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Applications of Mathematical Programming in Determining the Technical and Running Capacity of Mechanical Processing Sections in Machine-Building Enterprises

The article presents the numerical results concerning three models of technical and running capacity calculation; the first concerns the maximum technical capacity; the second and the third take into account the funds of equipment working operation (the second in three shifts, the third in two shifts).

The consistent promotion of our country's policy of socialist industrialization in the core of which is the predominant development of heavy industry and of its pivot — the machine building industry — assures the incessant progress of the level of technical endowment of production, as well as the constant high rate of its development.

The involvement of the process of production, the diversifying of the branches of national economy, in the first place of industry, and the broadening and deepening of the links beween the cells of national economy and between them and their higher bodies, contribute directly to the necessity of raising management and of organizing production along ever improved stages, corresponding to the more and more complex tasks which they must solve, both at the level of national economy or of its branches, and at the level of enterprises, production departments, etc. In the activity of production management, the most advanced methods which contemporary science places at the disposal of the staff, in economy, occupy an ever increasing place.

Taking into account the achievements obtained, the necessities and the possibilities offered by contemporary science, the IXth Congress of the Romanian Communist Party, as well as the plenum of the Central Committee of the Romanian Communist Party held on June 27 th — 28 th, 1966, granted particular attention to the problems of improving the managing activity and scientific organization of production. As stressed by Nicolae Ceauşescu, general secretary of the Central Committee of the Romanian Communist Party, in the closing speech made at the June plenum, "the increase of economy in amplitude and complexity demands the introduction of strict economic evidence, it requires the improvement of the economic information system and the wide-scale use of electronic computing technique in design, production programming, transportation and other economical activities".

Among the many problems in the organization of production we have chosen as subject the study of the optimum turning to account of technological equipment, which constitutes a major concern of our machine-building industry.

In view of achieving such a turning to account, studies of a methodological character are necessary, as well as concrete studies in every industrial unit, in order to find optimum solutions for technical-economical and operative schedule planning. It is well-known that the scientific basis for the substantiation of the technical, industrial and financial plan is constituted by technical-economical computations. Particularly, the scientific basis to substantiate the production plan—main chapter of the technical, industrial and financial plan—is constituted by the technical-economical computations referring to the possibilities of developing production during the plan period. Among them the main part is played by the computations referring to the determination of production capacity. It should be stressed from the very beginning that these studies generally lead to systems of equations having an infinite number of solutions, the optimum of which can only be determined by means of the modern methods of mathematic programming; in view of choosing the operative optimum variant it is necessary for these computations to be programmed on electronic computers.

In this study the authors present some results obtained by means of up-to-date methods of calculus, in comparison with the solutions recommended on basis of present-day practice relating to the maximum possible loading of the technological equipment in a machine building enterprise.

COMPUTATION BY LINEAR PROGRAMMING OF THE TECHNICAL CAPACITY OF MECHANICAL PROCESSING SECTION.

Let us assume that the section A has m machine tools which can be used to manufacture n types of parts. The processing time t_{ij} of part j on machine i is known, or as sometimes stated, the time standard per unit of product. Likewise, the fund of working time t of the equipment or of the machine i is known. The maximum technical capacity is sought, i.e. the potential production possibility of the section according to the list of products assigned to the respective section.

As it will be seen, no restrictive conditions of any kind and no limitations occurring on account of planned tasks are set. This computation should always be carried out before the plan tasks are received, and it should constitue a study on basis of which the production plan of the respective unit is to be established.

The mathematical model of the problem is:

Find the maximum of the function

$$F = \sum_{j=1}^{n} \tau_j x_j$$

in the conditions

$$\sum_{j=1}^{n} t_{ij} x_j \leqslant t_i \qquad i = 1, \ldots, m \qquad (a)$$

$$x_{ij} \geqslant 0$$

where $\tau_j = \sum_{i=1}^{m} t_{ij}$, and x_j will represent the number of parts j included in the assortment which assures the maximum use of the equipment.

