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A Mathematical Model for Organizing Production in an Enterprise Having Instrument Processes and a Continuous Production Flow.

The present paper presents a model for the optimization of production in an enterprise having instrument processes and a continuous production flow.

The model is built in a manner allowing it to be used both for an enterprise alone and for a group of enterprises, respectively an industrial branch (or sub-branch) having enterprises with the above characteristics. For solving the problem a spatial matrix is used, through the various plane cross-sections of which (horizontal, frontal or profile) the various aspects of the problem can be presented from various points of view.

0. INTRODUCTION

Starting from the search for a mathematical model for the optimization of production in an oil refinery, the authors have reached a model whose applications are much broader, thus constituting a generalization for any industry with instrument processes and a continuous production flow. Moreover, as it will be seen in the final part of the present paper, the application of the model presented may considerably exceed the domain shown in the title.

The basic documentary material used was the paper "Etude des problèmes économiques du raffinage a l'aide de la théorie des programmes linéaires et des calculateurs électroniques" presented by Ch. Berline and J. Wells in 1959 at the World Oil Congress in New York. It seems that from 1959 to this day the model presented has only suffered changes with respect to the sizes in which it is now used. Thus, in a paper published in 1964 in the "Revue française de l'énergie", by P. Vaur and J.S. Le Moigue "La programmation dans l'industrie du raffinage", no mention is made regarding any change, in principle, brought to the model presented in 1959.

The presentation of the model, the so-called "Refinery model" in the two papers mentioned above — the only materials at the authors' disposal in this problem — was not sufficiently detailed to allow its strict application and it therefore did not correspond integrally to the problem considered by the authors. It was therefore necessary to elaborate, in principle, starting from the models, a model to correspond to the aim in view.

1. SETTING THE PROBLEM AND DRAFTING IT

The main characteristics of production in an oil refinery are the following:

— *instrument processes with a continuous production flow occur*;

— *a small number of raw materials is processed into a large number of finished products having varied conditions of quality*;

— *production is flexible, this meaning that there are multiple possibilities to obtain, with the same instllations, the same product, by choosing the raw material, adjusting the installations, making the mixtures, etc.*

These characteristics are not typical only to oil refineries, but to all industries having instrument processes and a continuous production flow. Consequently, the domain of application of the model is, from the very beginning, broadened to include all these industries.

Further, it is required for the model to be usable both for an enterprise taken singly, and for a group of enterprises, respectively an industrial branch, (or sub-branch) having enterprises with the characteristics shown above. Obviously, for this last application, the model will have to be adequately completed.

A mathematical model is presented hereafter for the optimization of production in an enterprise having instrument processes and a continuous production flow, taken singly, and taking into account the requirement that this model should have the possibility to be extended to the entire branch (or sub-branch). In a general way, the problem is set as follows:

In an enterprise with intrument processes and a continuous production flow, having a set J of available installations, let the optimum flux of given raw materials and of intermediate products be established so as to obtain a required assortment of quantitatively and qualitatively finished products under the conditions of optimization of an objectively admitted function and the observance of the required limitations.

Starting from the requirement that it should be possible for the model, for an enterprise taken singly, to be extended to the entire branch (or sub-branch), the following generalized drafting is considered:

Let be a space R in which there is a closed contour Z which divides the space R into three disjointed domains: I (internal), E (external) and Z (contour). An exchange of products occurs between the domains I and E; it is made through the intermediary of the contour Z. According as we are dealing with an enterprises taken singly or with an entire branch (or sub-branch) we consider:

a) in the case of an enterprise taken singly, the set J of installations is located in the domain I, the contour Z bounding the enterprise with respect to the exterior *(domain E)*;

b) in the case of a branch (or sub-branch), the set of processing enterprises and transport, stocking and distributing installations outside the proces-

sing enterprises are located in domain I, the contour Z bounding the branch (or sub-branch) from the remaining national economy *(domain E)*.

Besides the above, a strainer S (negative source) is also considered in domain I; it uses up all the losses occurring in I.

This strainer plays the part of a fictitious installation (*in case a*) or of a fictitious enterprise *(in case b)* which consumes without producing or supplying anything.

Let be: M_{EI} the set of product assortment, given quantitatively and qualitatively, passing from domain E to domain I, and M_{EI} their total quantity, M_{IE} the set of product assortment, given quantitatively and qualitatively, which pass from domain I into domain E, and M_{IE} their total quantity.

