G. CRUȚU

A Model for Optimizing Production Organization by means of the Dynamic Programming Method

Oneof the effects of contemporary technical and scientific revolution is an increasing complexity of the problems connected with the organization and management of productive activities.

1. DEFINITION AND RESTRAINTS ON THE DECISION AND STATE VECTORS FOR PRODUCTION ORGANIZATION

The specialists' efforts have more and more been directed lately towards the analysis of the work process and the finding of some generally valid rules for achieving a higher efficiency with lower expenses of social labour. The concern for working out an optimal production plan pertains to this trend. The production plan can be divided into a finite number of stages, which leads to its approach as a discrete system by means of the dynamic programming method [1], [2], [3].

Consider the time interval necessary for organizing and achieving production as divided into a number of TT stages, (1, 2,...T,... TT), where T is a discrete time variable; the structure of a production plan for stages $T + 1$, T, $T - 1$ is represented as a diagram, shown in Fig. 1, where:

$X(T)$ = a decision variable, which is the quantity of products manufactured during stage T;

$S(T)$ = a decision variable, which is the stock during stage T;

$R(T)$ = a state variable, which is the quantity of products to be delivered at the end of stage T.

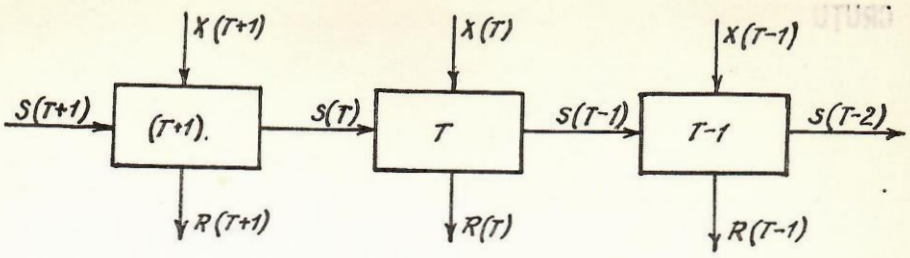

Fig. 1

Consequently, the examination of an optimal policy

$$X^* = (X^*(1),\ X^*(2),\ ...,\ X^*(TT)) \tag{1}$$

leads to a maximization (minimization) of the function-obiective of the transition from stage T — 1 to stage T, written under the form [3], [5].

$$W = w\,(T)\ [R(T-1),\ X(T)] \tag{2}$$

where X(T) is a decision vector, and R(T—1) is a stock vector.

The maximal (minimal) value of the function-objective will be yielded by relationship:

$$f(R(T-1),= \max_{X(T)}\ \{w(T)[R(T-1),\ X(T)]\} \tag{3}$$

This article, in accordance with the requirements of the Braşov Tractor Plant, is meant to find a recurrence relationship of the form (3) enabling the achievement of an optimal stock policy:

$$S^* = (S^*(1),\ S^*(2),\ ...,\ S^*(TT)) \tag{4}$$

under a production capacity C = constant. Consequently, the optimal policies (1) and (4) must lead to a minimization of the sum of production costs and stocking costs during each stage.

The mathematical model of production programming includes the restraints on the decision variables X(T) and S(T). The decision vector R(T) is assumed to be constant and represents the steady production plan, characteristic of socialist economy.

The steps in view of working up relationships expressing stock and production boundary values are given in detail in [8]. For the first two stages

taken together, the lower limit and the upper limit of the stock, respectively of production, are as follows:

$$LIS(2) = \max\begin{bmatrix} 0 \\ R(2) - C \end{bmatrix} \tag{5}$$

$$LSS(2) = \min\begin{bmatrix} C(TT-2) - \sum_{I=T}^{3} R(I) \\ \sum_{I=1}^{2} R(I) \end{bmatrix} \tag{6}$$

$$LIX(2) = \max\begin{bmatrix} 0 \\ R(2) - S(2) \end{bmatrix} \tag{7}$$

$$LSX(2) = \min\begin{bmatrix} C \\ R(2) + R(1) - S(2) \end{bmatrix} \tag{8}$$

For stages $T \geqslant 3$ we find the recurrence relationships:

$$LIS(T) = \max\begin{bmatrix} 0 \\ \sum_{I=2}^{T} R(I) - C(T-1) \end{bmatrix} \tag{9}$$

$$LSS(T) = \min\begin{bmatrix} C(T-1) - \sum_{I=T}^{T+1} R(I) \\ \sum_{I=1}^{T} R(I) \end{bmatrix} \tag{10}$$

$$LIX(T) = \max\begin{bmatrix} 0 \\ \sum_{I=2}^{T} R(I) - S(T) - C(T-2) \end{bmatrix} \tag{11}$$

$$LSX(T) = \min\begin{bmatrix} C \\ R(T) - S(T) + \min\begin{bmatrix} C(TT - T + 1) - \sum_{I=T}^{TT} R(I) \\ \sum_{I=1}^{T-1} R(I) \end{bmatrix} \end{bmatrix} \tag{12}$$

Relationships (5),..., (12) have a form enabling their programming on electronic computer. Thus, while computing the different values of the function-objective and minimizing it, pairs of discrete values S(T), X(T) can be called out of the set of values of the ranges comprised between the lower and the upper limit.

