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Non-Linear Programming Model for Resource Allocation*

The purpose of this paper is to present, on the basis of Chenery-Kretschmer and Chenery-Uzawa models a nonlinear programming technique that can be used as a guide for policy making in economic development.

I. INTRODUCTION

The problem of existence of structural disequilibrium in the less developed economies has long been recognized as a retarding factor in their economic development. It indicates the inefficiency of the pricing system in those countries as a means of allocating resources. Because of the imperfections in the market system, the central planning is meant to replace to a certain degree the markets and to guide the allocation of resources in order to achieve the desired goals specified in a development plan. To plan resource allocation for achieving desired goals planning agencies have been set up in many less developed countries with the hope that governmental actions and planning may speed up the development process. But the planning methods used in resource allocations in some of those countries are somewhat primitive and not based on rational calculus. The planners are to decide which investment project should be given precedence and priority over others, and these decisions are often based on noneconomic factors also.

Thus the need for a more efficient criteria for resource allocation has focused the attention of the economists on the use of inter-industry analysis. Input-output tables are now available for many countries, but the need for

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the alternative allocation possibilities has resulted in an emphasis on th use of some type of programming technique as a guide for economic polic decisions. There has been some research done on the use of linear progra mming in economic development but the basic assumption of linear progra mming that any commodity should be homogeneous in all its uses, has crea ted some statistical limitations on the use of linear programming in economi development.

Another technique which has produced some plausible results as a too for resource allocation and a guide for policy decision making, is the tech nique of non-linear programming. The contributions in this area, with possibl directions for application are the Chenery-Kretschmer and Chenery-Uzaw models [1], which present alternative pricing system and techniques of produc tion. It is therefore, the purpose of this paper to present, on the basis o Chenery-Kretschmer and Chenery-Uzawa models (1) a non-linear progra mming technique that can be used as a guide for policy making in econo mic development, and (2) to apply the technique to a four sector model o India for two different time periods. (1965 as a target year with price o foreign exchange (dollar exchange rate for Indian rupees) at $P_f =$ Rs 4.8 an the price of labor at $P_w = 0$; and for the target year 1970 with $P_f =$ Rs 7.5 and $P_w = 0$. For 1970 the same model is used to project the optimum level of prices. The price of labor is assumed to be zero on the ground and tha supply of labor is in abundance).

II. A. THE MODEL

The objective of the model is to find optimum levels of domestic prices production, imports and exports which would minimize the total investmen and at the same time satisfy a certain set of constraints on primary input and the total national output for the target year.

Mathematically:

$$Min \cdot C = \sum_{j=1}^{n} \left(\alpha j + \beta j \frac{Xj}{Xj + Mj} \right) Xj \qquad (1$$

$$\sum_{j=1}^{n} wj Xj \leqslant L \qquad (2$$

$$\sum_{j=1}^{n} \lambda_j M_j - \sum_{1}^{n} (\mu_j + \varepsilon_j E_j) E_j \leqslant \Omega \qquad (3$$

$$\sum_{j=1}^{n} Xj^{(t)} \geqslant Y^{(t)} \qquad (4$$

$$\sum_{j=1}^{n} \frac{cj}{\theta j} = L^* \qquad (5$$

$$Xj,\ E_j,\ M_j \geqslant 0 \qquad (6$$

[1] Chenery and Kretschmer, *Econometrica*, Vol. 23—24, No. 4, pp. 265—399, Octob 1956, and Chenery and Uzawa, in K. Arrow, et al, (ed.) *Studies in Linear and Non-Linear Pr grammming*, pp. 203—299.

II. B. DEFINITION OF SYMBOLS USED

C = total investment to be minimized
c_j = investment in sector j
$X_j^{(t)}$ = total output of sector j in the target year t
$Y^{(t)}$ = total output of the economy projected for the target year t
E_j = total export of sector j
M_j = total import of sector j
L = labor supply
Ω = foreign exchange deficit
Y_j = the net final demand for the output of sector j in the target year to be satisfied from the increase in domestic production or imports.
$\pi_j = \alpha_j + \beta_j\delta_j$ the capital output function in sector α_j and where and β_j are parameters indicating the variations in capital output function and where
δ_j = the ratio of domestic production plus imports or

$$\delta_j = \frac{X_j}{X_j + M_j}$$

$\varnothing j = \mu_j + \xi_j E_j$ the price of exports of sector j in foreign currency, where μ_j and ξ_j express the variations in demand for exports functions.
m_j = import coefficient in sector j where $m_j = \dfrac{1 - \delta_j}{\delta_j}$
a_{ij} = input-output coefficient
n_i = is the capital output coefficient for export sector
w_j = the marginal labor coefficient in sector j
θ_j = the net investment required per engaged person
L_* = the employment potential of C investment
λ_j = the ratio of import prices to domestic prices in sector j at a given exchange rate and
n = number of sectors

