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An Investigation of the Relative Velocity of Development of the Economic Phenomena Using *the Effect of Remanent Rates*

"After all, what matters in the last analysis is the result".

W. Leontief
(Input-Output Economics)

I. Consider $P_0; P_1; P_2; \ldots ; P_i; \ldots ; P_n$, to be the numerical indexes of an economic process development. The current mathematical model facilitating a most realistic characterization of the essence of the process under consideration will replace the numerical indexes P_i by a geometrical progression model, having as its original term value P_0 and as its final term value P_n.

Writing θ for the annual growth rate, its representative value will be yielded, under the above restriction, by equation:

$$P_n = P_0 \cdot \theta^n \cdot \ldots \theta = \sqrt[n]{\frac{P_n}{P_0}} \qquad \text{(I, 1)}$$

In this sense, an index $P_i^* = P_0\theta^i$ occurs instead of real index P_i. Thus, the sum of the absolute values of the differences between the real indexes and the computed indexes is $\sum_{n=1}^{n-1} | P_i - P_i^* |$.

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The lesser this difference as against P_n, respectively $P_0 + P_1 + .. + P_i + ... + P_n$, the more adequate is the theoretical model.

However, as a rule, this type of model does not reflect closely enough the real development of the economic process under consideration and, for this reason, some other models have been introduced, among which the best substantiated from the theoretical point of view have proved to be the parabolic mean method (Froda-Karasev) [1], [2], and the approximation model through the least squares method (also a geometrical progression, where the original index is not P_0 and the final index is not P_n) worked out by academician Gheorghe Mihoc and professor George Theiler [3].

A more careful analysis of the phenomena specific of our national economy will make clear that, although the development programmes contemplate an annual constant growth, yet, during their achievement some overfulfilments occur with certain regularity. In this way, we come to the idea that, beside the relative growth of the fundamental tendency of motion (the growth rate which is known in economic practice), there exists objectively *a second relative growth (the remanent rate)*. We have in view to describe here the new type of development model which is outlined under such a hypothesis and we mean to prove that this approximates better the real economic processes in comparison with the models we have just mentioned. We shall do this so much the more as it has been seen that also under some ideal conditions of achievement it is little probable that the dynamics of economic phenomena may constantly observe the growth law contemplated in a mathematical model of any type, in particular of the form pictured in (I, 1), because the development itself, considered as such, allows random deviations [4].

Thus, the sizing of the statistical rate requires a revaluation so that this rate shall illustrate simultaneously the two relative velocities of change. With this purpose in view, we shall investigate the entire relative growth velocity with the aid of what we call *the effect of the remanent rates*, pointing out its impact both in the discrete instance and in the continuous instance of the economic development [4].

II. *The discrete instance.* We consider P_i; $i = 0; 1; 2; ..; n$, the numerical indexes of a growth process which is achieved in n years and its dynamic chain-shaped indexes:

$$\frac{P_1}{P_0} = K; \frac{P_2}{P_1} = K(1 + \alpha_1); \frac{P_3}{P_2} = K(1 + \alpha_1)(1 + \alpha_2); \ldots; \frac{P_i}{P_{i-1}} =$$

$$= K(1 + \alpha_1) \cdot (1 + \alpha_2) \ldots (1 + \alpha_i); \ldots \frac{P_n}{P_{n-1}} = K(1 + \alpha_1)(1 +$$

$$+ \alpha_2) \ldots (1 + \alpha_i) \ldots (1 + \alpha_{n-1}),$$

where we have written K for the rate of the fundamental tendency of motion, and factors $(1 + \alpha_1); (1 + \alpha_2); \ldots; (1 + \alpha_i); \ldots (1 + \alpha_{n-1})$ for "the effect of the remanent rates", expressing the geometrical change of two successive indexes $\frac{P_i}{P_{i-1}} : \frac{P_{i-1}}{P_{i-2}}$.