M_{IS} the set of product assortment, given quantitatively and qualitatively (or unclassified), taken over from domain I by the strainer S, and M_{IS} the total quantity of losses.

With these notations the balance equation may be written, generally:

$$M_{EI} = M_{IE} + M_{IS} \tag{1}$$

without taking into consideration the processes occurring within the domain I.

This fact is important when passing from case *a)* shown above to case *b)*, where each processing enterprise only appears by its contour, without taking into consideration any longer the processes occurring within, processes which, however, are considered as taking place according to an optimum solution.

As shown, the mathematical model is expounded hereafter for case *a)* above, case *b)* being the the subject of another paper whose starting point is the present one.

2. SYSTEMATIZING THE INPUT DATA AND THE UNKNOWNS OF THE PROBLEM

In the case of an enterprise taken singly, this meaning that abstraction is made of the fact that it belongs to or co-operates within the framework of a branch (or sub-branch), in view of taking into consideration the processes occurring within this enterprise, hence in the domain I, it is considered that, in this domain, a finite set J of installations operates, which process products received also/or from contour Z (hence proceeding from domain E) or/also from the other installations within the domain. It is likewise considered that every installation $l \in J$ consumes without processing certain quantities of the products P*. These products are considered as taken from the contour, hence they can eventually also be integrally consumed from E (however, part of them will proceed from I).

We note:

M_k — total quantity of products consumed by the installation $k \in J$ for processing.

M_{Zk} — total quantity of products proceeding from E through the intermediate of contour Z and consumed in installation k for processing.

M_{Ik} — quantity of products proceeding from within the domain I and used for processing in the installation k.

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* P — set of products circulating within I and/or between Z and E or I.

M_{-Zk} — total quantity of products used from the contour, by the installation k, without being processed in it.

We shall therefore have the following relations which express:

— total consumed in installation k for processing

$$M_k = M_{Zk} + M_{Ik} \tag{2}$$

— total consumption of installation k

$$M_k + M_{-Zk} = M_k^{(t)} \tag{3}$$

From the processing of the quantity M_k a total quantity of products N_k is obtained, part of which is given to the other installations $l \in J$ for further processing, and part is given onto the contour Z.

Likewise, on processing, a series of losses occur; they are considered as given to the strain S.

We note by:

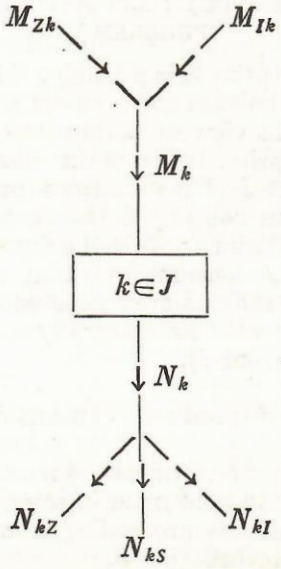
N_k — total quantity of products proceeding from the processing of the quantity M_k in the installation $k \in J$.

N_{kZ} — total quantity of products which the installation k gives onto the contour Z.

N_{kI} — quantity of products which the installation k gives to the other installations within I.

N_{kS} — total quantity of products which the installation k gives to the strainer S (losses).

Schematically, for an installation whatever $k \in J$, these notations have the following significance:



without, for the time being, also taking into consideration the eventual consumption in Z, which is not processed.

With the above notations, the following balance equation may be written:

$$M_{Zk} + M_{Ik} = M_k = N_k = N_{kZ} + N_{kI} + N_{kS} \tag{4}$$

It is theoretically assumed that any installation $k \in J$ can consume for processing all the respective products on the contour and from all the other installations $l \in J$ and can, likewise, produce all the products. Practically, for a concrete installation, many of these consumptions and products will be equal to zero.

Let P be the set of products, and $p_i \in P$ a product whatever, $i = 0.1 \ldots m$; $i = 0$ — unclassified losses, considered as a product in I which, however, is integrally consumed in I and exclusively in the strainer S.

The (quantitatively) unclassified products consumed from I for processing in an installation $k \in J$ are shown by a vector

$$\mathbf{m}_{Ik} = \| m_k^i \|_{(m \times 1)}$$

or, making the origin stand out:

$$\mathbf{m}_{lk} = \| m_{lk}^i \|_{(m \times 1)} \quad l,\ k \in J;\ p_i \in P.$$

where m_{lk}^i represents the quantity of products p_i proceeding from the installation l and consumed in the installation k.