II. C. THE METHOD OF SOLUTION

The technique presented here as stated earlier, follows the Chenery-Kretschmer and Chenery-Uzawa method of solution which incorporates some aspects of the techniques of linear programming, input-output, quadratic programming, the Seidel method of successive approximation to the solution and finally the concept of shadow prices.

The concept of shadow prices is used to determine the total cost of using the scarce resources in the economy to produce a given commodity. The shadow prices in effect determine the opportunity cost of each input. The nonegative shadow prices must be chosen so that if we increased one unit of the input to which the shadow price is assigned we would attain the largest possible increase in the investment cost of producing that commodity for which the input is being utilized. For example the shadow price of foreign exchange should be that amount such that if one unit of foreign exchange were to be

withdrawn we would obtain the largest possible increase in the investment cost. Similarly the labor should be imputed that value which would be equivalent to the largest possible increase in investment cost if one unit of labor were to be withdrawn. [2]

After imputing such shadow prices to labor and foreign exchange we find the optimum levels of production, exports, imports and also derive the shadow prices for each sector as follows:

II. C. 1. *Derivation of Shadow Prices.*

These prices should be derived so that they would determine the unit costs of production in each sector. Shadow prices so derived will also determine whether it is feasible to produce commodities domestically or to import them. To do this we have to first determine the investment cost of using other commodities as imputs to produce a particular commodity. Such analysis will take three possible forms.

Case I; where $P_j^{(t)} = \lambda_j p_f$(6) if it is determined that $\delta_j < 0$ (for the derivation of δ_j see below). This means that the domestic price for this particular sector must be as high as the import price, otherwise it is best to import rather than to produce domestically.

Case II. Where $$P_j^{(t)} = \lambda_j P_f - \frac{1}{4\beta_j}\delta_j^2 \tag{7}$$

if it is determined that $\sigma_j \leqslant 2\beta_j$ which would indicate that if it is determined that σ_j is less than the twice the capital coefficient in that sector, the price will be less than that of the price of importing the same commodity. And finally

Case III: where $$P_j^{(t)} = \lambda_j P_f - \sigma_j + \beta_j \tag{8}$$

if it is determined that $\sigma_j > 2\beta_j$ which means that σ_j is greater than twice the capital coefficient and thus the domestic prices must either be higher than the import prices in sector j, or that the country should import rather than to produce domestically. [3]

In all cases above σ_j is determined from the following formula:

$$\sigma_j = \lambda_j P_f - \sum_{i=1}^{n} aij P_i - w_j P_w - \alpha j \tag{9}$$

Where j is the sector for which the price is being computed and thus should remain constant.

II. C. 2. *Optimum Proportion of Domestic Production.*

As was mentioned previously equation (9) can be used as a criterion by which one can judge what kind of relationship the domestic prices in each sector should have to the import prices. With this in mind the fraction that

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[2]) *Econometrica*, Ibid., p. 376.

[3]) for mathematical proof refer to *Econometrica, Ibid.*, p. 379.

is being produced domestically must minimize the cost of producing the fraction δ_j and importing the fraction $(1 - \delta_j)$. Mathematically δ_j must minimize the following function.

$$T_j = \left(\sum_{i=1}^{n} a_{ij}P_i + w_jP_w + \alpha_j + \beta_j\delta_j\right)\ \delta_j + \lambda_jP_f(1 - \delta_j) \tag{10}$$

This function has a minimum since $T'' = 2\beta_j > 0$
Thus the minimum of T_j is obtained at

$$T'_j = \frac{\lambda_jP_f - \sum_{i=1}^{n} a_{ij}P_i - w_jP_w - \alpha_j}{2\beta_j} \tag{11}$$

The numerator here is nothing more than the σ_j of equation (9) above.
Thus (11) becomes

$$T'_j = \frac{\delta_j}{2\beta_j} \tag{12}$$

Here too we have three possible solutions:

I. $$\text{If } T'_j \leqslant 0 \tag{13}$$

$\delta_j = 0$ since it is not possible to produce a negative amount.

