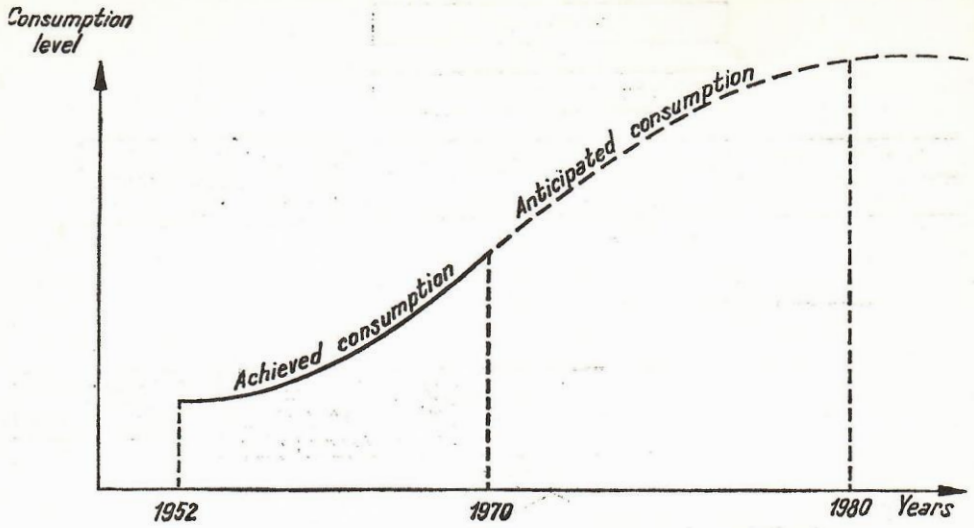

Consumption level
Anticipated consumption
Achieved consumption
1952
1970
1980
Years

Fig. 2

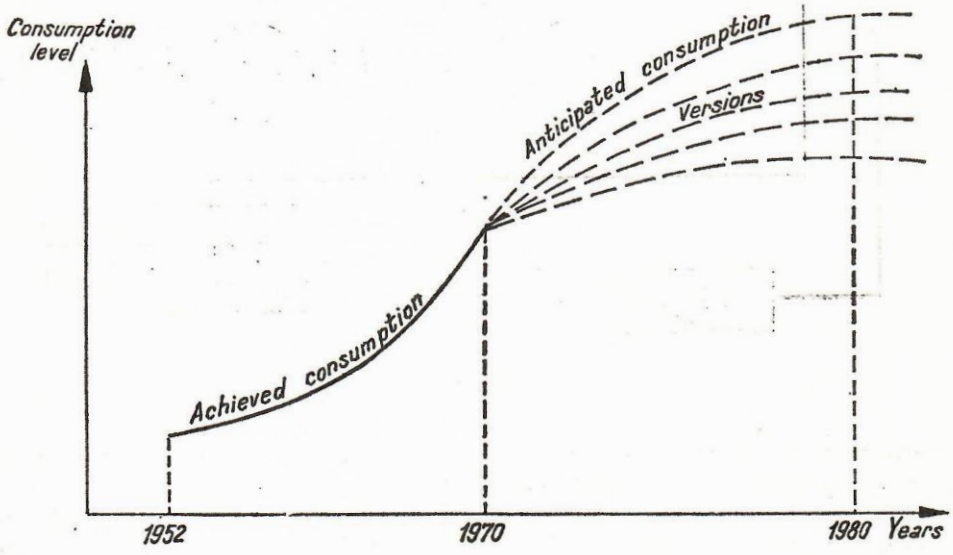

Consumption level
Anticipated consumption
Versions
Achieved consumption
1952
1970
1980
Years

Fig. 3

zer consumption increase), and the very physical limitation of the soil, which prescribes a certain consumption extreme limit. We can consider, in this hypothesis, that the „saturation level" coincides with value k. Consequently, we considered, for each consumption of the three fertilizing substances, a distinct form of logistic function. Denoting by $N(t)$ the nitrogen consumption, expressed in tons of active substance at the level of year t ($t = 1,2,...$), we look for values a_N, b_N, and k_N, such as function

$$N(t) = \frac{Sk_N}{1 + b_N e^{-a_N t}} \tag{1.1}$$

adjusts in the best way the data concerning nitrogen consumption. S is here the farming area. We can use, for estimating parameters, the method of least squares.

We estimate analogously the phosphorus consumption:

$$P(t) = \frac{Sk_p}{1 + b_p e^{-a_p t}} \tag{1.2}$$

and the potassium consumption:

$$K(t) = \frac{Sk_K}{1 + b_K e^{-a_k t}} \tag{1.3}$$

In this way, formulas (1.2) — (1.3) offer data on the dynamics of fertilizing substances consumption, data necessary in establishing the optimal profile in the chemical fertilizer branch.

II. DETERMINING THE OPTIMAL PROFILE OF THE CHEMICAL FERTILIZER BRANCH

The problem of selecting the directing of investments, consists, among others, in finding out the maximal total net efficiency of investment throughout the national economy, during a certain time period, e.g. during a year. This problem can be easily formulated as a dynamic problem, not for a single year, but for a series of successive years, e.g. for the long-term plan period.