There results:

$$\mathbf{m}_{Ik} = \| m_k^i \|_{(m \times 1)} = \sum_l \mathbf{m}_{lk} = \sum_l \| m_{lk}^i \|_{(m \times 1)} = \| \sum_l m_{lk}^i \|_{(m \times 1)}$$

and

$$M_{Ik} = \| 1 \|_{(1 \times m)} \left(\sum_l \| m_{lk}^i \| \right)_{(m \times 1)} = \| 1 \|_{(1 \times m)} \| \sum_l m_{lk}^i \|_{(m \times 1)}$$

hence

$$M_{Ik} = \| 1 \|_{(1 \times m)}\ \mathbf{m}_{Ik}$$

where M_{Ik}, as shown above, is the total quantity of products proceeding from within the domain I, consumed for processing by the installation k.

The (quantitatively) classified products consumed by E by intermediary of the contour Z, for processing in an installation $k \in J$ is represented by the vector

$$\mathbf{m}_{Zk} = \| m_{Zk}^i \|_{(m \times 1)}$$

and

$$M_K = \| 1 \|_{(1 \times m)} (\mathbf{m}_{Ik} + \mathbf{m}_{Zk}) = \| 1 \|_{(1 \times m)} \| \sum_l m_{lk}^i + m_{Zk}^i \|_{(m \times 1)}$$

represents the total of products consumed for processing (both from I and from E) by the installation k.

The products resulting from the processing of the mixture M_k are classified as follows:

— products given for processing in I

$$\mathbf{m}_{kI} = \| m_{kI}^i \|_{(m \times 1)} \qquad i \in P$$

— products given on the contour

$$\mathbf{m}_{kZ} = \| m_{kZ}^i \|_{(m \times 1)}$$

— losses

$$\mathbf{m}_{kS} = \| m^i_{kS} \|_{(m\times 1)}$$

By destination in I:

$$\mathbf{m}_{kI} = \| m^i_{kI} \|_{(m\times 1)}$$

wherefrom:

$$\mathbf{m}_{kI} = \sum_i \mathbf{m}_{kI}$$

The total production of an installation $k \in J$, classified by products and expressed as a balanced equation is:

$$\mathbf{n}_k = \mathbf{m}^i_{kI} + \mathbf{m}_{kZ} + \mathbf{m}_{kS} \tag{5}$$

or, written in another form:

$$N_k = N_{kI} + N_{kZ} + N_{kS} \tag{5 a}$$

where N_k is, as it was shown, the total quantity of products given by the installation k.

Quantitatively there results:

$$N_k = \| 1 \|_{(1\times (m\times 1))} \mathbf{n}_k \tag{6}$$

With the help of all these notations the following matrix of the order $(n + 2) \times (n + 2)$ can be defined, its elements being vectors of the order $(m + 1)$

$$\mathbf{N} = \left\| \begin{array}{ccccccccc}
\mathbf{m}_{ZS} & \mathbf{m}_{Z1} & \mathbf{m}_{Z2} & \dots & \mathbf{m}_{Zl} & \dots & \mathbf{m}_{Zn} & \dots & \mathbf{m}_{ZZ} \\
\mathbf{m}_{1S} & \mathbf{m}_{11} & \mathbf{m}_{12} & \dots & \mathbf{m}_{1l} & \dots & \mathbf{m}_{1n} & \dots & \mathbf{m}_{1Z} \\
\mathbf{m}_{2S} & \mathbf{m}_{21} & \mathbf{m}_{22} & \dots & \mathbf{m}_{2l} & \dots & \mathbf{m}_{2n} & \dots & \mathbf{m}_{2Z} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\mathbf{m}_{kS} & \mathbf{m}_{k1} & \mathbf{m}_{k2} & \dots & \mathbf{m}_{kl} & \dots & \mathbf{m}_{kn} & \dots & \mathbf{m}_{kZ} \\
\mathbf{m}_{nS} & \mathbf{m}_{n1} & \mathbf{m}_{n2} & \dots & \mathbf{m}_{nl} & \dots & \mathbf{m}_{nm} & \dots & \mathbf{m}_{nZ} \\
\mathbf{m}_{ZS} & \mathbf{m}_{-Z1} & \mathbf{m}_{-Z2} & \dots & \mathbf{m}_{-Zl} & \dots & \mathbf{m}_{-Zn} & \dots & 0
\end{array} \right\|_{(n+2)\times(n+2)} \tag{7}$$

What an installation whatever or the contour gives is shown on the lines, and what it receives, on the columns.