II. 1. From the dynamic indexes given in (II), by multiplying we infer:

$$\frac{P_n}{P_0} = K^n (1+\alpha_1)^{n-1} \cdot (1+\alpha_2)^{n-2} \ldots (1+\alpha_i)^{n-i} \ldots (1+\alpha_{n-1}) \qquad \text{(II.1.1)}$$

It is worth noticing that, if we assume a theoretical dynamics on considering during the first stage:

$\alpha_1 = \alpha_2 = \ldots = \alpha_i = \ldots = \alpha_{n-1} = \alpha$, that is $\frac{P_i}{P_{i-1}} : \frac{P_{i-1}}{P_{i-2}} = \alpha$, the model in (I, 1) is generalized.

At the same time we remark that, in contradistinction to (I, 1), $P_i^* = P_0 K^i (1+\alpha)^{\frac{i(i-1)}{2}}$ and, consequently, $P_n^* = P_0 K^n (1+\alpha)^{\frac{n(n-1)}{2}}$.

Therefore, in this relationship we ha e to find both the rate of the fundamental tendency of motion K and th rate of the stated effect $1+\alpha$ (a positive or a negative number). To this end we make use of two computation relationship). The first will assume the form of the one given in (I, 1), showing the connection between the numerical indexes P_0 and P_n, knowing that $P_n = P_n$, that is:

$$\frac{P_n}{P_0} = K^n \cdot (1+\alpha)^{\frac{n(n-1)}{2}} \qquad \text{(II.1.2)}$$

and the second relationship, connecting the product of the numerical indexes as a whole, will be $\prod_{i=1}^{n} P_i^* = \prod_{i=1}^{n} P_i$, which is equivalent to:

$$\prod_{i=1}^{n} \frac{P_i}{P_0} = K^{\frac{n(n+1)}{2}} \cdot (1+\alpha)^{\frac{n(n+1)(n-1)}{6}} \qquad \text{(II, 1.3)}$$

representing the result of some previous investigations into the optimal measuring of the mean growth rate [5]. Thus, we write:

$$\sqrt[n]{\frac{P_n}{P_0}} = K(1+\alpha)^{\frac{n-1}{2}} \qquad \text{(II, 1.4)}$$

and, accordingly:

$$\sqrt[\frac{n(n+1)}{2}]{\prod_{i=1}^{n} \frac{P_i}{P_0}} = K(1+\alpha)^{\frac{n-1}{3}} \qquad \text{(II, 1.5)}$$

By squaring (II, 1.4) and cubing (II, 1.5), we write further:

$$\sqrt[n]{\left(\frac{P_n}{P_0}\right)^2} = K^2(1+\alpha)^{n-1}; \qquad \sqrt[\frac{n(n+1)}{2}]{\left(\prod_{i=1}^{n} \frac{P_i}{P_0}\right)^3} = K^3(1+\alpha)^{n-1}$$

By dividing the second relationship by the first, we get:

$$K = \frac{\sqrt[\frac{n(n+1)}{2}]{\left(\prod_{i=1}^{n} \frac{P_i}{P_0}\right)^3}}{\sqrt[n]{\left(\frac{P_n}{P_0}\right)^2}} = \frac{\left(\prod_{i=1}^{n} \frac{P_i}{P_0}\right)^{\frac{6}{n(n+1)}}}{\left(\frac{P_n}{P_0}\right)^{\frac{2}{n}}} \qquad \text{(II, 1.6)}$$

After computing K, in order to find $1 + \alpha$ we shall use the same relationship (II, 1.2), this time under the form:

$$\left(\frac{P_n}{P_0}\right)^{\frac{2}{n(n-1)}} = K^{\frac{2}{n-1}} \cdot (1 + \alpha) \qquad \text{(II, 1.7)}$$