Asuming that, in view of satisfying various requirements of the national economy or contract demands, the plant is required, as a plan task, to manufacture certain quantities of products. In this case, the problem arises of determining the quantity of each type of product (j) that can be processed as a supplement, on basis of that part of the equipment working time fund, which is not used for manufacturing quantities of planned products on the compulsory list. These products can be supplied to third users, the optimum use of the equipment being thus assured even under these, more restrictive, conditions.

In this case, conditions of the type

$$x_j \geqslant b_j \geqslant 0, \qquad j = 1, \ldots, n \tag{b}$$

are added to the conditions (a).

As known, the value of this optimum decreases with the addition of new conditions.

Hereunder, numerical results will be presented regarding 3 computing models of technical and runing capacity. The initial data are presented in Tables 1 and 2 and concern the section of mechanical processing in a machine-building plant. Five products A, B, C, D, E, are included in the section manufacturing list. They must be processed on 18 groups of machines, interchangeable from the technological standpoint.

Model 1 determines the technical capacity and, concomitantly, the optimum assortment corresponding best to the structure of the existing equipment stock, taking into account the restrictions regarding the yearly technical working fund.

This is shown numerically hereunder:

The maximum of the following function is sought:

$$F = 47.97\, x_1 + 27.54\, x_2 + 47.59\, x_3 + 6.29\, x_4 + 69.73\, x_5$$

the unknowns x_i being subjected to the following restrictions:

$$\begin{aligned}
&4x_1 + 3.8x_2 + 4x_3 + 0.35x_4 + 1.2x_5 \leqslant 58\,240\\
&2x_1 + x_2 + 0.6x_3 + 0.5x_4 + 4x_5 \leqslant 41\,600\\
&28x_1 + 8.5x_2 + 32x_3 + 4x_4 + 21x_5 \leqslant 524\,160\\
&3x_1 + 2.2x_2 + 4x_3 + 0.8x_4 + 17x_5 \leqslant 83\,200\\
&5.5x_1 + 2.2x_2 + 3x_3 + 0.18x_4 + 6x_5 \leqslant 74\,880\\
&0.35x_1 + 0.19x_2 + 0x_3 + 0x_4 + 16x_5 \leqslant 24\,960\\
&0.5x_1 + 5x_2 + 0x_3 + 0x_4 + 0x_5 \leqslant 24\,960\\
&0.21x_1 + 0.09x_2 + 0.22x_3 + 0x_4 + 1.5x_5 \leqslant 16\,640\\
&0.5x_1 + 1.5x_2 + 0x_3 + 0x_4 + 0.5x_5 \leqslant 8\,320\\
&0.8x_1 + 0.25x_2 + 0.77x_3 + 0x_4 + 1.5x_5 \leqslant 24\,960
\end{aligned} \tag{a}$$

$$0.35x_1 + 0x_2 + 0.4x_3 + 0x_4 + 0.33x_5 \leqslant 8\,32$$
$$0.5x_1 + 0x_2 + 2x_3 + 0x_4 + 0.7x_5 \leqslant 33\,280$$
$$0.22x_1 + 0.5x_2 + 0.6x_3 + 0x_4 + 0x_5 \leqslant 16\,640$$
$$0.25x_1 + 0.45x_2 + 0.3x_3 + 0x_4 + 0x_5 \leqslant 8\,320$$
$$0.4x_1 + 0.28x_2 + 0.5x_3 + 0.11x_4 + 0x_5 \leqslant 16\,640$$
$$0.17x_1 + 0.2x_2 + x_3 + 0.36x_4 + 0x_5 \leqslant 16\,640$$
$$1.2x_1 + x_2 + 0.3x_3 + 0x_4 + 0x_5 \leqslant 24\,960$$
$$0.02x_1 + 0.4x_2 + 0.3x_3 + 0x_4 + 0x_5 \leqslant 8\,320$$
$$x_i \geqslant 0$$

where

$$\tau_j = \sum_{i=1}^{18} t_{ij}$$

The system of restrictions which in this case is a system of inequations may, as known, be transformed by adding the difference, or balance variables y_i in a system of equations:

$$\sum_{j=1}^{n} t_{ij}x_j + y_i = t_i \qquad (a')$$

In our case the system of restrictions leads to a system of 18 equations with $5+18 = 23$ unknowns, a system which generally has an infinite number of solutions. There results clearly the proper character, not to be replaced, of the method of mathematical programming in solving problems regarding the determination of production capacity; any empirical solution has only a very small probability of reaching the optimum solution.