The first line and the last column refer to the contour. The first column represents the losses (received by the strainer S) The last line represents what is given from the contour both as losses ($\mathbf{m}_{-ZS}$) and also in I for consumption of products without processing ($\mathbf{m}_{-Zk}$).

$\mathbf{m}_{kl} \neq 0$ for $k = l$ only if recycling of products occurs in the installation k, $l \in J$.

$\mathbf{m}_{ZZ} \neq 0$ only if a transit of products through domain I from E into E occurs, without being used up one way or another in I.

All the elements of the matrix $\mathbf{N}$ are vectors of the order $(m + 1)$ whose non-negative elements have three indices (one upper and two lower).

The lower indices belong to the set J and characterize the place (the first index the source, and the second the place of consumption). The upper index belongs to the set P and characterizes (classifies) the product.

As it can be seen, if the **m** vectors are considered by their scalars (their elements) ordered along the vertical, a spatial parallelepipedic matrix of the order $(n + 2) \times (n + 2) \times (m+1)$, is obtained. Any horizontal cross-section through this matrix gives a matrix of the order $(n + 2) \times (n + 2)$ with scalar elements representing the flow of a certain product $p_i \in P$, from which it is easy to deduce the balance equations for the respective products. In total, $(m + 1)$ such matrices per product will be available. As it can be seen the upper index gives the level of the horizontal plane in which the respective (scalar) element is to be found.

The matrix N may also be written in its partitioned form:

$$N = \left\| \begin{array}{c|c|c} & \mathbf{m}_{Zk} & \\ \hline \mathbf{m}_S & \mathbf{M} & \mathbf{m}_Z \\ \hline & \mathbf{m}_{_ZK} & \end{array} \right\| \qquad (8)$$

in which the spatial matrix M of the order $n \times n \times (m + 1)$ shows the processes that occur in the domain I. This matrix M is bordered on its four lateral sides:

— by the matrix $\mathbf{m}_S$, of the order $(n + 2) \times 1 \times (m + 1)$ showing all the losses;

— by the matrix $\mathbf{m}_Z$ of the order $(n + 2)\ 1 \times (m + 1)$ showing what the contour Z receives from the domain I and from itself (in the case of transport);

— by the matrix $\mathbf{m}_{Zk}$ of the order $1 \times (n + 2) \times (m + 1)$ showing what contour Z gives from domain E into domain I, for processing;

— by the matrix $\mathbf{m}_{_Zk}$ of the order $1 \times (n + 2) \times (m + 1)$ showing what contour Z gives into domain I for consumption without processing.

In this manner, with the help of the spatial matrix N, the set of data which enter in the composition of the problem has now been organized. The number of these data is equal to the size of matrix N, respectively to $(n + 2) \times (n + 2) \times (m + 1)$. Its data include both the input data (known) and the output data (unknown). However, due to the fact that very many of the scalar elements of matrix N are zero, the problem is considerably simplified. In final analysis matrix N, for which the values of all the elements have been found, represents the result sought for the optimization of the objective function in the framework of the restrictions set.

3. PROBLEM RESTRICTIONS

Since the problem is, in fact, a problem of mathematical programming, the restrictions and the objective function should be specified. With respect to restrictions, they are of several categories, viz:

3.1. *Restrictions regarding availabilities*

It should in general be admitted that unlimited quantities of all the products taken from domain E are not available. This implies a restriction of the form:

$$(\mathbf{m}_{ZS} + \mathbf{m}_{ZZ} + \| \mathbf{m}_{Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)})_{(m\times 1)\times 1} \leqq \mathbf{d}_{(m+1)\times 1}$$

where: $\mathbf{d}_{(m+1)\times 1} = \| d^i \| = \left\| \begin{matrix} d^o \\ d^1 \\ \cdot \\ \cdot \\ d^m \end{matrix} \right\}$; $d^o = 0$

$p_i \in P$

d^i = availability of product $p_i \in P$.