By an elementary algebraic transformation we can write directly, based on the previous findings:

$$1 + \alpha = \frac{\left(\frac{P_n}{P_0}\right)^{\frac{2}{n(n-1)}}}{K^{\frac{2}{n-1}}} = \frac{\left(\frac{P_n}{P_0}\right)^{\frac{6}{n(n-1)}}}{\left(\prod_{i=1}^{n} \frac{P_i}{P_0}\right)^{\frac{12}{n(n+1)(n-1)}}} \qquad \text{(II, 1.8)}$$

By introducing now the values corresponding to K and $1 + \alpha$ in (II, 1.4), we obtain a differentiated growth rate that we shall write:

$$\boxed{\theta = K \cdot (1 + \alpha)^{\frac{n-1}{2}}} \qquad \text{(II, 1.9)}$$

II. 2. So far, we have made use of one constant remanent factor in order to ascertain a model less different from the one given in (I, 1), particularly when α is very small.

We have to remark that, in general, we cannot talk of a sole constant remanent factor. A systematical overtopping, by the same quantity, of the rate characterizing the fundamental tendency of motion is in this respect questionable.

Therefore, it may be more advisable to assume a constant remanent rate (α) up to a certain moment of the development period under investigation and then introduce a second factor (β) of the class of these rates. Both α and β may express independetly of each other a quicker or slower remanent rate, contingent upon the general configuration of the quantitative changes.

As an illustration, we consider the given process as the development for $2n$ years, so that for values:

$$P_1; P_2; \ldots; P_i; \ldots P_n$$

where we recall that $P_n^* = P_n$ and $P_i \neq P_i^*$, the effect $(1 + \alpha)$ occurs, whereas for values:

$$P_{n+1};\ P_{n+2};\ \ldots;\ P_{n+i};\ \ldots\ P_{2n}$$

where, on the analogy with the above, $P_{2n} = P_{2n}^*$, the effect $(1 + \beta)$ occurs.

The finding of K and $1 + \alpha$ under such conditions is facilitated by relationships (II, 1.2) and, respectively, (II, 1.3). However, a third relationship is required in view of identifying $1 + \beta$.

Thus, we write:

$$\begin{aligned}
&\frac{P_{n+1}^*}{P_n^*} = K \cdot (1 + \alpha)^{n-1} \cdot (1 + \beta) \\
&\frac{P_{n+2}^*}{P_{n+1}^*} = K \cdot (1 + \alpha)^{n-1} \cdot (1 + \beta)^2 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\frac{P_{n+i}^*}{P_{n+i-1}^*} = K \cdot (1 + \alpha)^{(n-1)} \cdot (1 + \beta)^i
\end{aligned} \qquad \text{(II, 2.1)}$$

Their product will be:

$$\frac{P_{n+i}^*}{P_n^*} = K^i \cdot (1 + \alpha)^{i(n-1)} \cdot (1 + \beta)^{\frac{i(i+1)}{2}} \qquad \text{(II, 2.2)}$$

Hence, it follows further that:

$$\prod_{i=1}^{n} \frac{P_{n+i}^*}{P_n^*} = K^{\frac{n(n+1)}{2}} \cdot (1 + \alpha)^{\frac{n(n+1)(n-1)}{2}} \cdot (1 + \beta)^{\frac{n(n+1)(n+2)}{6}} \qquad \text{(II, 2.3)}$$

Consequently:

$$\left(\prod_{i=1}^{n} \frac{P_{n+i}^*}{P_n^*}\right)^{\frac{6}{n(n+1)(n+2)}} = K^{\frac{3}{n+1}} \cdot (1 + \alpha)^{\frac{3(n-1)}{n+2}} \cdot (1 + \beta) \qquad \text{(II, 2.4)}$$