II. $$\text{If } 0 \leqslant T'_j \leqslant 1 \tag{14}$$

our $\delta_j = T_j$ which means a positive amount.

III. $$\text{If } T'_j \geqslant 1 \text{ Then } \delta_j = 1 \tag{15}$$

which indicates that the economy can produce domestically and need not import any fraction of total output in sector j.

II. C. 3. *Optimum Export Levels.*

We assume that the only purpose for which the economy is exporting is to obtain enough foreign exchange to purchase imports. Thus the optimum export level in each sector must maximize the difference between the price received for exports and the cost of producing them. Mathematically E must maximize.

$$\theta_j = (\mu_j + \xi_jE_j^2)\ P_f - \left(\sum_{i=1}^{n} a_{ij}P_i + w_jP_w + n_j\right) E_j$$

since $\theta''_j = 2\xi_jP_f$ which is negative. Thus the function has a maximum which is obtained at

$$\theta'_j = \frac{\sum_{i=1}^{n} a_{ij}P_i + w_jP_w + n_j - \mu_jP_f}{2\xi_jP_f} \tag{16}$$

There are two possible cases here.

I. if $\theta'_j \geqslant 0$ then $E_j = \theta'_j$ (17)

II. if $\theta'_j < 0$ then $E_j = 0$ (18)

since E_j must be a positive amount.

II. C.4 *Optimum Levels of Domestic Production* $x_j^{(t)}$.

To find these levels we use the successive approximation to solution method. We first assume some $x_j^{(0)}$ such that they would satisfy the following condition:

$$x_j^{(0)} \geqslant \frac{1}{1 - a_{ij} + m_j} [Y_j + \Sigma_j a_{ij} E_j]$$

And with the results obtained from computing the values for σ_j, P_j, and E_j we find that production level which will satisfy the following equation: [4]

$$X_j^{(t)} = \frac{1}{1 - a_{ii} + m_j} \left[Y_j^{(t)} + \sum_{j=1}^{n} a_{ij} E_j + \sum_{j=1}^{i-1} a_{ij} X_j^{(t)} + \sum_{j=i+1}^{n} a_{ij} X_j^{(0)} \right] \quad (19)$$

It is evident from the above formula that each sector must produce enough to satisfy some specified net final demand in the target year and also supply other sectors which use the former's output as input to produce both for domestic production and exports.

II. C. 5 *Optimum Import Level*

We defined

$$\delta_j = \frac{X_j}{X_j + M_j}$$

and

$$m_j = \frac{1 - \delta_j}{\delta_j}$$

from these two definitions we derive

$$M_j = m_j X_j \quad (20)$$

After finding the optimum levels of export, import, domestic production and also the prices for each sector we have to see whether the solution satisfies the objective function and the constraints of the problem. If the solution satisfies the objective function and does not violate the constraints we try one or more iteration with higher or lower shadow prices for labor and foreign exchange to see if a better solution is possible.

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4) This formula is recursive; for mathematical proof and discussion refer to *Econometrica, Ibid.*, p. 382.

If, however, the constraints are violated we have to successively vary and revise the shadow prices of the primary resources until we find two successive answers which equally satisfy the requirements of the problem, this then would be the final solution to the programming problem.

III. APPLICATION OF THE MODEL TO THE INDIAN ECONOMY

III. 1. *For target year 1965*

The Problem: Based on the given input coefficients we want to find such price leves, and such levels of exports, imports, and domestic production that will minimize the total investment required without exceeding the primary input constraints and also satisfying the total output constraint in the target year. Mathematically:

$$Min\ C = \sum_{j=1}^{n} \left(\alpha_j + \beta_j \frac{X_j}{X_j + M_j} \right) X_j \tag{21}$$

Subject to:

$$\sum_{j=1}^{n} w_j X_j \leqslant 1{,}600 \tag{22}$$

$$\sum_{j=1}^{n} \lambda_j M_j - \sum_{j=1}^{n} (\mu_j + \xi_j E_j)\, E_j \leqslant 3{,}200 \tag{23}$$

$$\sum_{j=1}^{n} X_j^{(t)} \geqslant 20{,}000 \tag{24}$$

$$\sum_{j=1}^{n} \frac{c_j}{\theta_j} = 3.5 \text{ mil. per year or } 17.5 \text{ Million for the plan period} \tag{25}$$

$$x_j,\ M_j,\ E_j \geqslant 0 \tag{26}$$

Sources of Data: The input coefficients given in Table I is based on an aggregation of 1952—1953 input-output flow table and coefficient table for India. The employment, capital, foreign exchange, export demand function coefficients and Y_j the net final demand, have been obtained or estimated from the various publications of the government of India. $X_j^{(0)}{}'s$ of table IV. have been assumed as the first approximation to solution such that they would satisfy the following criterion. [5]

$$X_j^{(0)} \geqslant \frac{1}{1 - a_{ii} + m_j} \left(Y_j + \sum_{j=1}^{n} a_{ij} E_j \right) \text{ for } j = 1 \ldots 4 \tag{27}$$