3.2. *Restrictions regarding balance*

The material balance should be respected at all points in domain I, starting from balance equation (1) viz.:

$$M_{EI} = M_{IE} + M_{IS}$$

$$M_{EI} = \| 1 \|_{((m+1)\times 1)} \mathbf{m}_{EI}$$

where:

$$\mathbf{m}_{EI} = \mathbf{m}_{ZI} + \mathbf{m}_{ZS} + \mathbf{m}_{ZZ} = [\| \mathbf{m}_{Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)} + \mathbf{m}_{ZS} + \mathbf{m}_{ZZ}]$$

hence:

$$M_{EI} = \| 1 \|_{((m+1)\times 1)} [\| \mathbf{m}_{Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)} + \mathbf{m}_{ZS} + \mathbf{m}_{ZZ}]$$

$$M_{IE} = \| 1 \|_{((m+1)\times 1)} \mathbf{m}_{IE}$$

where:

$$\mathbf{m}_{IE} = \mathbf{m}'_Z \| 1 \|_{((n+2)\times 1)} - \| \mathbf{m}_{-Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)} - \mathbf{m}_{-ZS}.$$

and:

$$M_{IS} = \| 1 \|_{((m+1)\times 1)} [\mathbf{m}'_S \| 1 \|_{((n+2)\times 1)}]$$

Hence, the above balance equation may be written in the form:

$$\| 1 \|_{((m+1)\times 1)} [\| \mathbf{m}_{Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)} + \mathbf{n}_{ZS} + \mathbf{n}_{ZZ}] = \| 1 \|_{((m+1)\times 1)} [\mathbf{m}'_Z \| 1 \|_{((n+2)\times 1)}$$

$$- \| \mathbf{m}_{-Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)} - \mathbf{m}_{-ZS}] + \| 1 \|_{((m+1)\times 1)} [\mathbf{m}'_S \| 1 \|_{((n+2)\times 1)}]$$

Now taking into account what happens in I, one can write

$$\| 1 \|_{(1\times (m+1))} [(\mathbf{m}_{ZS} + \mathbf{m}_{ZZ}) + \| \mathbf{m}_{Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)}] =$$

$$\| 1 \|_{(1\times (m+1))} [(\| \mathbf{m}_{Zk} \|_{(1\times n)} + \| 1 \|_{(1\times n)} \mathbf{M}) \| 1 \|_{(n\times 1)} + (\mathbf{m}_{ZS} + \mathbf{m}_{ZZ})]$$

or

$$\| \mathbf{m}_{Zk} \|_{(1\times n)} \| 1 \|_{(n\times 1)} = \| \mathbf{m}_{Zk} \|_{(1\times n)} + (\| 1 \|_{(1\times n)} \mathbf{M}) \| 1 \|_{(n\times 1)}$$

3.3. *Restrictions regarding capacities*

Every $k \in J$ installation has a known processing capacity C_k. Consequently, the total quantity of products subjected to processing in installation k cannot exceed this capacity. There results herefrom:

$$\| 1 \|_{1\times (m+1)} (\| 1 \|_{1\times n} \mathbf{M}_{n\times n} + \| \mathbf{m}_{Zk} \|_{1\times n})_{(m+1)\times n} \leqq \mathbf{C}_{1\times n} = \| C_k \|_{(1\times n)} k \in J$$

3.4. *Restrictions regarding technology*

The balance equations shown at point *3.2* are necessary but not also sufficient, since they do not also contain the conditions imposed by the technologic process that is specific for each installation. The technologic process taking place in an installation whatever $k \in J$ is reflected by the processing output that can be obtained in the installation considered, and it is a function of the composition and characteristics of the mixture subjected to processing and of the adjustment of the respective installation.

A certain mixture, characterized by a vector $\mathbf{m}_k^i$; $k \in J$; $i \in P$ will have to be given to each installation for processing. As it results from matrix N that an installation whatever k receives for processing a mixture given by the relation:

$$\mathbf{m}_{Zk} + \sum_l \| \mathbf{m}_{lk} \| = \left\| \sum_l \mathbf{m}_{lk} \right\| + \mathbf{m}_{Zk}$$

there results a first restriction of a technologic character:

$$\mathbf{m}_{Zk} + \left\| \sum_l \mathbf{m}_{lk} \right\| = \mathbf{m}_k^i;\ i = 1,\ 2,\ ...,\ m... \tag{9}$$