2. FINDING THE COSTS

The function-objective of the model suggested is a sum of the stocking costs and production costs. In this instance, the optimality criterion lies in the minimization of this sum, on condition that the delivery plan is observed and the existing production capacity is fully employed.

a) Stocking costs

During a stage T

$$S_M(T) = 1/2 \cdot (S(T) + S(T-1)) \tag{13}$$

units are on the average stocked. The stocking cost of a unit during a stage (i.e., time unit) shall be:

$$CD(X(T)) = KD \cdot 1/2 \cdot (S(T) + S(T-1)) \tag{14}$$

where:

KD stands for the stocking cost of a unit during a time unit.
In accordance with Fig. 1, we can write:

$$S(T-1) = S(T) - X(T) + R(T). \tag{15}$$

We consider that at the beginning of the time interval the stock is nil, that is $S(O) = 0$, whereas the sum of the stocked products and of those manufactured during stages 1 and 2 is at least equal to the sum of the delivered products; then,

$$S(2) + X(2) + X(1) = R(2) + R(1). \tag{16}$$

Hence, the stocking costs for the first two stages will be expressed by the formula:

$$SCD(X(2)) = KD \cdot 1/2 \cdot (2S(2) - X(2) + R(2)) + KD \cdot 1/2 \cdot S(1). \tag{17}$$

b) Production costs

The structure of production costs of a finished product P at the Braşov Tractor Plant and their dependency on the volume of production is experimentally analyzed in [8]. The necessary formulas for making the calculation on digital computer are found. For a stage T, these costs are given by successive relationships of the form:

$$CP(X(T)) = KP1 \cdot C1 + KP2(C2 - C1) + KP3(C3 - C2) + \\ + KP4(X(T) - C3) + CP(0) \tag{18}$$

where KP1, KP2, KP3 and KP4 stand for the production cost factors corresponding to the production volume expressed by the partial production capacities C1, C2, C3,... The functions of the production costs [8] written in FORTRAN are given in the Appendix.

c) Sum total of costs

For stages T = 2 and T = 1, the sum total of stocking costs and production costs will be:

$$STCDP(2) = KD \cdot 1/2 \cdot (2S(2) - X(2) + R(2)) + KD \cdot 1/2 \cdot S(1) + \\ + KP(X(2) + X(1)) \tag{19}$$

Taking into account relationship (16), relationship (19) becomes:

$$STCDP(2) = KD \cdot 1/2 \cdot (2S(2) - X(2) + R(2)) + KD \cdot 1/2 \cdot S(1) + \\ + KP(R(1) + R(2) - S(2)). \tag{20}$$

The sum total of stocking costs and production costs for the first two stages will be:

$$STCDP(T) = KD \cdot 1/2 \cdot (2S(T) - X(T) + R(T)) + \\ + KP \cdot X(T) + MSTCDP(T - 1). \tag{21}$$

The minimum sum total of costs, written MSTCDP (T — 1) in (21) includes the components of costs calculated for the previous stages comprising the optimal values of the decision variables. It follows that, for the first three stages, the value MSTCDP(2) is computed in accordance with variables:

$$S(2) = S(3) - X(3) + R(3)$$

$$S(1) = S(2) - X^*(2) + R(2) \tag{22}$$

$$X(1) = R(1) + R(2) - X^*(2) - S(2)$$

Relationship (21) is the function-objective. This mathematical model is to be minimized by a succession of computation sequences, starting with the first two stages jointly taken, continuing with the first three stages, etc.

The mechanism of finding the optimal policies is rather elaborate. We have successively cleared it up and cheacked at the Brașov Tractor Plant. The rather big volume of calculi requires an IBM 360/30 computer for about 8 minutes.

3. MINIMIZING THE FUNCTION-OBJECTIVE

The optimal variant for organizing production during a certain stage T will be the one for which the sum of stocking costs and production costs is a minimum [4], [6].