From where:

$$1 + \beta = \frac{\left(\prod_{i=1}^{n} \frac{P_{n+1}^*}{P_n^*}\right)^{\frac{6}{n(n+1)(n+2)}}}{K^{\frac{3}{n+2}} \cdot (1 + \alpha)^{\frac{3(n-1)}{n+2}}} \qquad \text{(II, 2.5)}$$

and we find the differentiated rate, under the hypothesis of the action of two constant remanent factors by means of relationships:

$$\frac{P_{2n}}{P_n} = K^n \cdot (1 + \alpha)^{n(n-1)} \cdot (1 + \beta)^{\frac{n(n+1)}{2}} \cdot \text{(relationship II, 2.2 } i = n)$$

and:

$$\frac{P_n}{P_0} = K^n \cdot (1+\alpha)^{\frac{n(n-1)}{2}} \quad \text{(relationship II, 1.2)}$$

By multiplying we get:

$$\frac{P_{2n}}{P_0} = K^{2n} \cdot (1+\alpha)^{\frac{3n(n-1)}{2}} \cdot (1+\beta)^{\frac{n(n+1)}{2}}$$

From where:

$$\boxed{\theta = \sqrt[2n]{\frac{P_{2n}}{P_0}} = K \cdot (1+\alpha)^{\frac{3(n-1)}{4}} \cdot (1+\beta)^{\frac{n+1}{4}}}, \qquad \text{(II, 2.6)}$$

expressing this time the product of three factors.

As an illustration, we shall take the dynamic series of the electric power output in Romania during 1965—1970.*

Table I

Year	Electric power output (in thousands of millions kWh) *) P_i	Indexes of the form $\frac{P_i}{P_0}$	Indexes of the form $\frac{P_i}{P_{i-1}}$	Geometrical change of two successive dynamic indexes $\frac{P_i}{P_{i-1}} : \frac{P_{i-1}}{P_{i-2}}$
A	0	1	2	3
1965	17.2	—	—	—
1966	20.8	1.209	1.209	—
1967	24.7	1.436	1.187	0.98
1968	27.8	1.616	1.125	0.95
1969	31.5	1.831	1.133	1.00
1970	35.0	2.035	1.111	0.98

Based on the data contained in Table I, we compute the rate of the fundamental tendency of motion:

$$K = \frac{\left(\prod_{i=1}^{n} \frac{P_i}{P_0}\right)^{\frac{6}{n(n+1)}}}{\left(\frac{P_n}{P_0}\right)^{\frac{2}{n}}} = \frac{\left(\frac{20.8}{17.2} \cdot \frac{24.7}{17.2} \cdot \frac{27.8}{17.2} \cdot \frac{31.5}{17.2} \cdot \frac{35.0}{17.2}\right)^{\frac{6}{30}}}{\left(\frac{35.0}{17.2}\right)^{\frac{2}{5}}} = 1.203$$

*) Source: Anuarul Statistic al R. S. România 1971, p. 199.

and the remanent effect:

$$1+\alpha=\frac{\left(\frac{P_n}{P_0}\right)^{\frac{6}{n(n-1)}}}{\left(\prod_{i=1}^{n}\frac{P_i}{P_0}\right)^{\frac{12}{n(n+1)(n-1)}}}=\frac{\left(\frac{35.0}{17.2}\right)^{\frac{6}{20}}}{\left(\frac{20.8}{17.2}\cdot\frac{24.7}{17.2}\cdot\frac{27.8}{17.2}\cdot\frac{31.5}{17.2}\cdot\frac{35.0}{17.2}\right)^{\frac{12}{120}}}=0.9788$$

It follows, then, as a rate for the extreme values of the dynamic series under investigation:

$$\theta=K\,(1+\alpha)^{\frac{n-1}{2}}=1.203\cdot 0.9788^2=1.152$$

Computing on the other hand also the geometrical mean rate, which is exclusively related to the extreme values of the dynamic series, we get the same $\theta = 1.152$ since this, as well as the differentiated rate, are originally based on the same analytical relationship, represented by $\sqrt[n]{\frac{P_n}{P_0}}$. However, the rate in model (I, 1) appears in our model under the form of a product of two factors (the rate of the fundamental tendency of motion and the remanent rate), which enables a deeper investigation of the relative velocity of change.

In order to clear up this problem we re-form the dynamic series with the aid of the model of the differentiated rate, as against the model of the geometrical means, starting from the original real value.