The problem has been solved both manually — by means of the simplex method — and by means of a type BESM electronic computer. The results and solving time are shown in the table hereunder.

	Optimum assortment		Time used in the conditions the computation was made (minutes)		
	Computed manually	Computed on computer		Manually	By computer
x_1	9 884	10 018	Plant data systematized for computation	10	25
x_2	0	0	Band punching	—	10
x_3	1 477	1 457	Problem solving	900	2
x_4	30 756	30 650	Total	910	37
x_5	1 363	1 340			

This solution leads to a coefficient of equipment loading

$$k = \frac{\sum_{j=1}^{n} \tau_j x_j}{\sum_{j=1}^{m} t_i} = \frac{835255}{1015040} \cdot 100 = 82.3\%$$

This loading coefficient represents the maximum possible loading in the framework of the list of products A, B, C, D, E and it is particularly interesting in view of estimating any other variant.

It is worth noticing that the task established by plan: $x_1 = 5\,300$ parts/year, $x_2 = 1\,700$, $x_3 = 100$, $x_4 = 700$, $x_5 = 563$ leads to a loading coefficient

$$K = \frac{32\,341}{1\,015\,040}\,100 = 31.8\%$$

It might be of interest to know if this planned task is the consequence of requirements or if it has been considered "according to earlier experience", that the section is unable to produce better.

Regarding the efficiency with which such computations, manually very elaborate, can be carried out by machine, the data shown are edifying.

Models 2 and 3 add to the type (*a*) restrictive conditions, other restriction conditions of type (*b*), which represent conditions for the achievement of a normal assortment, compulsory by plan. These conditions are the same for both models. In both models t in the conditions (*a*), represent (in comparison with model 1 where they represented technical working funds of the machines), normal running funds. The methods differ from each other by the fact that in model 2 three daily shifts are considered, while in model 3, only two.

The numerical aspect of model 2 is the following:

$$\max F = 55.07952x_1 + 31.47136x_2 + 55.3413x_3 + 7.221487x_4 + 77.67902x_5$$

$$x_j \geqslant 0$$

in the conditions

$$4.3863x_1 + 4.2045x_2 + 4.3181x_3 + 0.5x_4 + 1.5x_5 \leqslant 49\,000$$

$$1.9611x_1 + 1.3888x_2 + 0.6888x_3 + 0.6111x_4 + 4.1055x_5 \leqslant 35\,000$$

$$33.19368x_1 + 9.18269x_2 + 34.72527x_3 + 4.450549x_4 + 23.26236x_5 \leqslant 441\,000$$

$$5.3464x_1 + 2.9583x_2 + 4.1904x_3 + 0.9702x_4 + 18.1011x_5 \leqslant 70\,000$$

$$6.02335x_1 + 2.5412x_2 + 3.85989x_3 + 0.26785x_4 + 7.01923x_5 \leqslant 63\,000$$