Through the mathematical model presented as follows, we intended to determine the optimal profile of the chemical fertilizer production, in other words, the amount to be produced of each fertilizer type. We determine also, in this way, the new capacities bound to be put into service, at the national economy level, at each time period; we also implicitly determine the optimal spreading out in time of the opening of financings. We mention, as a last general characteristic of the problem, the fact that the results supplied by this model can be utilized as such, or modified on the basis of some requirements of economic policy, not liable to be quantized in the framework of the model. In this last situation, we can compute, with the help of the optimal solution delivered by the model, the „regret" which is the difference between the purpose-function for the optimal solution and the

one corresponding to the adopted solution. The model has been worked out with a view to a maximal elasticity, to the effect that certain conception changes, which could occur meanwhile, de not principially affect the model structure, hence the latter can be used without essential alterations.

Concerning the optimization of investments in the chemical fertilizer branch, we must have in view that the production, at each year's level, covers the domestic necessaries with regard to dynamics, and ensures the fulfilment of the exportation tasks. At the same time the model indicates the fact that the investment value cannot exceed the investment fund allotted for the development of the sector on the fixed planning horizon. A specific feature of the chemical industry is the fact that, for each manufacturing plant, the production capacities of nitrogen, phosphorus and potassium (fertilizing substances), can take values only from a predetermined values range, corresponding to the capacity moduli, existing in the chemical industry. This aspect will finally lead to the solving of the model with the help of mixed programming. The model itself actually belongs to the class of dynamic programming problems.

The purpose of optimization is to minimize the equivalent expenses. Thus, the purpose-function of the problem reflects the index „equivalent expenses", given by expression

$$C + EI$$

where C represents the production expenses, E — the normal coefficient of investment efficiency, and I is the specific investment. This index of estimating the investment efficiency is settled by the existing legistlation in our country, and is also highly utilized in the world praxis (see e.g., the COMECON methodology concerning investment efficiency).

As the dynamic programming model refers to a wide planning horizon, an updating coefficient was introduced, to make comparable investment expenses, in various years.

Let us suppose that the investment fund at our disposal for the whole planning period is I, and this fund can be invested within a number M of distinct activities. We consider as activities all types of chemical fertilizers planned to be manufactured on the whole planning horizon. Consequently, it is necessary to specify the number and the kind of assortments, as well as the magnitude of the planning horizon, before solving any problem of dynamic determination of the optimal profile in the chemical fertilizer branch.

Further on, we generally admitted that investment fund I is destined to activities whose financing opening extends on a horizon of N years (we consider only time intervals equal to one year). Let us denote by 1, 2, ..., N the beginning of each year in the planning horizon; we shall generically denote by t the beginning of each year in the planning horizon. Hence t takes the values $t = 1, 2, ..., N$. Alike, we attach subscript i to any magnitude referring to assortment number i. Hence i takes the values $i = 1, 2, ..., M$.

Concerning each activity type, two moments deserving mention result, i. e. the year of financing opening for the respective activity, and the year of putting into service. We shall denote by $\Delta i (i = 1, 2, ..., M)$ the time interval (measured in years), elapsing between the financing opening and

the moment of putting into function the relevant objective. Hence, if concerning the financing opening, the planning horizon includes N years, from year 1 to year N, as concerns the moment of putting into service, the planning horizon will also include N years, but from year $1 + \Delta_i$ to year $N + \Delta_i$. For the same reason (the lagging existing between the moment of financing opening and the moment of putting into service), a new aspect appears, which must be taken into account: namely, within the time period between the first year of the planning horizon and the year $1 + \Delta i$ ($i = 1, 2, ..., M$), objectives can be put into service, whose financing was opened before the first year of the considered planning horizon. For these objectives, investments are carried out from the investment fund allotted to the planning period, previous to the considered time period. We shall take into account that these investments do not affect the investment fund, I, and that their amount is known.

After having specified the above mentioned, let us establish the variables and restrictions which build up the model. When establishing restrictions, we considered that investment should be done so as to take into account following requirements:

— ensuring the domestic need of fertilizing substances (nitrogen, phosphorus, potassium);

— observing the exportation tasks;

— not-exceeding the allotted investment fund;

— including within the possible moduli of technologic capacity.

If we denote by $x_i(t)$ — the value of the investment allotted to type i-fertilizer, having the putting into service of the objective in year t, by $y_i(t)$ — the amount to be produced from type i-fertilizer at year's t level, expressed in tons of physical substance, and by e_i — the specific investment for the type i-fertilizer (lei/ton of physical substance), we can describe the evolution in time of the capacities of production, by equation:

$$y_i(t) = y_i(t-1) + \frac{x_i(t)}{e_i} \quad (i = 1, 2, ..., M;\ t = 1, 2, ..., N + \Delta_i) \tag{2.1}$$

We associate to this equation initial conditions which result from the existing situation at the beginning of the planning period;

$$y_i(0) = y_i^0 \quad (i = 1, 2, ..., M) \tag{2.2}$$

$$x_i(t) = x_i^0(t) \ (i = 1, 2, ..., M;\ t = 1, 2, ..., \Delta_i - 1) \tag{2.3}$$

Here y_i^0 represents the production volume of the type i-fertilizer at the level of the planning-period beggining year. This is known, and so is also x_i^0 (t), which represents the investment value for the type i-fertilizer, and the investment is put into service at moment $t (t = 1, 2, ..., \Delta_i - 1)$, hence resulting from funds allotted previously to the planning period.