For the first two stages T = 2 and T = 1, the value STCDP(2) is computed in accordance with the pair of values S(2) and X(2) of the range of values found in relationships (5),... ,(8). Values STCDP(2) can be entered in a Fig. 2. The lines of this table are characterized by S(T) = constant. The variable X(T) assumes a variable number of discrete values within this line, but different from a line to another.

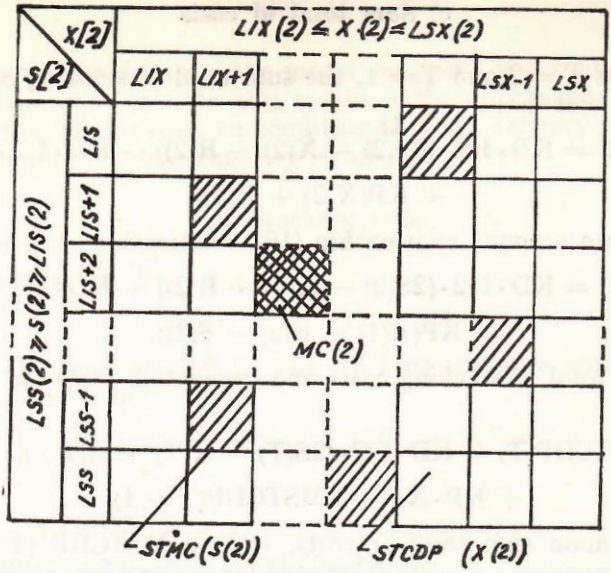

Fig 2

The minimal sum total of costs for S(2) = constant will be:

$$STMC(2) = \min_{X(2)} \{STCDP(2)\} \tag{23}$$

Variable X(2) minimizing sum (23) is an optimal partial value written VOPX(2) in the program. Costs STMC(2) on each line of Fig. 2 are subsequently minimized. A new minimization for all the combinations of values S(2) and X(2) in Fig. 2 leads to the least value of costs for the first two stages:

$$MC(2) = \min_{S(2)} \{STMC(2)\}. \tag{24}$$

In accordance with this sum (MC(2)), the optimal sub-policies X*(2), S*(2) and X*(1), S*(1) are ascertained.

The minimization of the function-objective for the first T stages is performed following the same course. Thus,

$$STMC(T) = \min_{X(T)} \{STCDP(T)\}. \tag{25}$$

The optimal solution for stage T will correspond to the optimal decisions

X*(T) and S*(T), etc.

4. COMPUTING PROGRAM

The computing program written in FORTRAN includes among the initial assignment instructions the values of the cost factors and of the productive capacities, as well as the production plan for product P at the Brașov

Tractor Plant, for a time interval of one year divided into 12 stages. From the computation sample comprising values of the parameters in production with a certain approximation degree, the results written at the printer were obtained. (Table I). The program makes use of a number of auxiliary variables necessary for temporary storage of some intermediate values in the computer, but they do not render more difficult the course of the calculation

Table I

T	X(T)	S(T)
1	53.2	0
2	53.0	0
3	49.0	6
4	62.0	10
5	47.0	6
6	45.0	6
7	56.0	6
8	42.0	2
9	46.0	0
10	54.0	0
11	55.0	0
12	55.0	0

logic. The results put down in Table I supply reasons for optimizing the production organization through the dynamic programming method. The subpolicies X*(T) have values close to the value of the maximum capacity C = 55 products yielded during a stage (a calendar month).

The solution S(T) = 0 for T = 9, 10, 11 and 12 is ideal but inacceptable in practice; all this means that both the structure of the model and, particularly, the values of the parameters, must be improved while the method is being experimented.

The value of the investigation carried out and expounded in this article and in [8], [9], has been corroborated through a comparison with the policies achieved in practice, on watching the product organization during the time interval afore stated.