$P_j^{(0)}$ have been obtained from an article by Ramachandran and Sengupta [6] and are used here as the first approximation to the price solution. Since labor

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[5]) *Econometrica, Ibid.*, p. 382

[6]) Ramachandram, M. and Sengupta, J.K. „*A Programming Application of the Input-Output Model in India*" Indian Economic Journal; Vol. 9, No. 4, April 1962.

is in abundant supply in India, the shadow price of labor is assumed to be zero. The shadow price of foreign exchange is assumed to be the exchange rate prevailing at 4.8 .

Conclusions about the reasonableness of the solution and the result obtained therefrom can be deduced from the economic projections made for 1965—1966 in the Quarterly Economic Report of the Indian Institute of Public Opinion. If a different set of constraints is desired or formulated the shadow prices for labor and foreign exchange should be successively varied until the constraints are satisfied. A summary of final solutions is given in Table V. Tables III and IV are also given for the purpose of checking and varifying the solutions if it is so desired. Column c_j of table V shows the sectoral investment requirement.

The solutions obtained are contigent upon the fact that prices in each sector should be as follows:

$$P_1 = 4.0103$$

$$P_2 = 3.5168$$

$$P_3 = 3.1971$$

$$P_4 = 2.4500$$

The investment requirement obtained from these solutions is somewhat lower than the investment required for the same time period by the plans for the comparable periods. The investments envisaged by the second and third five year plans covering the period 1956—1966 add up to Rs. 17,150 Crores (public and private investment combined), whereas the period of study covers 1952 to 1965 with the investments of only Rs. 17,384 Crores. It is also concluded that if the total investment is allocated according to the specifications obtained in the problem the employment potential of Rs. 17,384 Crores of investment is about 3.5 million per year or 17.5 millions for the plan period.

III.2. *For target year 1970:*

For the purpose of this section the basic data of the last section with the exception of Y_j will be used. The constraints again have been obtained, estimated and derived from the same official sources of last section. The P_j however has been increased to its present level of Rs. 7.5 Y_j or the final demand in 1970 that is to be satisfied from increase in domestic production or imports are estimated from those of 1965—66 levels of Table II and are consistent with the annual growth rate projected for 1970 in Official Indian Plan documents.

These Y_j's are estimated to be:

Sectors	Y_j
Agriculture	6095.0
Industry	7628.0
Transport	1159.2
Services	2921.4

The shadow prices for this part of the study were assumed to be the same as used in the last section. It was also assumed that the result of the change in P_f has not yet reached the import sector, since the absence of such an assumption would result in total self-sufficiency of the economy by 1970. Thus m_j for this section were assumed to be those of the 1965—66 study.

The first approximations for quantity solutions $X_j^{(0)}$ were estimated so that, both for 1965 and 1970 study, they would meet Chenery's Criterion (27).

These $X_j^{(0)}$'s (in Rs. Crores) were computed to be:

Sector	$X_j^{(0)}$
Agriculture	8584
Industry	8988
Transport	870
Services	3099

The optimum levels derived from the use of the technique is given in Table VI. The optimum growth rate is found to be about 5% per annum and the study shows an increase in foreign exchange deficit for 1970. The agricultural sector is found still to be the sector with highest employment potentials. The industrial sector is found to be the most capital intensive sector. The total employment potential of 17,213 Crores of Rupees investment is found to be about 3.1 million per year or 15 million over the five year period.

The employment potential of investment for the target year 1970 is 3.1 million persons as against the potential of 3.5 million employment for the target year 1965. Therefore if we could assure the average annual employment to be about 3.3 million during the period 1965 to 1970, the total employment during the fourth plan (1965—1970) could be put at 15.5 million approximately.