Consequently, for the first instance we have:

$$\begin{aligned}
P_0&=P_0 && =17.2 && =17.2\\
P_1&=P_0\cdot K && =17.2\times 1.203 && =20.7\\
P_2&=P_0\cdot K^2(1+\alpha) && =17.2\times 1.203^2\times 0.9788 && =24.4\\
P_3&=P_0\cdot K^3\cdot(1+\alpha)^{\frac{3(3-1)}{2}} && =17.2\times 1.203^3\times 0.9788^3 && =27.2\\
P_4&=P_0\cdot K^4\cdot(1+\alpha)^{\frac{4(4-1)}{2}} && =17.2\times 1.203^4\times 0.9788^6 && =31.5\\
P_5&=P_0\cdot K^5\cdot(1+\alpha)^{\frac{5(5-1)}{2}} && =17.2\times 1.203^5\times 0.9788^{10} && =35.0
\end{aligned}$$

Whereas for the second instance we have:

$$\begin{aligned}
P_0&=P_0 && =17.2 && =17.2\\
P_1&=P_0\cdot\theta && =17.2\times 1.152 && =19.9\\
P_2&=P_0\cdot\theta^2 && =17.2\times 1.152^2 && =22.7\\
P_3&=P_0\cdot\theta^3 && =17.2\times 1.152^3 && =26.8\\
P_4&=P_0\cdot\theta^4 && =17.2\times 1.152^4 && =30.3\\
P_5&=P_0\cdot\theta^5 && =17.2\times 1.152^5 && =35.0
\end{aligned}$$

By computing the absolute deviations of the terms of the two series obtained above with respect to the real numerical indexes, the part played by the remanent rates becomes obvious:

Table II

Year	Electric power output (in thousands of millions kWh) P_i	Re-forming with the aid of the geometrical mean of the dynamic indexes $P_0 \cdot \theta^i$	Re-forming with the aid of the differentiated rate $P_0 \cdot K^i \cdot (1+\alpha)^{\frac{i(i-1)}{2}}$	Absolute deviations	
				$\lvert P_i - P_0\theta^i \rvert$	$\left\lvert P_i - P_0 K^i \cdot (1+\alpha)^{\frac{i(i-1)}{2}} \right\rvert$
A	0	1	2	3	4
1965	17.2	17.2	17.2	0	0
1966	20.8	19.9	20.7	0.9	0.1
1967	24.7	22.7	24.4	2.0	0.3
1968	27.8	26.8	27.2	1.0	0.6
1969	31.5	30.3	31.5	1.2	0
1970	35.0	35.0	35.0	0	0
Total	—	—	—	5.1	1.0

It follows from the data contained in Table II that the model described in this paper (column 2) is closer to the development of the real phenomenon than to the model generated by the geometrical mean (see column 1). Indeed, the sum of the absolute deviations in the first instance represents only 20 per cent of the sum of the respective deviations yielded by the model of the geometrical mean.

III. *The continuous instance.* In ponting out the effect of the remanent rates and their economic significance, we started, naturally, from the discrete instance. Mathematically, the solving of the problems described and the computation technique becomes simpler with the continuous instance. It is true that by passing from the discrete instance to the continuous instance θ becomes less precise, but this is usually counterbalanced by the possibility of expressing more detailedly the various aspects involved by the development rate.

Let us see, then, further how *the effect of the remanent rates* occurs under the continuous instance.

III. 1. It is well-known that the growth model corresponding to the geometrical mean has been extended to the continuous instance [6], the respective formula being yielded by:

$$P_t = P_0 \, e^{kt} \qquad \text{(III, 1.1)}$$

Now we shall refer again to the problem tackled before in order to see how the differentiated rates model described in the first section of this paper works in this instance.

Consider:

$P(t)$ the representative value of an economic phenomenon at moment t and $P(t+h)$ its values at moment $t+h$, where h represents a small increase.