Here also we took into account the capacity moduli for nitrogen, phosphorus and potassium. If we denote by $q_k^N (k = 1, 2, ..., C_N)$ the nitrogen stan-

dard capacities, by $q_k^P (k = 1, 2, ..., C_P)$ and $q_k^K (k = 1, 2, ..., C_K)$ the standar phosphorus and potassium capacities, respectively, we can express this b

$$\sum_{i=1}^{M} p_i^N y_i(t) = \sum_{k=1}^{C_N} \alpha_k(t) q_k^N$$

$$\sum_{i=1}^{M} p_i^P y_i(t) = \sum_{K=1}^{C_P} \beta_k(t) q_k^P \quad (t = \Delta, \Delta + 1, ..., N + \max_i \Delta_i) \qquad (2.4$$

$$\sum_{i=1}^{M} p_i^K y_i(t) = \sum_{k=1}^{C_K} \gamma_k(t) q_k^K$$

where p_i^N, p_i^P, p_i^K are the percentages of nitrogen, phosphorus and potassium respectively, in the type i — fertilizer, and $\alpha_k(t)$, $\beta_k(t)$, $\gamma_k(t)$ can take value 0, 1, 2, ... observing the condition

$$\alpha_k(t) \geqslant \alpha_k(t-1), \ \beta_k(t) \geqslant \beta_k(t-1), \ \gamma_k(t) \geqslant \gamma_k(t-1) \qquad (2.5$$

Equations (2.1) — (2.5) describe the evolution of the fertilizer produc tion on the planning horizon. We added to them certain restrictions, whic should reflect the way the fertilizers are used. If we denote by $E_i(t)$ th exportation tasks for the type i-fertilizer in year t, then it is necessar that:

$$N(t) \leqslant \sum_{i=1}^{M} (y_i(t) - E_i(t) p_i^N$$

$$P(t) \leqslant \sum_{i=1}^{M} (y_i(t) - E_i(t)) p_i^P \quad (t = 1, 2, ..., N + \max_i \Delta_i) \qquad (2.6$$

$$K(t) \leqslant \sum_{i=1}^{M} (y_i(t) - E_i(t)) p_i^K$$

The model is completed by the restriction of not exceeding the allotte investment fund:

$$\sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} x_i(t) \leqslant I \qquad (2.7$$

and by conditions of non-negativity for variables:

$$x_i(t) \geqslant 0 \ (t = 1, 2, ..., N + \Delta_i) \qquad (2.8$$

By virtue of what was stated in the previous paragraph, the optimiza tion consists in minimizing the functional

$$\sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i y_i(t) + \frac{x_i(t)}{T} (1 + \delta)^{N - (t - \Delta_i)} \right] \qquad (2.9$$

where we denoted by c_i the costs of assortment i, by T — the normed time of recovering investments, and by δ — the updating rate.

In conclusion, the problem consists in finding the time functions $x_i(t)$ which satisfy (2.3) and (2.8), and which, introduced in equation (2.1) and condition (2.2), would permit to compute the time functions $y_i(t)$ which satisfy restrictions (2.4) — (2.7), and together with them to minimize the functional (2.9) in the class of functions consistent with above established restrictions. One can notice that restrictions (2.4) and (2.5) render very difficult the solving of the model. This is why we do transform the problem in another equivalent one, of a type treated in the speciality literature.

II. 1. Solving the model

Equation (2.1) allows to eliminate variables $y_i(t)$ from all the other equations, By using a known procedure, we find from (2.1) and (2.2):

$$y_i(t) = y_i^0 + \sum_{i=1}^{t} \frac{x_i(j)}{e_i} \quad (i = 1,2, ..., M;\ t = 1,2, ..., N + \Delta_i)$$

This relation, introduced in (2.4), (2.6) and (2.9) leads to:

$$\begin{cases} \sum_{i=1}^{M} p_i^N \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} \right) = \sum_{k=1}^{C_N} \alpha_k(t) q_k^N \\ \sum_{i=1}^{M} p_i^P \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} \right) = \sum_{k=1}^{C_P} \beta_k(t) q_k^P \quad (t = 1, 2, ..., N + \max_i \Delta_i) \\ \sum_{i=1}^{M} p_i^K \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} \right) = \sum_{k=1}^{C_K} \gamma_k(t) q_k^K \end{cases} \tag{2.10}$$

$$\begin{cases} N(t) \leqslant \sum_{i=1}^{M} \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} - E_i(t) \right) p_i^N \\ P(t) \leqslant \sum_{i=1}^{M} \left(y_i^0 + \sum_{j=1}^{j} \frac{x_{ii}^{e_i}(j)}{e_i} - E_i(t) \right) p_i^P \quad (t = 1, 2, ..., N + \max_i \Delta_i) \\ K(t) \leqslant \sum_{i=1}^{M} \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} - E_i(t) \right) p_i^K \end{cases} \tag{2.11}$$

$$\min \sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i \sum_{j=1}^{t} \frac{x_i(j)}{e_i} + \frac{x_i(t)}{T} (1 + \delta)^{N-(t-\Delta_i)} \right] \tag{2.12}$$

As the minimizing of functional (2.9) is involved, we gave up constant $N \sum_{i=1}^{M} c_i y_i^0$ when (2.2) was written.