IV. CONCLUSION

The technique discussed in previous sections, is a single-time period non-linear programming technique applicable to countries, for which a detailed and reliable input-output table is available. The technique appears to be more useful for the problem of resource allocation in developing economics than either of the two techniques, linear programming and input-output analysis. The following reasons would perhaps make the point more clear:

(1) The method does not require that each commodity be homogeneous in all its uses;

(2) it provides for a non-equiproportional capital coefficient, and permits the economy to produce those commodities which require least capital first;

(3) it makes the price of exports an increasing function of the amount exported, because by so doing it makes the amount exported from the economy, a function of the foreign countries demand for the products of the economy in question; i.e.: $\varphi_j = \mu_j + \xi_j E_j$*

* see page 69 for the definition of φ_j.

(4) the amount exported and imported are determined within the system; and

(5) it modifies the pricing system within the economy in question.

There are, however, certain limitations associated with the present method. The estimation of the parameters in the model, demand functions for exports, and capital coefficients require some careful prior studies, and a detailed sectoral accounting record, and an input-output table. In the present study the estimates are obtained from five year plans for India and other official and non-official publications. Even the input-output table for India is condensed to four sectors. The study is illustrative and is in the direction of demonstrating the definite use of the technique in planning for economic development.

Nevertheless, the model is extremely sensitive, and errors of estimation or specifications would make the results sometimes unrealistic. Notwithstanding these limitations, the technique seems to be useful for resource allocation, and suggests the need for more detailed studies to follow with a larger model and bigger input-output table and using the modern electronic computers.

Table I

Input-Output Coefficients

(A)

Sectors		Agriculture	Industry	Transport	Services
1	Agriculture	0.2797	0.1382	0.1483	0.0217
2	Industry	0.0216	0.2593	0.2125	0.0282
3	Transport	0.0012	0.0122	0.0185	0.0105
4	Services	0.0018	0.0478	0.0756	0.0415

(I-A)

Sectors		Agriculture	Industry	Transport	Services
1	Agriculture	$0.7203 + m_1$	-0.1382	-0.1483	-0.0217
2	Industry	-0.0216	$0.7407 + m_2$	0.2125	-0.0282
3	Transport	-0.0012	-0.0122	$0.9805 + m_3$	-0.0105
4	Services	-0.0018	-0.0478	-0.0756	$0.9585 + m_4$

Table II

Other Basic Data

Sectors	α_j	β_j	η_j^*	ξ_j
(1)	2.9900	0.8000	0.8000	−0.001300
(2)	1.6843	1.6670	1.6670	−0.000259
(3)	0.1035	3.0000	3.0000	−2.500000
(4)	0.2000	2.2220	2.222	−3.000000

* We assume that the Commodities produced for exports have the same capital-output Coefficients as the Commodities produced for domestic market.

λ_j	θ_j	μ_j	γ_j	w_j
1.0000	6,250.0	1.0000	5300.0191	0.528900
3.20000	15,000.0	1.0000	5568.3344	0.147300
1.50000	10,000.0	1.0000	962.0974	0.035500
0.0000	3,750.0	1.0000	2355.9846	0.2885600

Price Solution (For $P = 4.800$ $Pw = 0.000$)

Sectors	$P_i^{(0)}$	$P_i^{(t)}$	$-a_{i1P_i^{(t)}}$	$-a_{i1P_i^{(t)}}$
	(1)	(2)	(3)	(4)
(1) Agriculture	0.6001	4.0103	−0.169700	−1.1217
(2) Industry	0.1683	3.5168	−0.002120	−0.0443
(3) Transport	0.0861	2.1971	−0.001000	−0.0038
(4) Services	0.1455	2.4500	−0.000300	−0.0044

$-a_{i2P_i^{(0)}}$	$-a_{i2P_i^{(t)}}$	$-a_{i3P_i^{(0)}}$	$-a_{i3P_i^{(t)}}$	$-a_{i4P_i^{(0)}}$	$-a_{i4P_i^{(t)}}$
(5)	(6)	(7)	(8)	(9)	(10)
−0.0829	−0.5542	−0.0890	−0.5947	−0.0130	−0.0870
−0.0436	−0.9119	−0.0358	−0.7463	−0.0081	−0.1695
−0.00105	−0.0390	−0.0016	−0.0592	−0.0009	−0.0336
−0.0070	−0.1171	−0.0101	−0.1872	−0.0060	−0.1017

