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On Modelling the Fundamental Correlation of the National Income Utilization

"One of the most interesting directions of development of the economic theory in the last few decades has undoubtedly been the interpretation of the macro-economic processes in the terms of optimum programming."

The formation of the national fund of economic-social development is a cardinal problem of the progress of society; it was the object of an ample analysis in the Report presented by Nicolae Ceauşescu at the National Conference of the Romanian Communist Party in July 1972, which has established both the optimum proportion of this fund in the future five-year plans for the development of our country and the ways and means for its most efficient use.

In this study, I shall present some considerations on the criterion and the period for the optimization of the correlation between accumulation and consumption, the determining of the parameters of the prognosis models and the development of the models presented in the examined work.

The criterion of the macro-economic optimum in socialism.

Under the conditions of socialism, the general criterion of the macro-economic optimum is the maximization of the consumption fund—a specific form of manifestation of the fundamental aim of this system. Naturally, this maximization must be interpreted in a dynamic way, to the effect that a harmonious blending of the immediate and future requirements for raising living standards, is achieved.

Firstly, the problem is raised in connection with the diversity of the optimization criteria. Professor R. Moldovan pointed out: "... the optimum can be determined only in a direct way with the objectives we propose in our conscious construction; only on this basis can the relative macro-economic

optimum be determined, which should adequately fulfil the functions in the modelling action and in the model" (Economic Problems, Bucharest, no. 4, 1972, page 101). The blending of the various partial optimum criteria is one of the most difficult problems of planning. The more important is the following characteristic of the general criterion of the macro-economic optimum in the form of the maximization of the consumption fund for a certain period: in the vecinity of the optimum point there are a number of other points very close as regards level. This permits that setting out from the general criterion of the macro-economic optimum, other criteria should also be taken into consideration, thus ensuring a complex outlook on the requirements of development. *Thus, the objective function of the consumption fund totalled for a certain period of time does not only exclude the simultaneous use of other criteria (with a temporary or partial character), but, on the contrary, it can be organically blended with a wide system of such criteria.*

Secondly, it is worth examining the remark professor Em. Florescu made in his study published in Economic Problems, Bucharest, no. 3, 1972, page 82. In connection with the definition of the relative economic optimum (in which the minimizing criterion is the social labour expenses), Em. Florescu considers that, for limited periods, the criterion can be the minimization of expenses in a resource other than labour, especially when there is no acute deficit in the respective resource; in exchange, he expresses his reserves in connection with the affirmation contained in the work that such situations have a particular character. "Such situations are exceptional at most for a certain resource but not for the necessity of minimizing instead of social labour certain resources showing a deficit which may vary in the course of economic development. Obviously, the minimization of the social labour expenses always remains a guide mark, a reference asymptotic limit, but, it will prevail as such only in the future".

Yet, I take the liberty of mentioning once again the fact that social labour is a resource of a special kind: on the one hand, because spare time is an essential element of living standards and, on the other hand, because a number of material resources may be substituted by work, on the basis of international economic exchanges. Moreover, in our country too, the problem of the ensurance of labour starts becoming an important restriction of economic growth. The studies on the drawing up of a preliminary macro-economic prognosis for the 1976—1990 period show that both in industry and in agriculture, the raising of labour productivity — thus, the reduction of social labour expenses per product — will represent the main element for increasing production. Therefore, I consider that *the outlining of the general character of the criterion of minimizing the social labour expenses is corresponding not only on a theoretical plane, but also from the point of view of practical requirements.* The time horizon and the up-to-dating factor.

The difficulties linked with the establishment of the period for maximizing the total consumption fund are not avoided by introducing an infinite time horizon but only transposed on another plane. We shall illustrate this with the aid of simple models.

Noting by F — the production funds, V — the national income, e — the efficiency of the production funds (the national income per fund unit) and by k — the part of the national income destined for development (the accu-

mulation rate) and considering e constant, we can write the following obvious relations:

$$F_t = F_{t-1} + kV_{t-1} = F_{t-1}(1 + ke) = F_1(1 + ke)^{t-1}. \quad (1)$$

$$V_t = eF_1(1 + ke)^{t-1} \quad (2)$$

Introducing the consumption fund B_t to which an up-to-dating coefficient is applied $Q_t = \frac{1}{a^t}$, in which $a > 1$, we obtain:

$$B_t = (1 - k)\, eF_1(1 + ke)^{t-1} \quad (3)$$

$$B_tQ_t = \frac{B_t}{a^t} = \frac{eF_1}{a}(1 - k)\left(\frac{1 + ke}{a}\right)^{t-1}. \quad (4)$$

We formulate the macro-economic optimum criterion by:

$$\sum_{t=1}^{n} B_tQ_t = \frac{eF_1}{a}(1 - k)\sum_{t=1}^{n}\left(\frac{1 + ke}{a}\right)^{t-1} = eF_1(1 - k)\,\frac{\left(\frac{1 + ke}{a}\right)^n - 1}{1 + ke - a} \quad (5)$$

Not only the total consumption but also the attained economic potential — as the basis of a subsequent economic growth — are important for society. Of course, in this case too, an up-to-dating coefficient can be admitted: for example $\frac{1}{b}$, where b is super-unitary and marks an exponential dynamics. However, we do not consider it justified to adopt the same principle as in the consumption fund, because the limit hypothesis is the simple reproduction. In other words, *for the attained potential, the up-to-dating rate should be constant.* Therefore, the decreasing up-to-dating rate should not be applied over the entire existing potential at a given moment, but, only over its increase, on the basis of the resources alloted for the productive funds (through them, we shall express the development of the economic potential). Thus, the up-to-dating coefficient b is applied for F which we determine as follows:

$$\Delta F_t = F_t - F_{t-1} = F_1(1 + ke)^{t-1} - F_1(1 + ke)^{t-2} = F_1(1 + ke)^{t-2}\, ke \quad (6)$$

As F_1 is given (depending on F_0 and k_0), we do not apply the up-to-dating coefficient on it. For the subsequent increases of the production funds, this coefficient is applied in the form $\frac{\Delta F_t}{b^t}$.