Consequently, the problem was reduced to a mixed programming problem, with unknowns $x_i(t)$, $\alpha_k(t)$, $\beta_k(t)$, $\gamma_k(t)$. For solving such a problem, we recomend, e. g. (2). Part of the unknowns are fixed by relations (2.3). Besides this, other unknowns can be also fixed due to some relations of economic policy; we refer to unknowns $\alpha_k(t)$, $\beta_k(t)$ or $\gamma_k(t)$.

II. 2. The existence of the solution

The model presented in paragraph 1 can be utilized only in the case when we have the certainty that there exists the possibility that, from the investment fund, objectives can be built, whose capacity could cover the demand for fertilizers. This means, of course, that we do not always expect the existence of a solution.

We present, in the conditions, a new version of the problem, for which there is a solution in any situation. This version is suggested by economic considerations, and its solving reduces to a series of problems, of the type presented in the preceding paragraphs. Principially, the modifying of the problem consists in finding that strategy, which allows to cover a percentage as large as possible, of the domestic and exportation requirements.

Further on, by keeping the notations used up to now, let us denote by λ_t the percentage of the domestic requirement which will be covered in year t, and by μ_t the percentage of the exportation task for year t, which will be achieved. It is certain that

$$0 \leqslant \lambda_t \leqslant 1; \quad 0 \leqslant \mu_t \leqslant 1 \tag{2.13}$$

In this situation, restrictions (2.11) are replaced by:

$$\begin{aligned}
\lambda_t N(t) &\leqslant \sum_{i=1}^{M} \left[y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} - \mu_t E_i(t) \right] p_i^N \\
\lambda_t P(t) &\leqslant \sum_{i=1}^{M} \left[y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} - \mu_t E_i(t) \right] p_i^P \\
\lambda_t k(t) &\leqslant \sum_{i=1}^{M} \left[y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} - \mu_t E_i(t) \right] p_i^K \\
&\qquad (t = 1, 2, \ldots, N + \max_i \Delta_i)
\end{aligned} \tag{2.14}$$

and purpose-function (2.12) will be replaced by:

$$\max_{\lambda_t \mu_t} x_i(t), \quad \alpha_k^{(t)} \min_{\beta_k^{(t)}} \gamma_k^{(t)} \sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i \sum_{j=1}^{t} \frac{x_i(j)}{e_i} + \right.$$

$$\left. + \frac{x_i(t)}{T} (1 + \delta)^{N-(t-\Delta_i)} \right] \tag{2.15}$$

For the remaining, we keep restrictions (2.3), (2.4), (2.5), (2.7) and (2.8). The sense of this optimization is as follows; for each value of λ_t and μ_t, which satisfies (2.13), one must solve the problem composed of restric-

tion (2.3), (2.5), (2.7), (2.8), (2.10) and (2.11), so obtaining an optimal value of the purpose-function; we select, from all these values, the highest one, and its corresponding stategy will be the optimal solution of the proposed problem. If there is a solution for all values of λ_t, μ_t which satisfy relation (2.13), the highest value of the purpose-function will be obtained for $\lambda_t = 1$ and $\mu_t = 1$. In this case, the solution of the modified version of the models coincides with the solution of the unmodified version. In other words, the unmodified version is a special case of the second one. There exists, of course, the risk that the modified problem has no solution for all λ_t and μ_t values which satisfy (2.13), a situation which justifies the modified version of the model. We must mention that the unmodified version is more easily solved, but we must have the certainty that it admits a solution. Otherwise, we use the modified version of the model. Some of the parameters λ_t and μ_t can be settled by economic policy considerations.

Let us analyze further on the conditions of existence for the model solution.

Let x be a vector with dimension $(M + C_N + C_p + C_k) \times (N + \max_i \Delta_i)$, the first $M \times (N + \max_i \Delta_i)$, $\alpha_k(t)$ components having $x_i(t)$, the following $C_p \times (N + \max_i \Delta_i)$ components having $\alpha_k(t)$, the following $C_p \times (N + \max_i \Delta_i)$ components having $\beta_k(t)$, and the last $C_K \times (N + \max_i \Delta_i)$ components having $\gamma_k(t)$. For each vector couple (λ, μ) with components λ_t, μ_t; $t = 1, 2, \ldots, N + \max_i \Delta_i$; $(0 \leqslant \lambda_t \leqslant 1,\ 0 \leqslant \mu_t \leqslant 1)$, we define the set:

