Topkis, Donald M.

A NOTE ON CUTTING – PLANE METHODS WITHOUT NESTED CONSTRAINT SETS
In: Operation research 18(1970), 1216–1220.

In this note the author eliminates the inactive constraints in cutting-plane algorithms. A theorem in weekened conditions is established. Let $S = \{x: G(x) \geqslant 0,\ x \in T\}$, be. $G(x)$ is concave and continuous on T. There exists $t \in S$ with $G(t) > 0$. For $x \in T \sim S$ define $\lambda(x) = \alpha = \sup\{\lambda : \lambda x + (1 - \lambda)\ t \in S\}$, and set $w(x) = \alpha(x)\, x + [1 - \alpha(x)]\ t$ for any $\alpha(x) \in [\lambda(x), 1]$. There exists a function $\mu(w)$ from $T \sim S$ into E^n with $|\mu(w)| \leqslant k$ for all $w \in T \sim S$. Let $G(y) \leqslant G(w) + \mu(w)\,(y - w)$ be for all $w \in T \sim S$ and $y \in S$. Let $H_k = \{x : 0 \leqslant G[w(x_k)] + \mu[w(x_k)] \cdot [x - w(x_k)]\}$ be. If f is strongly concave on T and differentiable on S, and $\bar{x}$ is the unique maximum of f on S, then for $k \geqslant 1$, $f(x_k) - f(\bar{x}) \leqslant \dfrac{1}{a_1 k}$ and $x_k - \bar{x} \leqslant \dfrac{1}{a_2 \sqrt{k}}$ where

$$a_1 = 2\gamma \left| \frac{G(t)}{bdk} \right|^2, \qquad a_2 = \frac{2\gamma G(t)}{bdk}$$

$d = max\ \{|\nabla f(y)| : y \in S\}$, $b = max\ \{|y - t| : y \in T\}$, and is, as in the definition, of strong concavity.

M. Stoica