As a rule, a relationship of the form:

$$P(t+h) = P(t)[1 + Kh + 0(h)] \qquad \text{(III, 1.2)}$$

is expected to occur.

We make clear that in (III, 1.2) we have to do with a linear increase, and K should depend on $t[K(t)]$. The simplest instance in this respect is to assume $K =$ constant.

In this sense, relationship in (III, 1.1) can be written under the form:

$$\frac{P(t+h)}{P(t)} = 1 + Kh + 0(h) \Rightarrow \frac{P(t+h)}{P(t)} - 1 = Kh + 0(h)$$

$$\Rightarrow \frac{\dfrac{P(t+h) - P(t)}{h}}{P(t)} = K + \frac{0(h)}{h} \qquad \text{(III, 1.3)}$$

Making $h \to 0$, we get:

$$\frac{P'(t)}{P(t)} = K \Rightarrow ln\, P(t) = Kt + ln\, P(0) \Rightarrow P(t) = P(0)\, e^{Kt} \qquad \text{(III, 1.4)}$$

Yet, if K is not constant but a function of t, formula (III, 1.4) becomes:

$$P(t) = P(0)\, e^{\int_0^t K(t)\, \alpha t} \qquad \text{(III, 1.5)}$$

Assuming a linear increase in relation to h along a short time interval, we have in view only the fundamental tendency of motion, but not the variations appearing on the way. In order to make clear in what manner the introduction of *the effect of the remanent rates* in the continuous instance works, we shall use a more extensive computation relationship comprising also terms of the second degree:

$$P(t+h) = P(t)[1 + Kh + sh^2 + 0(h^2)] \qquad \text{(III, 1.6)}$$

We also write relationship:

$$P(t+2h) = P(t)[1 + 2Kh + 4sh^2 + 0(h^2)] \qquad \text{(III, 1.7)}$$

The equivalent of the effect analyzed in the discrete instance will be in our case:

$$\left| \frac{P(t+2h)}{P(t+h)} : \frac{P(t+h)}{P(t)} \right|$$

On the basis of the relationships found above, it follows that:

$$\frac{P(t+2h)\,P(t)}{P^2(t+h)} = \frac{[1+2Kh+4sh^2+(0h^2)]}{[1+Kh+sh^2+0(h^2)]^2} = 1+(2s-K^2)h^2+0(h)^2 \qquad \text{(III, 1.8)}$$

Hence, as a conclusion:

$$\boxed{\frac{P(t)\,P(t+2h)}{P^2(t+h)} = 1+rh^2+0(h^2)} \qquad \text{(III, 1.9)}$$

We want to find the law concerning an infinitesimal increase of the type described in (III, 1.9), assuming r to be constant, bearing in mind that it stands for *the effect of the remanent rates.*

We shall then write in turn the following relationships:

$$\frac{P(t)\,P(t+2h)}{P^2(t+h)} - 1 = rh^2+0(h^2) \qquad \text{(III, 1.10)}$$

$$\frac{P(t)\,P(t+2h)-P^2(t+h)}{P^2(t+h)} = rh^2+0(h^2) \qquad \text{(III, 1.11)}$$

$$\frac{P(t)[P(t+2h)-2P(t+h)+P(t)]-[P(t+h)-P(t)]^2}{P^2(t+h)} = rh^2+0(h^2) \qquad \text{(III, 1.12)}$$

and:

$$\frac{P(t)\left[\dfrac{P(t+2h)-2P(t+h)+P(t)}{h^2}\right]-\left[\dfrac{P(t+h)-P(t)}{h}\right]^2}{P^2(t+h)} = r+\frac{0(h^2)}{h} \qquad \text{(III, 1.13)}$$

Making $h \to 0$, we get:

$$\frac{P(t)\,P''(t)-p'^2(t)}{P^2(t)} = r \therefore \left[\frac{P'(t)}{P(t)}\right]' = r \Rightarrow$$

$$\frac{P'(t)}{P(t)} = rt+K \Rightarrow ln\,P+(t) = \frac{r}{2}t^2+Kt+ln\,P(0) \Rightarrow P(t) = P(0)e^{\frac{r}{2}t^2+Kt} \qquad \text{(III, 1.14)}$$

Certainly, if $r = 0$, we also get here the growth law characteristic of the accepted development model.