$$
R_{\lambda 0 \mu} = \left\{ \begin{array}{l}
\displaystyle\sum_{i=1}^{M} p_i^N \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} \right) = \sum_{k=1}^{C_N} \alpha_k(t) q_k^N \\
\displaystyle\sum_{i=1}^{M} p_i^p \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} \right) = \sum_{k=1}^{C_P} \beta_k(t) q_k^p \\
\displaystyle\sum_{i=1}^{M} p_i^k \left(y_i^0 + \sum_{j=1}^{t} \frac{x_i(j)}{e_i} \right) = \sum_{k=1}^{C_K} \gamma_k(t) q_k^K \\
\alpha_k(t),\ \beta_k(t),\ \gamma_k(t) = 0,\ 1,\ 2,\ \ldots \\
\alpha_k(t) \geqslant \alpha_k(t-1), \beta_k(t) \geqslant \beta_k(t-1), \gamma_k(t) \geqslant \gamma_k(t-1) \\
x_i \displaystyle\sum_{i=1}^{M} y_i^0 p_i^N + \sum_{i=1}^{M} \sum_{j=1}^{t} \frac{x_i(j)}{e_i} p_i^N \geqslant \lambda_t N(t) + \mu_t \sum_{i=1}^{M} E_i(t) p_i^N \\
\displaystyle\sum_{i=1}^{M} y_i^0 p_i^P + \sum_{i=1}^{M} \sum_{j=1}^{t} \frac{x_i(j)}{e_i} p_i^P \geqslant \lambda_t P(t) + \mu_t \sum_{i=1}^{M} E_i(t) p_i^P \\
\displaystyle\sum_{i=1}^{M} y_i^0 p_i^K + \sum_{i=1}^{M} \sum_{j=1}^{t} \frac{x_i(j)}{e_i} p_i^K \geqslant \lambda_t K(t) + \mu_t \sum_{i=1}^{M} E_i(t) p_i^K \\
\displaystyle\sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\max_i \Delta_i} x_i(t) \leqslant I \\
x_i(t) \geqslant 0
\end{array} \right.
$$

In other words $R_{\lambda,\mu}$ is the set of vectors x whose components verify (2.10), (2.5), (2.14), (2.7), and (2.8). For each vector couple (λ, μ) we define the following mathematical programming problem $(P_\lambda; \mu)$: Minimize

$$\sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i \frac{x_i(t)}{e_i} + \frac{x_i(t)}{T} \times (1+\delta)^{N-(t-\Delta_i)} \right]$$

over $R_{\lambda,\mu}$.

With these reformulations, following lemmas are valid:

Lemma 1. Let functional

$$F(\lambda, \mu) = \min_{x \in R_{\lambda,\mu}} \sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i \frac{x_i(t)}{e_i} + \frac{x_i(t)}{T} (1+\delta)^{N-(t-\Delta_i)} \right] \text{ be.}$$

If $(\bar{\lambda}, \bar{\mu})$ is a vector couple, such that $(\bar{\lambda}, \bar{\mu}) \geqslant (\lambda, \mu)$ (i. e. $\bar{\lambda}_t \geqslant \lambda_t$; $\bar{\mu}_t \geqslant \mu_t$; $t = 1, 2, \ldots, N + \max_i \Delta_i$), then the set $R_{\bar{\lambda},\bar{\mu}}$, is included in the set $R_{\lambda,\mu}$.

The demonstration is immediate, if we make the remark that, when the vector x components verify restrictions (2.10), (2.5), (2.14), (2.7) and (2.8), corresponding to values $\bar{\lambda}_t$ and $\bar{\mu}_t$, they automatically verify the same restrictions for $\bar{\lambda}_t$ and $\bar{\mu}_t$.

Consequence. It results from Lemma 1 that:

$$\min_{x \in R_{\lambda\mu}} \sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i \frac{x_i(t)}{e_i} + \frac{x_i(t)}{T} (1+\delta)^{N-(t-\Delta_i)} \right] \leqslant$$

$$\min_{x \in R_{\bar{\lambda}\bar{\mu}}} \sum_{i=1}^{M} \sum_{t=\Delta_i}^{N+\Delta_i} \left[c_i \frac{x_i(t)}{e_i} + \frac{x_i(t)}{T} (1+\delta)^{N-(t-\Delta_i)} \right]$$

as the minimum in the right hand member is taken only for part of the elements of set $R_{\lambda,\mu}$ In other words, we showed that

$$F(\lambda, \mu) \leqslant F(\bar{\lambda}, \bar{\mu}).$$

It results, from Lemma 1 and its consequence, that the model can be considered as solved in the moment when couple (λ, μ) is determined, for which the maximum of function $F(\lambda, \mu)$ is achieved, and for which the mathematical programming problem $(P_{\lambda,\mu})$ has a meaning. It is possible that, for certain vector couples (λ, μ), the set $R_{\lambda,\mu}$ either be void or contain the zero vector of the space with $(M + C_N + C_P + C_K) \times (N + \max_i \Delta_i)$ dimensions. In the first case, the corresponding programming problem $(P_{\lambda,\mu})$ has no sense, and in the second case, the solution of problem $(P_{\lambda,\mu})$ will be the trivial solution. If this takes place for all vectors (λ, μ) of coordinates

λ_t, μ_t included in interval [0,1], then the model's solution will be the zero solution.

But if, for at least one couple (λ, p), the corresponding problem $(P_{\lambda, \mu})$ admits a solution with at least a non-zero component, then model will also admit such a solution. This means that we must demonstrate that to zero vector in the vectorial space with $(M + C_N + C_P + C_K)\ x(N + max_i \Delta_i)$ dimensions do not belong to set $R_{\lambda, \mu}$ and that this set is non-void.