Appendix

Production cost functions

```
// JOB DINAMIC
// OPTION LINK, NOLOG
INTEGER C, TT, T, RIND, C1, C2, C3, C4, VOS(12), SS(60), S(12)
REAL KD, KP1, KP2, KP3, KP4
DIMENSION R(12), ZWX(12, 60), VVX(12, 60), CPX(12), X(12),
CDX(12)
DIMENSION CD(12), VOX(12), CDXX(60), CP(12)
TT = 12
KD = 16.
```

```
      KP1 = 1.97
      KP2 = 1.49
      KP3 = 1.
      KP4 = 1.97
      C = 55
      C1 = 18
      C2 = 36
      C3 = 55
      C4 = 65
      R(1) = 53.7
      R(2) = 52.5
      R(3) = 56.0
      R(4) = 52.5
      R(5) = 42.2
      R(6) = 41.5
      R(7) = 51.2
      R(8) = 40.7
      R(9) = 46.8
      R(10) = 54.6
      R(11) = 55.0
      R(12) = 55.0
      T = 2
    1 H = 0
      D0  10 I = 2, T
   10 H = H + R(I)
      N = C*(T - 1)
      IF(H - N) 100, 100, 101
  100 LIS = 0
      GOTO  102
  101 LIS = H - N
  102 V = 0
      D0  11 I = 1, T
   11 V = V + R(I)
      W = C*(TT - T)
      K = T + 1
      D0  12  I = K, TT
   12 W = W - R(I)
      IF(V - W) 103, 103, 104
  103 LSS = V
      GOTO  105
  104 LSS = W
  105 S(T) = LIS
      RIND = 0
      MC = 9999999
    2 RIND = RIND + 1
      LIX = R(T) - S(T)
      K = T - 1
      IF(T - 2) 106, 106, 107
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```
107  LIX = LIX − C*(T − 2)
     D0  13  I = 2, K
 13  LIX = LIX + R(I)
106  IF(LIX) 108, 108, 109
108  LIX = 0
109  Z = R(T) − S(T)
     IF(Z) 110, 110, 111
110  Z = 0
111  L = 0
     M = 0
     DO 14 I = 1, K
     Z = Z + R(I)
 14  L = L + R(I)
     IF (T − 2) 112, 112, 113
113  DO 15 I = T, TT
 15  M = M + R(I)
     M = C*(TT − T + 1) − M
     IF (L − M) 114, 114, 115
115  Z = Z − L + M
     GOTO 112
114  Z = R(T) − S(T) + L
112  IF(Z − (C + 10) )116, 117, 117
117  LSX = C + 10
     GOTO 118
116  LSX = Z
118  X(T) = LSX
     STMC = 9999999
     SS(RIND) = S(T)
  3  CDX(T) = KD*0.5* (2* S(T)− X(T) + R(T))
     IF (CDX(T)) 204, 205, 205
204  CDX(T) = 0
205  IF(T − 2) 119, 119, 120
119  S(1) = S(2) − X(2) + R(2)
     CDX1 = KD*0.5*S(1)
     IF(CDX1) 206, 207, 207
206  CDX1 = 0
207  GOTO 121
120  S(T − 1) = S(T) − X(T) + R(T)
     IF(S(T − 1) 208, 209, 209
208  S(T − 1) = 0
209  IF(T − 3) 123, 123, 122
122  I = T − 1
127  I = I − 1
     S(I) = S(I + 1) − ZWX(I + 1, RIND) + R(I + 1)
     IF (I − 2) 124, 124, 127
123  S(1) = S(2) − ZWX (2, RIND) + R(2)
     IF(S(1)) 125, 126, 126
126  CDXX (RIND) = KD*0.5* S(1)
     GOTO 124
```

```
125  CDXX(RIND) = 0
124  CD(T — 1) = 0
     I = T
129  I = I — 1
     WL = 2*S(I) — ZWX(I, RIND) + R(I)
     IF (WL) 128, 210, 210
128  WL = 0
210  CD(T — 1) = CD(T — 1) + KD*0.5* W1
     IF(I — 2) 121, 121, 129
121  IF(X(T) — C1) 130, 130, 131
130  CPX (T) = KP1*X(T)
     GOTO 200
131  IF(X(T) — C2) 132, 132, 133
132  CPX(T) = KP1* C1 + KP2* (X(T) — C1)
     GOTO 200
133  IF(X(T) — C3) 134, 134, 135
134  CPX(T) = KP1* C1 + KP2* (C2 — C1) + KP3* (X(T) — C2)
     GOTO 200
135  CPX(T) = KP1* C1 + KP2* (C2 — C1)+ KP3* (C3 — C2)+KP4*
     (X(T) — C3)
200  IF(T — 2) 136, 136, 137
137  IF(T — 3) 138, 138, 139
138  X(1) = R(1) + R(2) — ZWX(2, RIND) — S(2)
     IF(X(1) — C1) 140, 140, 141
140  CPX1 = KP1* X(1)
     GOTO 201
141  IF(X(1) — C2) 142, 142, 143
142  CPX1 = KP1* C1 + KP2* (X(1) — C1)
     GOTO 201
143  IF (X(1) — C3) 144, 144, 145
144  CPX1 = KP1 * C1 + KP2* (C2 — C1) + KP3* (X(1) — C2)
     GOTO 201
145  CPX1 = KP1* C1 + KP2* (C2 — C1) + KP3* (C3 — C2) + KP4*
     (X(1) — C3)
201  VVX (1, RIND) = CPX1
139  CP(T — 1) = 0
     I = T
146  I = I — 1
     CP(T — 1) = CP(T — 1) + VVX(I, RIND)
     IF(I — 1) 202, 202, 146
136  SX1X2 = R(1) + R(2) — S(2)
     IF(SX1X2 — C1) 147, 147, 148
147  CP1 = KP1 * SX1X2
     GOTO 202
148  IF(SX1X2 — C2) 149, 149, 150
149  CP1 = KP1* C1 + KP2* (SX1X2 — C1)
     GOTO 202
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150  IF(SX1X2 — C3) 151, 151, 152
151  CP1 = KP1* C1 + KP2* (C2 — C1) + KP3* (SX1X2 — 2C)
     GOTO 202
152  CP1 = KP1* C1 + KP2* (C2 — C1) + KP3* (C3 — C2) + KP4*
     (SX1X2 — C3)
202  IF (T — 2) 153, 153, 154
154  PP = T* (R(T) + R(T — 1) — 2*S(T)
     STCDP = CDX (T) + CD (T — 1) + CDXX(RIND) + CPX(T)+
     + CP(T — 1) + PP
     GOTO 203
153  STCDP = CDX (2) + CDX1 + CPX(2) + CP1
203  IF(STMC — STCDP) 155, 155, 156
156  STMC = CTCDP
     VVX(T, RIND) = CPX(T)
     ZWX(T, RIND) = X(T)
155  X(T) = X(T) — 1
     IF(X(T) — LIX) 157, 3, 3
157  IF (MC — STMC) 158, 158, 159
159  MC = STMC
     VOX (T) = ZWX(T, RIND)
     VOS (T) = SS (RIND)
158  S(T) = S(T) + 1
     IF (S(T) — LSS) 2, 2, 160
160  IF (RIND — 60) 161, 162, 162
161  K = RIND + 1
     IF(T — 3) 163, 164, 163
164  DO 16 I = K, 60
     CDXX(I) = CDXX(RIND)
 16  VVX (1, I) = VVX(1, RIND)
163  D0 17 I = K, 60
     VVX(T, I) = VVX (T, RIND)
 17  ZWX(T, I) = ZWX(T, RIND)
162  IF(T — 2) 165, 165, 166
165  X(1) = R(1) + R(2) — VOX (2) — VOS (2)
     S(1) = VOS (2) — VOX (2) + R(2)
     WRITE (3, 501) X(1), S(1)
166  WRITE (3, 502) VOX (T), VOS (T)
     T = T + 1
167  IF(T — 3) 170, 169, 170
169  READ (1, 503) KP1, KP2, KP3, KP4, KD
501  FORMAT (/// F7. I, I7)
502  FORMAT (// F7. 1, I7)
503  FORMAT (5F10.2)
170  IF (T — 12) 1, 1, 172
172  STOP
     END
```