$$0.2651x_1 + 0.1893x_2 + 18.106x_5 \leqslant 21\,000$$

$$0.4464x_1 + 5.739x_2 \leqslant 21\,000$$

$$0.21291x_1 + 0.08929x_2 + 0.21978x_3 + 2.00549x_5 \leqslant 14\,000 \qquad (a)$$
$$0.6726x_1 + 1.7500x_2 + 0.5416x_5 \leqslant 7\,000$$
$$0.80357x_1 + 0.23351x_2 + 0.76923x_3 + 1.90247x_5 \leqslant 21\,000$$
$$0.4924x_1 + 0.5x_3 + 0.3333x_5 \leqslant 7\,000$$
$$0.6776x_1 + 2.0165x_3 + 0.8016x_5 \leqslant 28\,000$$
$$0.2196x_1 + 0.681x_2 + 0.7196x_3 + 0x_5 \leqslant 14\,000$$
$$0.47x_1 + 0.59311x_2 + 0.3826x_3 \leqslant 7\,000$$
$$0.49450x_1 + 0.28159x_2 + 0.69368x_3 + 0.10989x_4 \leqslant 14\,000$$
$$0.16995x_1 + 0.26864x_2 + 1.38706x_3 + 0.36184x_5 \leqslant 14\,000$$
$$1.3409x_1 + 1.1515x_2 + 0.4166x_3 \leqslant 21\,000$$
$$0.02060x_1 + 0.46703x_2 + 0.45329x_3 \leqslant 7\,000$$

$$\left.\begin{aligned} x_1 &\geqslant 5\,300 \\ x_2 &\geqslant 1\,700 \\ x_3 &\geqslant 100 \\ x_4 &\geqslant 700 \\ x_5 &\geqslant 563 \end{aligned}\right\} \qquad (b)$$

The optimum solution is

$$x_1 = 5\,300\,;\ x_2 = 1\,700\,;\ x_3 = 488.948,\ \text{thus } 489$$
$$x_4 = 30\,438\,;\ x_5 = 849.453,\ \text{thus } 849$$
$$\max F = 658273 \cdot 88$$

The loading coefficient of the equipment for this assortment is

$$k_1 = \frac{658273.88}{1015040}\,100 = 64.8\%$$

with respect to the technical time fund and

$$k' = \frac{658\,273.88}{854\,000}\,100 = 77\%$$

with respect to the normal running fund.

Therefore, even in the conditions where the plant must achieve, by plan, a certain given assortment, it still has the possibility to contract — with other users — delivery of products over and above the nominal task, thus leading to a much better employment of the equipment (or leading to suggest a change in the plan to the competent authorities).

Model 3, which we do not write since it is similar to model 2 (only the coefficients t_i are changed in comparison to model 2, according to column 3 in Table 2), has led us to put into discussion a problem that is difficult for enterprises, viz. that of the "narrow points".

Solutions for eliminating the "narrow points"

Effectively, in the example taken and in the conditions of work in 2 shifts, the carrying out of the compulsory plan is not possible. From the mathematical standpoint this means that the system of restrictions is incompatible and this incompatibility appears in inequation 9. From the technical-economical viewpoint, this means that the plant must acquire additional equipment of the type $i = 9$.

Even in the conditions where the problem is compatible with respect to the global yearly limitations t, we may reach "narrow points" in the efficient planning of production, which leads us to solve some problems regarding the process, in several stages (1).

Generally, the problems with reference to narrow points lead to problems of dynamic programming.

One of the simplest forms for solving this type of problems — but not always the best — is to suppress this condition.

Table 2

TECHNICAL AND RUNNING OPERATION FUND OF MACHINE-TOOL GROUPS

Machine group	Yearly technical working fund (machine-hours/year)	Yearly running operation fund	
		Three shifts per day	Two shifts per day
1	58 240	49 000	32 662
2	41 600	35 000	23 330
3	524 160	441 000	293 958
4	83 200	70 000	46 660
5	74 880	63 000	41 994
6	24 960	21 000	13 998
7	24 960	21 000	13 998
8	16 640	14 000	9 332
9	8 320	7 000	4 666
10	24 960	21 000	13 998
11	8 320	7 000	4 666
12	33 280	28 000	18 664
13	16 640	14 000	9 332
14	8 320	7 000	4 666
15	16 640	14 000	9 332
16	16 640	14 000	9 332
17	24 960	21 000	13 998
18	8 320	7 000	4 666
	1 015 040	854 000	569 252

By solving the problem without condition (9) we reached the following solution:

$$x_1 = 5\,300\,;\ x_2 = 1\,700\,;\ x_3 = 100\,;\ x_4 = 1\,634.76,\ \text{thus } 1635$$

$$x_5 = 677.71\,;\ \max F = 415415$$

$$K = \frac{415415}{569252} = 72.97\%$$

on condition that, by purchasing additional machine-tools of the type $i = 9$, a time fund $t_9 = 6\,907$ should be assured.