III, 2. Referring further to the same growth law in (III, 1.1) we see that:

$$\frac{P'(t)}{P(t)} = K$$

and for the continous instance:

$$\frac{P'(t)}{P(t)} = K + rt$$

where K and r have the same content as the corresponding constants of the discrete instance (K and $1 + \alpha$).

Since:

$$\frac{P(t+h)}{P(t)} = \frac{P(0)\, e^{\frac{r}{2}(t+h)^2 + K(t+h)}}{P(0)\, e^{\frac{r}{2}t^2 + Kt}} = e^{rht + \frac{r}{2}h^2 + Kh} \qquad \text{(III, 2.1)}$$

and:

$$\frac{P(t+2h)}{P(t+h)} = \frac{P(0)\, e^{\frac{r}{2}(t+2h)^2 + K(t+2h)}}{P(0)\, e^{\frac{r}{2}(t+2h)^2 + K(t+2h)}} = e^{\frac{r}{2}[(t+2h)^2 - 2(t+h)^2] + K[(t+2h) - (t+h)]} =$$

$$= e^{rht + \frac{3}{2}rh^2 + Kh} \qquad \text{(III, 2.2)}$$

we shall have in the last analysis:

$$\frac{P(t+2h)}{P(t+h)} : \frac{P(t+h)}{P(t)} = e^{rh^2} = 1 + rh^2 + 0(h) \qquad \text{(III, 2.3)}$$

which carries out *the effect of the remanent rates* in the context of the mathematical law governing it.

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RECHERCHE SUR LA VITESSE RELATIVE DU DEVELOPPEMENT DES PHENOMENES ECONOMIQUES PAR L'EFFET DES RYTHMES REMANENTS

Observant que les programmes de développement prévoient, en moyenne, une augmentation constante et que durant la réalisation des programmes, il existe en général des dépassements qui présentent une certaine tendance de régularité, l'auteur aboutit à l'idée, que, outre la vitesse relative de la direction fondamentale du mouvement (le rytme de développement bien connu dans la pratique économique), il y a, de façon objective, une seconde vitesse relative dénommée par lui — le rythme remanent.

Dans une telle hypothèse, l'auteur définit et exprime par voie analytique la notion de rythme remanent, matérialisant son effet dans un nouveau type de modèle du développement et, à l'aide duquel, on cherche à donner une meilleure approximation des processus économiques réels. Dans ce contexte, „l'effet des rythmes remanents" favorise une investigation complexe de la vitesse relative de développement, tant dans les conditions de la variation discrète que dans celles de la variation continuelle.

ИССЛЕДОВАНИЕ ОТНОСИТЕЛЬНОЙ СКОРОСТИ РАЗВИТИЯ ЭКОНОМИЧЕСКИХ ЯВЛЕНИЙ ПРИ ПОМОЩИ "ЭФФЕКТА ОСТАТОЧНЫХ ТЕМПОВ"

Отмечая, что программы развития предусматривают в среднем постоянный рост и что в течение их выполнения имеют место обычно перевыполнения, имеющие определенную тенденцию регулярности, автор приходит к выводу, что помимо относительной скорости основного направления движения (темпов развития, известных в экономической практике) объективно существует вторая относительная скорость, которую он именует остаточными темпами. При таком предположении автор определяет и выражает аналитическим путем понятие остаточных темпов, выражая их эффект в новом типе модели развития, при помощи которой достигается лучшее приближение к реальным экономическим процессам. В этой связи «эффект остаточных темпов» облегчает комплексное исследование относительной скорости развития, как в условиях дискретной, так и непрерывной вариации.