Lemma 2. In order that the vector should not belong to the set $R_{\lambda, \mu}$, it is sufficient that at least one of the following conditions be satisfied:

1. There is at least an index i and an index t such as:

$$y_i^0 < \mu_t E_i(t).$$

2. For at least one index t one of following relations is satisfied:

$$\lambda_t N(t) > \sum_{i=1}^{M} [y_i^0 - \mu_t E_i(t)] p_i^N$$

$$\lambda_t P(t) > \sum_{i=1}^{M} [y_i^0 - \mu_t E_i(t)]\, p_i^P$$

$$\lambda_t K(t) < \sum_{i=1}^{M} [y_i^0 - \mu_t E_i(t)]\, p_i^K$$

We remark that, if we put into the first inequality of condition 2, $\lambda_t = 1$ and $\mu_t = 0$, we obtain:

$$N(t) > \sum_{i=1}^{M} y_i^0 p_i^N. \tag{2.16}$$

This inequality expresses the fact that, in year t, the domestic nitrogen consumption exceeds the production capacity of the initial year. As the domestic consumption increases together with t, in the case of a sufficiently long planning period, there will be a year t_0, starting from which inequality (2.16) is verified. An analogous discussion can be done also about the inequality concerning the potassium and phosphorus consumptions.

We shall denote by λ_0 and μ_0 the vectors:

$$\lambda_0 = (\underbrace{0, 0, ..., 0, 1}_{t_0}, 0, ..., 0) : \mu_0 = (0, 0, ..., 0)$$

Lemma 3. The set R_{λ_0, μ_0} is non-void, if there is at least one vector x, whose components verify relations:

$$\sum_{i=1}^{M} y_i^0 p_i^N + \sum_{i=1}^{M} \sum_{j=1}^{t_0} \frac{x_i\ (j)}{e_i} p_i^N \geqslant N(t_0)$$

$$\sum_{i=1}^{M} y_i^0 p_i^P + \sum_{i=1}^{M} \sum_{j=1}^{t_0} \frac{x_i(j)}{e_i} p_i^P \geqslant P(t_0)$$

$$\sum_{i=1}^{M} y_i^0 p_i^K + \sum_{i=1}^{M} \sum_{j=1}^{t_0} \frac{x_i(j)}{e_i} p_i^K \geqslant K(t_0)$$

$$\sum_{i=1}^{M} y_i^0 p_i^N + \sum_{i=1}^{M} \sum_{j=1}^{t_0} \frac{x_i(j)}{e_i} p_i^N = \sum_{k=1}^{C_N} \alpha_k(t_0) q_k^N$$

$$\sum_{i=1}^{M} y_i^0 p_i^P + \sum_{i=1}^{M} \sum_{j=1}^{t_0} \frac{x_i(j)}{e_i} p_i^P = \sum_{k=1}^{C_P} \beta_k(t_0) q_k^P$$

$$\sum_{i=1}^{M} y_i^0 p_i^K + \sum_{i=1}^{M} \sum_{j=1}^{t_0} \frac{x_i(j)}{e_i} p_i^K = \sum_{k=1}^{C_K} \gamma_k(t_0) q_k^K$$

$$0 < \sum_{i=1}^{M} \sum_{j=1}^{t_0} x_i(j) \leqslant I$$

$$x_i(t) \geqslant 0$$

The conditions required by above inequalities represent precisely the possibility that, by a certain utilization of the investment fund (or of part of it), the complete satisfying of the domestic requirement be carried out in year t_0. It is obvious that this condition is not artificial, and its satisfying depends exclusively on the way the investment fund I was established, on the basis of domestic consumption, forecast for each year.

Founding ourselves upon the property of function $F(\lambda, \mu)$, of being increasing as against each vector component, it results that, in order to find the model solution, we must determine λ_{MAX}, μ_{MAX} such that problem ($P\lambda_{MAX}$, μ_{MAX}) have non-trivial solutions, and for any vectors λ, μ with property $\lambda > \lambda_{MAX}$ and $\mu > \mu_{MAX}$, problem ($P_{\lambda, \mu}$) has at least the trivial solution. It goes without saying that $\lambda_{MAX}$and μ_{MAX} can be vectors whose components are all equal to 1. Once determined λ_{MAX} and μ_{MAX}, we solve problem ($P\lambda_{MAX}$, μ_{MAX}) as a mixed programming problem. Hence the solving will be done by iterations, on succesively increasing λ and μ by some step, until the moment when we arrive in the situation that there is no more trivial solution. This means a finite number of reoptimizations.