On the other hand, since for the (known) processing output ρ_k^i the following relations should be satisfied:

$$\sum_{i=0}^{m} \rho_k^i = 1$$

and, from matrix N, there results for an installation k in a frontal section of the column vector:

$$\mathbf{n}_k = \begin{Vmatrix} m_{kS}^0 & m_{k1}^0 \dots m_{kn}^0 & m_{kZ}^0 \\ m_{kS}^{'} & m_{k1}^{'} \dots m_{kn}^{'} & m_{kZ}^{'} \\ \vdots & \vdots & \vdots \\ m_{kS}^i & m_{k1}^i \dots m_{kn}^i & m_{kZ}^i \\ \vdots & \vdots & \vdots \\ m_{kS}^m & m_{k1}^m \dots m_{kn}^m & m_{kZ}^m \end{Vmatrix}_{(m+1)\times(n\times 2)} \times \| 1 \|_{(n+2)\times 1} = \| n_k^i \|_{m+1)\times 1}$$

which shows the products obtained through processing, a second restriction of a technologic character can be stated:

$$\| 1 \|_{1x(n+1)} \| m_k^{il} \|_{(m+1)\times 1} \| \rho_k^i \|_{(m+1)\times 1} = \| n_k^i \|_{(m+1)\times 1}$$

or

$$\mathbf{M}_K \| \rho_k^i \|_{(m+1)\times 1} = \| n_k^i \|_{(m+1)\times 1} \tag{10}$$

3.5. *Regarding outside deliveries*

The (quantitative and qualitative) assortment of products to be delivered outside (from Z to E) is considered given. Two further restrictions result from this, the one regarding the quantities and the second regarding the quality of the products.

With regard to the quantities the following restriction may by written.

$$(\| 1 \|_{1\times(n+2)} \mathbf{m}_Z - \| 1 \|_{1\times n} \mathbf{m}_{-Zk})_{m\times 1} \geqslant \mathbf{P}_{m\times 1} = \| p_k^i \|_{m\times n} \| 1 \|_{n\times 1} = \| p^i \|$$

$$p^i \in P; \quad i = 1, 2, \ldots, m \tag{11}$$

With regard to quality, for every product p_k^i, $p^i \in P$ and $k \in J$, a quality index is defined, represented by a vector γ_k^i specific for each $k \in J$ installation from where it proceeds. All the γ_k^i vectors are considered known.

The product p^i delivered outside, being a mixture of all the p_k^i will have a quality index resulting from the weighted average of the indices γ_k^i and, the vector Γ which represents the minimum qualities required being known, there results the restriction regarding quality:

$$\| \gamma^i \| = \left\| \frac{\sum_k \gamma_k^i p_k^i + \gamma_z^i p_z^i}{\sum_k p_k^i + p_z^i} \right\| > \Gamma$$

With regard to vector $\mathbf{m}_{-Zk}$, it is determined as a function of the specific consumptions α_k^i, considered known and specific for each installation in part:

$$\mathbf{m}_{-Zk} = \alpha_k^i \mathbf{M}_k$$

4. OBJECTIVE FUNCTION

One considers the objective function to be the total production costs, and their minimum is sought. The production costs for each installation in part are considered known, viz.:

$$F_k = A_k + B_k M_k; \quad k \in J$$

respectively, the coefficients A_k (constant conventional costs) and B_k (specific costs per unit quantity of product subjected to processing, corresponding to the conventional variable costs) as well as the general costs F_0 for the enterprise, are considered known.

With these notations the objective function will be:

$$F = \sum_{k\in J} (A_k + B_k M_k) + F_0$$

or

$$F = \sum_{k\in J} B_k M_k + \left(\sum_{k\in J} A_k + F_0 \right) = \sum_{k\in J} B_k M_k + \text{ct.}$$

wherefrom there results that it is sufficient to set minimum conditions for

$$\sum_{k\in J} B_k M_k = \min. \tag{12}$$

or, in the matricial form:

$$\| B_k \|_{(1\times n)} \cdot \| M_k \|_{(n\times 1)} = \min. \tag{12. a}$$

5. CONCLUSIONS

As it was shown above the present study represents a first approach to the domain subjected to operational investigation, namely a first stage in which — for the time being, theoretically — we have determined the organization of the known (input) and unknown (output) data, the restrictive relations and the objective function, all as component parts of a problem of mathematical programming, in the present case linear programming, in view of optimizing the production of an enterprise having instrument processes and a continuous production flow.