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UN MODELE POUR OPTIMISER L'ORGANISATION DE LA PRODUCTION A L'AIDE DE LA METHODE DE LA PROGRAMMATION DYNAMIQUE

L'auteur élabore un plan optime de production, divisé en un nombre fini de phases traité comme un système discret en quatre étapes.

Etant donné les nécessités des Usines de Tracteurs de Brasov on cherche à établir une relation de récurrence ayant la forme

$$f(R)T-1) = \max_{X(T)} \{W(T)\, R(T-1),\ x(T)\},$$

qui puisse permettre d'obtenir une politique optime des stocks dans les conditions d'une capacité, de production constante.

On fait l'analyse de la détermination des coûts d'emmagasinage et de production et on détermine la minimisation de la fonction-but.

Dans la partie finale on présente le programme de calcul écrit en langage FORTRAN.

МОДЕЛЬ ОПТИМИЗАЦИИ ОРГАНИЗАЦИИ ПРОИЗВОДСТВА ПРИ ПОМОЩИ МЕТОДА ДИНАМИЧЕСКОГО ПРОГРАММИРОВАНИЯ

Автор разрабатывает оптимальный план производства с разбивкой на конечное число фаз, ассматриваемый как дискретная система с четырьмя этапами.

С учетом потребностей Брашовского тракторного завода преследуется цель установления рекурентного отношения формы:

$$f(R(T-1) = \max_{X(T)} \{W(T)\, [R(T-1),\ x(T)]\},$$

которое делало бы возможным и установление оптимальной политики запасов в условиях постоянства производственных мощностей.

Рассматривается определение расходов хранения и производства и определяется минимизация целевой функции.

В заключение приводится программа расчета, написанная на языке Фортран.