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FILTRAGE OPTIMAL DES SYSTEMES LINEAIRES

(Optimum Filtering of Linear Systems)

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In 1961, with the publication of the article "New Results in Linear Filtering and Predication Theory" by R. E. Kalman and R. S. Bucy, very important changes have taken place in the filtering and prediction methods used until then.

The problem raised is the following: take $Y(t)$ and $Z(t)$ two random time functions between which there is a relation. We have the possibility to note the evolution of $Z(t)$, but we cannot note the evolution of $Y(t)$. The estimation of the best $Z(t)$ is pursued, with the aid of which $Y(t)$ is achieved.

More precisely, the estimation at the moment T of $Y(t)$ with the aid of $Z(t)$, for $t \in [0, T]$, is called filtering.

Another two very important notions are defined in the work:

— prediction, which is a better estimation of $Y(T_1)$ for $T_1 > T$ with the aid of $Z(t)$ for $t \in [0, T)_i$.

— levelling, which is a better estimation of $Y(T_1)$ for $T_1 < T$ with the aid of $Z(t)$ for $t \in [0, T)$.

These problems can be found especially in the theory of communications, in econometrics, in the recognition of shapes, in medical applications, etc.

Theoretically, the problem was solved by N. Wiener. A better estimation $\hat{Y}(T)$ of $Y(T)$ with the aid of $Z(t)$ for $t \in [0, T]$ is being obtained with the aid of Wiener's filter. Yet, Wiener's filter has a great inconvenience: suppose that at the moment T, $\hat{Y}(T)$ was calculated based on data supplied by $Z(0, T)$ and that the calculation of $Y(T_1)$ is wished with the aid of $Z(Y, T_1)$ at the moment $T_1 > T$. In this case, Wiener's algorithm must be again applied for the interval $[0, T_1]$, making new observations on $Z(0, T_1)$, for estimating the best $\hat{Y}(T_1)$. We note that the results obtained on the interval $(0, T)$ cannot be used, that is we cannot take advantage of the previous calculus which supplied $\hat{Y}(T)$. In this way, Wieners's filter has not the property of recurrence.

This property — recurrence — exists only if known algorithm f is applied, so that:

$$\hat{Y}(T_1) = f(Y(T), Z(T, T_1), T, T_1 \text{ for } T_1 > T. \qquad (1)$$

The recurrence property is fundamental in the so-called real time problems, where an extremely short time interval exists for a change in estimation, for example, of the position of a flying satellite.

Recurrence thus allows a considerably economy of computation time.

The proposed methods by Kalman and Bucy have allowed the filter to have recurrence, on the basis of several verified hypotheses.

The idea is the following: suppose that $Y(t)$ is a Markov process deriving from the integration of a linear stochastic differential equation:

$$\begin{cases} \dfrac{dy}{dt} + A(t)\, y = B(t)\ \xi(t) \\ y(0) = y_0 + \xi \end{cases} \tag{2}$$

where

(t) is a stochastic process of the "white noise" type, that is it verifies:

$$\mathrm{E}\zeta(t_1)\ \xi(t_2) = \delta(t_1 - t_2) \tag{3}$$

and that $Z(t)$ is linked to $Y(t)$ by

$$\mathrm{Z}(t) = C(t)\ Y(t) + \eta(t) \tag{4}$$

where $\eta(t)$ is also of a "white noise" type.

From the mathematical point of view, this modelling is raising some problems, because $\zeta(t)$ and $\eta(t)$ are not really stochastic processes. The currently used method for solving this problem is to replace (2) and (3) by stochastic differential equations in the direction ITO:

$$\begin{cases} dy + A(t)\ ydt = B(t)\ d\zeta(t) \\ y(0) = y_0 \end{cases} \tag{5}$$

$$dZ(t) = C(t)\ y(t)\ dt + d\eta(t) \tag{6}$$

where $\zeta(t)\eta$ and (t) are, this time, processes of a Wiener type.

In these conditions, it can be demonstrated that the best estimator $Y(T)$ of $Y(T)$ with the aid of $Z(0, T)$has the property of recurrence. The same results can be reached using other ways too: either by replacing the linear differential equation (2) by a nonlinear equation, or by generalizing model (2), (4) by considering the equations with partial linear derivates. This second way is dealt with in the work by A. Bensouassan who apreciates it particularly for its random interest, due to the fact that the problems of estimation of an random field $y(t, x)$ with the aid of another random field $Z(t, x)$ may be considered similarly too. The problem concerns many sectors of optics, meteorology (in case of short term prediction), aeronautics, etc.

A similar way of introducing recurrence is to consider a model with equations with partial stochastic derivates of a progressive type.

The work presents a model based on the Kalman-Bucy theory which solves the filtering problem for a wide class of representations with the aid of equations with partial linear derivates.

The discretization of the filter equations is presented by setting out from the optimum control of equations with partial derivates, with the aid of the quadratic criterion. The methods of numerical solving of Riccati's equation and of the filter equations are thus presented.

The work concludes with a few notes on the problems which will have to be solved, constituting eventual guiding indications for researches concerned with the theory of linear systems.

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