Both methods were solved by means of Romanian computers through the method of the pilot variables. Problem 2 was solved on the CET 500 computer in less than 15 minutes, and the preparation and introduction of the data lasted 1 hour. Problem 3 was solved by means of the CIFA 102-Economist computer, the time of computation being approximately 1 hour.

Before concluding the account of these results we must point out that we consider these computations altogether necessary for the estimated determination of these essential indicators; we are aware that, in reality, matters are much more complex.

These indicators are, however, very useful in showing the direction towards which the efforts to organize and rationalize work should be made. In the second place, we mention that other models may also be considered, in which, for instance, certain economic indicators should be considered explicitly, such as the (minimum) cost price, the (maximum) profit, taking into account the sales price, taxes, raw material, machine amortizing, repairs, insurance, etc.

BIBLIOGRAPHY

1 Bellman Richard: Dynamic Programming. Princeton University Press, 1957, pag. 207.

2 Kern W.: Die Messung industrieller Fertigungskapazitäten und ihre Ausnutzung, Köln, 1952.

3 Rottner Jean: Le service opeıationnel de production de la SACM à Mulhouse. Automatisme IX, nr. 10/1964.

4 Bîrlea St., Homoş T., Mărgărit G.: Unele aspecte ale folosirii calculelor capacităţii de producţie în organizarea şi planificarea producţiei întreprinderilor constructoare de maşini. Metalurgia şi construcţia de maşini, nr. 5, mai 1961.

APPLICATIONS DE LA PROGRAMMATION MATHÉMATIQUE POUR LA DÉTERMINATION DE LA CAPACITÉ DE PRODUCTION DES ENTREPRISES CONSTRUCTRICES DE MACHINES

Dans le but de donner un fondement au plan de production, il convient de déterminer dans le cadre des calculs technique et économiques la capacité technique maximale des outillages, c'est-à-dire les possibilités potentielles de production en fonction de la liste des produits qui doivent être fabriqués selon la spécialisation de l'entreprise. Ces investigations mènent en général à des systèmes d'équations avec un nombre infini de solutions dont la solution optimale ne peut être déterminée qu'à l'aide de la programmation mathématique. Le modèle mathématique du problème est:

$$\max F = \sum_{j=1}^{n} \tau_j x_j$$

dans les conditions:

$$\sum_{i=1}^{n} t_{ij} x_j \leqslant t_i$$

$$x_{ij} \geqslant 0$$

L'article présente les résultats numériques concernant trois modèles de calcul de la capacité technique et de régime ; le premier concerne la capacité technique maximale ; le deuxième et le troisième concernent les fonds de régime de travail des outillages (le deuxième dans trois tours et le troisième dans deux tours).

ПРИМЕНЕНИЕ МАТЕМАТИЧЕСКОГО ПРОГРАММИРОВАНИЯ ПРИ ОПРЕДЕЛЕНИИ ПРОИЗВОДСТВЕННОЙ МОЩНОСТИ НА МАШИНОСТРОИТЕЛЬНЫХ ПРЕДПРИЯТИЯХ

В целях обоснования производственного плана в процессе технико-экономических расчетов необходимо определение наибольшей технической мощности, то есть потенциальных производственных возможностей в соответствии с номенклатурой продукции, подлежащей выпуску по профилю данного предприятия. Это изучение приводит в общем, к системам уравнений с бесконечным числом решений, из которых оптимальное решение можно определить только при помощи математического програмированиям. Математическая модель задачи является следующей:

$$\max F = \sum_{j=1}^{n} \tau_j x_j$$

в условиях:

$$\sum_{j=1}^{n} t_{ij}\, x_j \leqslant t_i$$

$$x_{ij} \geqslant 0$$

В статье приводятся числовые результаты 3 моделей расчета технической и режимной мощности; первая модель относится к наибольшей технической мощности; вторая и третья модели относятся к фондам режима работы оборудования (вторая модель — в 3 смены, а третья — в 2 смены).