III. FIXING THE LOCATION OF OBJECTIVES AND THE VERSION OF SUPPLYING FERTILIZERS TO BENEFICIARIES

After fixing the objectives which must be put into service at the level of each year of the planning period, their optimal location is to be established. To do this, it was necessary that the experts of the chemical industry

VARIABLES AT INTERMEDIATE LEVEL

VARIABLE	SOLUTION ACTIVITY	UPPER BOUND	COST/UNIT INCREASE	INCREASED ACTIVITY	LOWEST COST
TYPE	CURRENT COST	LOWER BOUND	COST/UNIT DECREASE	DECREASED ACTIVITY	HIGHEST COST
L8BAZ	82.696	147.800	3.741	86.960	−3.741
B*	0.0	0.0	6.260	76.976	6.260
L9VAZ	0.0	16.000	200.741	4.263	−200.741
B*	0.0	0.0	*********	0.0	********
L22BMMP	0.014	72.000	42.721	3.155	−42.721
B*	0.0	0.0	1818.380	0.000	1818.380
L24LUMP	13.993	72.000	74.426	14.714	−74.426
B*	0.0	0.0	42.721	10.852	42.721
L27 TGCON	193.840	209.830	18.145	195.814	−18.145
B*	0.0	0.0o	10.841	192.369	10.841
L28GCOP	100.000	100.00	********	100.000	********
B*	0.0	0.0â	711.961	96.140	711.961
L146TRTK	5.354	********	4695.445	5.426	−4695.445
B*	0.0	1.180	488.777	5.199	488.777
L191IRMH	42.212	********	3.596	42.349	−3.596
B*	0.0	20.200	3.369	38.352	3.369
L192IRDJ	91.060	********	5.608	92.261	−5.608
B*	0.0	74.200	2.691	90.788	2.691
L194IRVL	10.400	********	60.244	13.307	−60.244
B*	0.0	7.700	92.875	10.278	92.875
L195IRO	67.400	********	0.776	68.601	−0.776
B*	0.0	36.600	0.517	65.830	0.517
L196IRAG	26.600	********	16.665	29.507	−16.665
B*	0.0	18.900	141.291	26.062	141.291

