E. René Giraud

SELF – REGRESSIVE PROCESSES IN DISCRETE TIME

(Processus autoregressifs à temps discret)

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Considering a self-regressive model of the k order and especially case $k = 1$, the author examines the deviations ε_t of the calculated models for a certain variable x_t as compared to the noticed ones. The author proposes to present a synthesis of the results obtained so far by various specialists. This, however, does not exclude the critical and competent intervention of René Giraud who repeatedly dissects with much attention the known results and submitts them to a laborious and fruitful analysis also presenting his own results and conclusions.

So, consider a self-regressive function of the first degree:

$$x_1 = a\ x_{t-1} + \in_t \qquad (1)$$

with "a" unknown parameter and $\in_t$ random independent variables with a no value hope and a finite variation σ^2. Expressing the function in relation to $\in_t$ for each year $t \in [1, t]$, we have

$$x_t = \in_t + a \in_{t-1} + \ldots + a^{t-1} \in_1 + a^t x_0$$

so that the distribution of x_t is determined not only by that of $\in_t$ but also by that specific of x. Three cases can be distinguished for x.

a) x_0 — constant

b) x_0 — normally distributed to a null mean and variation $\dfrac{\sigma^2}{1-a^2}$

c) $x_0 - x_T$ ("circular" distribution)

The author is dealing only with the first case, approaching it from three distinct points of view as regards parameter "a".

1. $|a| < 1$ the process is called "stable"
2. $|a| > 1$ the process is called "explosive"
3. $|a| = 1$ considered by many authors as less interesting, so less studied.

The cases of stability, the "explosive" case and case $|a| = 1$ are dealt with on an equation of the first degree in the first three chapters of the work. The author insistently dwells on the study of the de-

viations of estimator a^* from the exact-ideal value of parameter "a" which intervenes in the considered equation. Taking into consideration the expression formula of parameter "a" (by the method of the smallest squares), for $a^* - a$, we obtain the form:

$$a^* - a = \frac{\sum_{t=1}^{T} x_t x_{t-1} - a \sum_{t=1}^{T} x_{t-1}^2}{\sum_{t=1}^{T} x_{t-1}^2} = \frac{A_T}{B_T}$$

or

$$\sqrt{T}\,(a^* - a) = \frac{A_T/\sqrt{T}}{B_T/T}$$

and

$$\sqrt{\sum_{t=1}^{T} x_{t-1}^2}\,(a^* - a) = \frac{A_t/\sqrt{T}}{\sqrt{B_T/T}}.$$

The study of the component parts of A_T and B_T is presented separately, very analytically, the properties and behaviours of each element of their structure being discussed. The conclusions are the following:

$$\frac{B_T}{T} \xrightarrow[T \to \infty]{P} \frac{\sigma^2}{1 - a^2}$$

$$\frac{A_T}{\sqrt{T}} \xrightarrow{l} \mathcal{N}\left(0; \frac{\sigma^2}{1 - a^2}\right)$$

$$\sqrt{T}\,(a^* - a) \xrightarrow{l} \mathcal{N}(0; 1 - a^2)$$

$$\sqrt{\sum_{t=1}^{T} x_{t-1}^2}\,(a^* - a) \xrightarrow{l} \mathcal{N}(0; \sigma^2)$$

As regards the explosive case ($|a| > 1$), the author criticizes the opinions according to which "nothing is to be expected from this case". René Giraud considers that this case is sometimes more realistic than others and approaches it from the mathematical point of view in several ways, after the methods of Quenouille, White, Anderson, Chow and Levitan. In connection with this problem, the author points to the possibility of introducing a stationary "component part" into the "explosive" model, which still more brings to reality the studied case and gives it more interest and utility.

Consider $(a^* - a)$ normalized by $\sqrt{\sum_{t=1}^{T} x_{t-1}^2} \Big/ \sigma^{2*}$ in which σ^{2*} is the estimator of σ^2. The deviation of a^* from a, respectively $(a^* - a)$

has a limit distribution $N(0; 1)$. The author approaches the expansion of these results over equations of higher degreee, making the presentation on a quadratic equation.

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \in_t \tag{2}$$

The author gives a form of transformation of this equation into one of the first degree:

$$x_t = \rho x_{t-1} + V_t \tag{3}$$

with

$$V_t = \in_t + \lambda_2 (m_2 - \rho) x_{2,\, t-1}$$

and with

$$x_{2,\, t-1} = \sum_{r=1}^{t-1} m_2^{t-r} \in_r$$

and m_2 a root of the characteristic equation associated to equation (2).

Parameter ρ is to be estimated in equation(3). In connection with this estimation, the author makes an ample analysis of the convergence of estimator ρ^* , of its deviation from the exact-ideal value of ρ (respectively $\varphi^* - \varphi$) and of the distribution of this deviation. The points of view of other authors are also presented.

The author reserves ample space, having the character of a synthesis to the study of the moments of the first and second degrees of estimator a^* of equation (1), presenting in a critical way and with ample comments and examples the results obtained in this respect by Hurwicz, White, R. Shenton and W. L. Johnson. A series of studies on estimator a in models of the form (1) or of the form

$$x_t = a x_{t-1} + b + \in_t \quad \text{and} \quad x_t = a x_{t-1} + b Z_t + c + \in_t,$$

with a Z_t exogene variable, (for example, the studies made and the results obthined by J. B. Copas, Guy H. Orcutt, H. S. Winokur Jr. and E. Malinvaud).

The estimation of the parameters of a quadratic equation is approached in the last two chapters of the work. Worth mentioning is the orientation of the author in tackling this problem on the case of a stationary process, a case which is very frequently used by economists and having large practical applications. The author is developing this problem on the model of a self-regressive process of the second degree, of the form:

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \in_t \tag{4}$$

The estimation of parameters a_1 and a_2 is made with the aid of the serial correlation coefficients

$$r_{st} = \frac{\sum_{t=1}^{T-s} x_t \cdot x_{t+s}}{\sqrt{\sum_{t=1}^{T-s} x_t^2 \sum_{t=1+s}^{T} x_t^2}}$$

in which the index s represents the order of the serial coefficient.

The author is also outlining other forms for calculating the serial correlation coefficients and in this connection he analyses the effect of these various forms on the estimators of the parameters of a quadratic equation. These effects are illustrated on concrete examples and graphical methods are also given for the estimation of parameters and possibilities for simulating the variable x_t in relation to time.

The work proves a useful instrument both for economists and for specialists in various domains who will try to outline the causal links explaining a certain phenomenon. It is worth mentioning that the author gives special attention to the estimation of the parameters in case of short series, which are so often met with in the economic analysis, the more so as the statistical and statistical-mathematical theories apply to infinite series in the approach to this problem. The observation that in case of a finite T number of data, a number of complications appear in the estimation of parameters and in the study of the distributions of estimators having the form a^* is fully justified and the merit of the work is greater as the author devoted wide space within it to the analysis of estimators for short statistical series and to the methods allowing the reduction of the deviation of the estimators.

Radu Stroe