Table IV

Quantity Solution

Sectors	$X_j^{(0)}$	$X_j^{(t)}$	$-a_{1j}X_j^{(0)*}$	$-a_{1j}X_j^{(t)}$
	(1)	(2)	(3)	(4)
(1) Agriculture	9078	9078.9300	+6597.8904	+6598.5663
(2) Industry	7113	7113.7800	−983.0166	−983.1244
(3) Transport	929	929.1200	−137.7707	−137.7885
(4) Services	2880	2880.8800	−62.4960	−62.5151

$-a_{2j}X_j^{(0)}$	$-a_{2j}X_j^{(t)}$	$-a_{3j}X_j^{(0)}$	$-a_{3j}X_j^{(t)}$	$-a_{4j}X_j^{(0)}$	$-a_{4j}X_j^{(t)}$
(5)	(6)	(7)	(8)	(9)	(10)
−196.0848	−196.1049	−10.8936	−10.8948	−16.3404	−16.3421
+6262.2852	+6262.9719	−86.7786	−86.7881	−340.0014	−340.0387
−197.4125	−197.4380	+1252.7565	+1252.9183	−70.2324	−70.2415
−138.8160	−138.8584	−30.2400	−30.2493	+2760.4800	+2761.3239

Table V

Summary of Optimum Solutions + for target year 1965

$P_f = \text{Rs. } 4.8 = P_w = 0$

Sectors	Price Solution	Quantity Solutions +	Optimum + Import Levels
Agriculture	4.0103	9078.9300	150.0000
Industry	3.5168	7113.7800	993.7951
Transport	3.1971	929.1200	341.9162
Services	2.4500	2880.8800	3.4571

Optimum + Export Levels	Investment Per Sector + c_j	L_j^* = Employment Potentials
227.8526	5,335.5414	2.1410
607.2910	9,432.4254	0.6738
0.0088	131.7495	0.0088
0.0068	2,466.4884	0.6577

+the unite are in Crores of Rupees.

Investment requirement $C = 17.384.2047$

Labor Requirement $L = 1,537.4555$

Foreign Exchange Deficit $\Omega = 3,170.8765$

Total employment potential of C investment

$L^* = 3.4713$ mil.

Table VI

Summary of Optimum Solutions-Target Year 1970

$P = 7.500$ $^{Rs}P_w = 0$

Sectors	Price Solution	Quantity Solution	Optimum Import Levels
Agriculture	4.0128	10,413.4028	68.1037
Industry	3.5366	9622.9817	1344.3305
Transport	3.4151	1139.9543	419.5032
Services	2.4500	3687.3187	3.4571

Optimum Export Levels	Investment Per Sector	L_j^* Employment Potentials
283.3080	5051.6205	2.0265
1084.8641	7770.0133	0.5550
0.0775	484.1810	0.0323
0.1085	1953.5983	0.5210

Investment Requirement $C = 17{,}213.0114$
Labor Requirement $L = 1{,}315.5502$
Foreign Exchange Deficit $\Omega = 4088.3992$
Total Employment Potential of C Investment
$L^* = 3.13478$ mil.

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UN MODELE DE PROGRAMMATION NONLINEAIRE POUR L'AFFECTATION DES RESSOURCES

Une autre technique utilisée en tant qu'instrument pour affecter les ressources et comme guide pour la politique de la prise des décisions, qui a donné des résultats plausibles est la programmation nonlinéaire.

Des contributions dans ce domaine, ayant diverses possibilités d'applications, sont les modèles de Chenery-Kretschmer et Chenery-Uzawa, qui présentent alternativement le système des prix et les techniques de production.

Le but de cet article est celui de présenter, utilisant les modèles, une technique de programmation nonlinéaire qui peut être employée comme guide dans la politique du développement économique et être appliquée à un modèle à quatre secteurs de l'Inde, pour deux périodes différentes.

МОДЕЛЬ НЕЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ ДЛЯ ВЫДЕЛЕНИЯ РЕСУРСОВ

Другой техникой, давшей определенные допустимые результаты в качестве орудия для выделения ресурсов и руководства для политики принятия решений, является техника нелинейного программирования.

Вклад в эту область с возможными направлениями применения внесли модели Ченери-Кречмера и Ченери-Узава, представляющие систему оценки и технику производства.

Целью настоящей статьи является изложить на основе моделей Ченери-Кречмера и Ченери-Узава технику нелинейного программирования, которая может быть использована: 1) в качестве руководства для политики принятия решений в экономическом развитии; 2) для применения техники секториальной модели Индии для двух различных периодов времени.