Some new aspects have resulted therefrom in operational investigation, aspects which the authors have not met with in the literature available to them. Moreover, these very aspects have determined the conclusion of this first paper, limited to the theoretical part only, without being also completed by a concrete example ; this in view of maintaining its general character.

The application of the model elaborated, to a concrete example, will form the object of a subsequent, immediate, study. Likewise, starting from the present mathematical model a further paper will endeavour to establish objectively the cost price for every finished product in part by solving a quasi dual problem of the present programming problem considered as primary.

The new aspects having appeared are the following:

a) The use of a special (or multi-level) matrix which can, by its various plane sections (horizontal, frontal or profile), present the aspects of the problem from different standpoints. In the present case from the standpoint of a certain product — horizontal section, from the standpoint of the products consumed by a certain installation — profile section, or from the standpoint of the products resulting from a certain installation — frontal section.

Since the matrix, having column vectors as elements, presents formally quite a good analogy with the matrix of a balance of connections between branches, perhaps the approach to the balance of connections between branches with the help of such a spatial matrix, could lead, without too much complication to its further elucidation. The horizontal sections might refer to the territorial distribution or to the specification of the products by assortments or by factories.

b) The introduction of the intermediate contour Z as an intermediary for the exchanges between the domains I and E increases the range in which the model can be applied regarding the organization of the data, to a series of problems in which an exchange with the outside takes place, as for instance the relations of foreign trade or for the study of biological metabolism — in the case of the cell, the contour playing the part of its membrane.

c) The organization of the data as suggested, including the central matrix M representing the processes in the domain I, with its bordering by the four side-matrices representing the contour (losses — strainer — and collateral consumptions), should permit the elaboration of a programme to simulate the phenomena investigated, allowing, to this end, the transfer from digital to analogous computers.

In conclusion, the authors consider that the present study, although it is still theoretical, will be of interest both through the approach to a problem that is not only large with regard to its size, but also of particular complexity (further stressed by the generalization adopted), and by the fact that it may lead the way to new investigations along unaccustomed new paths.

MODÈLE MATHÉMATIQUE EN VUE D'OPTIMISER LA PRODUCTION D'UNE ENTREPRISE À FLUX CONTINU ET DISCONTINU DE FABRICATION

Cette étude présente un modèle d'optimisation de la production d'une entreprise à flux continu et discontinu de fabrication.

Pour l'élaboration de ce modèle, on a tenu compte des caractéristiques de ce type d'industrie, à savoir:

— un nombre réduit de matières premières sont traitées au moyen d'un grand nombre de produits chimiques à conditions qualitatives variées;

— la production est „élastique", en ce sens qu'il y a multiples possibilités d'obtenir, avec les mêmes installations, le même produit, en raison du choix de la matière première, du réglage des installations, des mélanges effectués, etc.

Le modèle est construit de manière à pouvoir servir à une entreprise isolée aussi bien qu'à un groupe, respectivement une branche (ou sous-branche) industrielle, dont les entreprises auraient les caractéristiques citées ci-dessus. Le problème se résout au moyen d'une matrice spaciale qui, par ses différentes sections planes (horizontale, frontale ou de profil) peut présenter les divers aspects du problème sous des angles différents.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДЛЯ ОПТИМИЗАЦИИ ПРОИЗВОДСТВА НА ПРЕДПРИЯТИИ С АППАРАТНЫМИ ПРОЦЕССАМИ И ПРОДОЛЖИТЕЛЬНЫМ ПОТОКОМ ПРОИЗВОДСТВА

Настоящая работа представляет модель оптимизации производства в предприятии с аппаратным процессом и продолжительным потоком производства.

При разработке модели были учтены характеристики этого типа промышленности, а именно:

— *гибкость производства, в том смысле, что существуют различные возможности получения на тех же самых установках одного и того же продукта, путем выбора сырья, регулирования установок, получения смесей и т. д.*

Модель построена таким образом, что она может быть использована как на одном, отдельно взятом, предприятии, так и на целом ряде предприятий, соответственно в промышленной отрасли (или подотрасли), в которую входят предприятия с вышеуказанными характеристиками. При решении вопроса используется пространственная матрица, которая различными своими плоскими секциями (горизонтальные, фронтальные или в профиль) может демонстрировать различные стороны вопроса с разных точек зрения.