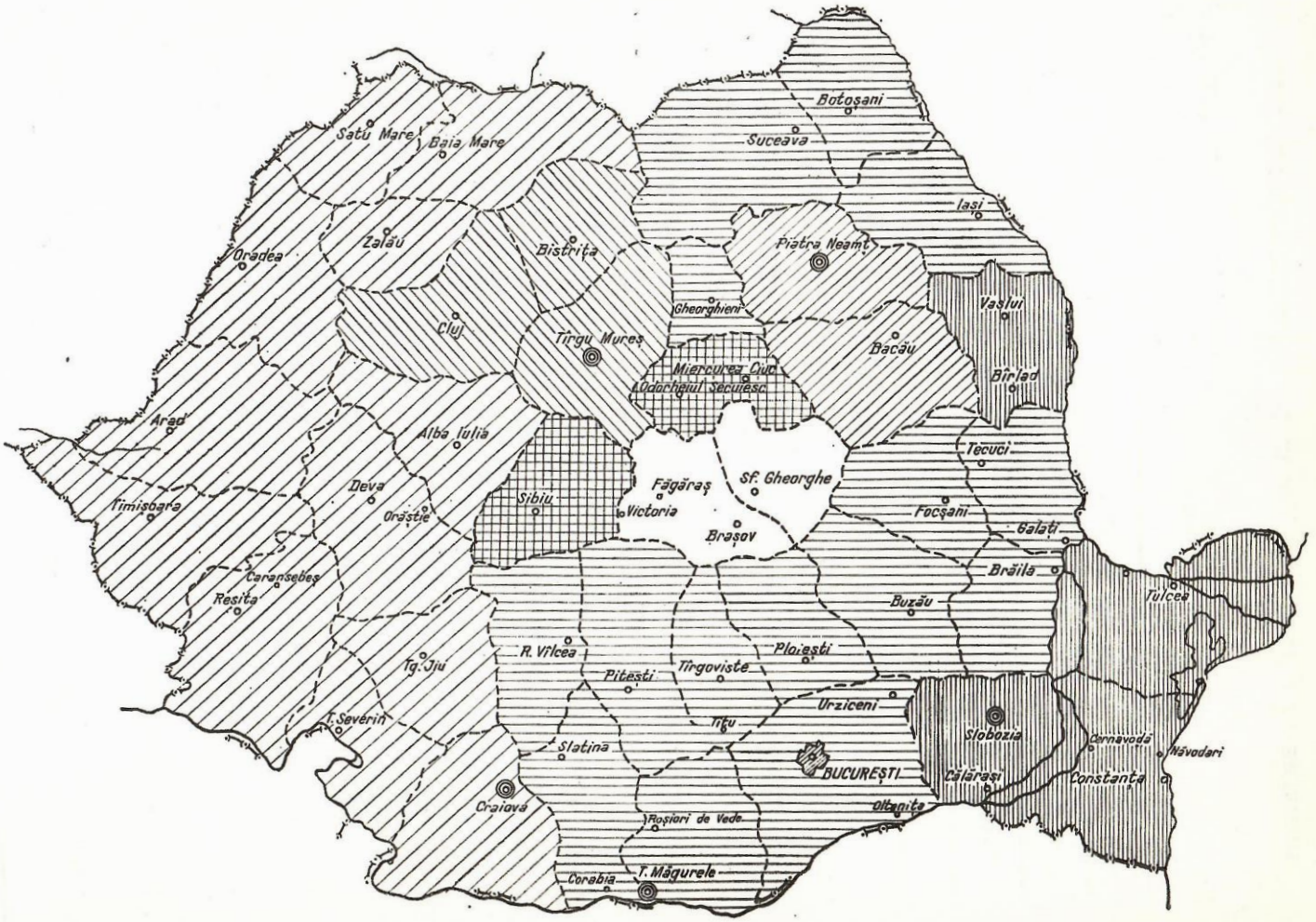

Botoşani
Satu Mare
Baia Mare
Suceava
Iaşi
Zalău
Oradea
Bistriţa
Piatra Neamţ
Gheorghieni
Vaslui
Cluj
Tirgu Mureş
Bacău
Miercurea Ciuc
Odorheiul Secuiesc
Bîrlad
Arad
Alba Iulia
Tecuci
Deva
Făgăraş
Sf. Gheorghe
Sibiu
Victoria
Timişoara
Oraştie
Braşov
Focşani
Galaţi
Caransebeş
Brăila
Reşiţa
Buzău
Tulcea
R. Vîlcea
Ploieşti
Tg. Jiu
Pitesti
Tirgoviste
T. Severin
Urziceni
Tîrgu
Slobozia
Slatina
Cernavodă
Măvodari
BUCUREŞTI
Călăraşi
Constanţa
Craiova
Oltenita
Roşiori de Vede
T. Măgurele
Corabia

Fig. 4

indicate the probable location versions. The model for fixing locations answers to following problems:

— the optimal location for each fertilizer plant, whose construction was determined by the preceding model;

— the distribution of production by consumers;

— the optimal recipe of covering the needs of consumers; this refers to the determination of the fertilizer types which co-operate to the ensuring of the needs in fertilizing substance of each consumer.

This model was worked out as a linear programming model. It was solved in each stage in which the model of determining the optimal profile stated that a new objective is put into service.

The model restrictions were built by starting from the covering of the domestic fertilizer requirement and from the utilization of the exportation possibilities.

Another category of restrictions refer to the capacity balances of each fertilizer plant.

Owing to the high number of consumers, the model has very large dimensions.

At the same time, a high number of versions was taken into account, which resulted from the multiple locating possibilities of the objectives, as well as from the technical and technological possibilities of production. As a result, when solving the considered versions, the technique of post-optimization was used, which allowed the substantial reducing of the necessary computer-hours and, implicitly, of the cost of the study.

In conclusion, the three presented models answer the main problems arising when an investment fund in the branch of chemical fertilizers is to be distributed. The model does not need intricate input data, but merely routine coefficients, usual in the sector's managing and planning practice, starting from designing, until the dispatching of products to beneficiaries. By its construction the model was provided with a high elasticity and adaptability to the modification of the conditions of setting the problem. All these contributed to ensure an immediately operative character of the model, on the basis of which decisions for the current planning period are already formulated.

We estimate, in the same time, that certain aspects presented here can be used for other branches, taking, of course, into account their specific characteristics.

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MODELES POUR OPTIMISER LES INVESTISSEMENTS DANS L'INDUSTRIE DES ENGRAIS CHIMIQUES

Dans cet article l'auteur présente un modèle pour optimiser les investissements dans l'industrie des engrais chimiques. Le modèle appartient à la classe des problèmes de programmation mathématique, étant donc, accessible aux spécialistes familiarisés à ces sortes de problèmes. Le modèle a été résolu de façon effective, constituant le fondement de la planification dans l'industrie des engrais chimiques.

Le modèle comprend trois parties distinctes (trois sous-modèles) qui sont résolus de façon successive, de telle manière que les résultats du premier sous-modèle représentent les données d'entrée pour le second, et les résultats de ce dernier — les données d'entrée pour le troisième sous-modèle.

La première partie de l'article présente la façon de calculer le nécessaire d'engrais chimiques de l'agriculture à base d'une prognose économétrique. Dans la suite, l'article présente un modèle dynamique pour établir la structure optime dans l'industrie des engrais chimiques et utilisant cette structure, on établit, à la fin, la façon optime d'emplacer les objectifs et de satisfaire les nécessités de l'agriculture.

МОДЕЛИ ДЛЯ ОПТИМИЗАЦИИ КАПИТАЛЬНЫХ ВЛОЖЕНИЙ В ПРОМЫШЛЕННОСТИ ХИМИЧЕСКИХ УДОБРЕНИЙ

В настоящей статье приводится модель для оптимизации капитальных вложений в отрасли химических удобрений. Модель входит в класс задач математического программирования и, следовательно, доступна специалистам, занимающимся такими задачами. Модель была фактически решена и является основой планирования промышленности химических удобрений.

Модель содержит три отдельные части (три подмодели), которые решаются последовательно таким образом, что результаты решения первой подмодели представляют собой вводные данные для второй подмодели, а результаты решения последней являются вводными данными для третьей подмодели.

В первой части статьи приводится способ расчета потребности сельского хозяйства в удобрениях на основе эконометрического прогноза. Далее в статье приводится динамическая модель установления оптимальной структуры отрасли химических удобрений, а на основе этой структуры устанавливается, в конечном итоге, оптимальный способ размещения объектов и обеспечения потребностей сельского хозяйства.