Proceeding in this way, we obtain:

$$F_nQ_t = F_1 + \frac{F_1ke}{b} + \frac{F_1ke(1 + ke)}{b^2} + \ldots + \frac{F_1ke(1 + ke)^{n-2}}{b^{n-1}} =$$

$$= F_1 - \frac{F_1ke}{1 + ke} + \frac{F_1ke}{1 + ke}\sum_{i=0}^{n-1}\left(\frac{1 + ke}{b}\right)^i =$$

$$F_1 + \frac{F_1ke}{1 + ke}\left[\frac{b\left(\frac{(1 + ke)}{b}\right)^n - (1 + ke)}{1 + ke - b}\right]. \quad (7)$$

For $\sum_{t=1}^{\infty} B_t Q_t$ to have a limit, the condition $a > 1 + ke$ is necessary; as regards $\Sigma F_n Q_t$, for $n = \infty$, similarly, the condition $b > 1 + ke$ is necessary. Admitting these conditions, we obtain:

$$\Sigma B_t Q_t = \frac{eF_1(1-k)}{a-(1+ke)}. \tag{8}$$

$$F_n Q_t = \frac{F_1(b-1)}{b-(1+ke)}. \tag{9}$$

We shall formulate the objective function by maximizing the sum $\Sigma B_t Q_t + F_n Q_t$. Evidently, in this case the balance coefficients should be introduced, which should express the relation between the preferences of the society for consumption and its preferences for the development of the economic potential. We shall thus presume that the balance of $\Sigma B_t Q_t$ is equal to p, and that of $F_n Q_t$ to $1 - p$, where $0 < p < 1$.
Thus,

$$\Phi = p \sum_{t=1}^{\infty} B_t Q_t + F_n Q_t (1-p) = max. \tag{10}$$

or

$$\Phi = \frac{peF_1(1-k)}{a-(1+ke)} + \frac{(1-p)F_1(b-1)}{b-(1+ke)} = max. \tag{10 bis}$$

$$\frac{\partial \Phi}{\partial k} = \frac{[a-(1+ke)][-peF_1] + [peF_1(1-k)e]}{[a-(1+ke)]^2} +$$

$$+ \frac{(1-p)F_1(b-1)e}{[b-(1+ke)]^2} = \frac{peF_1[e-ke-a+1+ke]}{[a-(1+ke])^2} +$$

$$+ \frac{F_1 e(1-p)(b-1)}{[b-(1+ke)]^2} = eF_1\left[\frac{p(1+e-a)}{[a-(1+ke)]^2} + \frac{(1-p)(b-1)}{[b-(1+ke)]^2}\right]. \tag{11}$$

We shall determine the second derivate:

$$\frac{\partial^2 \Phi}{\partial k} = eF_1\left[\frac{2ep(1+e-a)\,[a-(1+ke)]}{[a-(1+ke)]^1} + \right.$$

$$\left. + \frac{2e(1-p)(b-1)[b-(1+ke)]}{[b-(1+ke)]^1}\right].$$

The denominators of both fractions are positive. The numerator of the first fraction is negative in case that $a > 1 + e$, because it was admitted that $a > 1 + ke$; as regards the numerator of the second fraction, it is positive because we supposed that $b > (1 + ke)$. Consequently, the second deri-

vate is negative (which would demonstrate that the function has maximum) only in certain conditions, which results from the following transformations:

$$\frac{\partial^2\Phi}{\partial k} = 2e^2F_1\left[\frac{p(1-a+e)}{[a-(1+ke)]^3} + \frac{(1-p)(b-1)}{[b-(1+ke)]^3}\right].$$

As $2e^2F_1 > 0$, inequality $\frac{\partial^2\Phi}{\partial k} < 0$ is achieved, on condition that

$$\frac{p(1-a+e)}{[a-(1+ke)]^3} + \frac{(1-p)(b-1)}{[b-(1+ke)]^3} < 0$$

or, replacing $(b-1)$ by $(1-b)$:

$$\frac{p(1-a+e)}{[a-(1+ke)]^3} < \frac{(1-p)(1-b)}{[b-(1+ke)]^3}$$

from where we deduce

$$\frac{p(1-a+e)}{(1-p)(1-b)} < \left[\frac{a-(1+ke)}{b-(1+ke)}\right]^3$$

As $0 < k < 1$, the term on the right acquires values between $\left[\frac{a-1}{b-1}\right]^3$ and $\left[\frac{a-1-e}{b-1-e}\right]^3$.

If $a < b$, that is the preference for the development of the economic potential decreases more rapidly than that for consumption, then $\left[\frac{a-1}{b-1}\right]^3 > > \left[\frac{a-1-e}{b-1-e}\right]^3$. In this case, for Φ the following condition is sufficient to admit a maximum:

$$\frac{p(1-a+e)}{(1-p)(1-b)} < \left[\frac{a-(1+e)}{b-(1+e)}\right]^3$$

If $a > b$, that is the preference for consumption decreases more rapidly than that for the development of the economic potential, then $\left[\frac{a-1}{b-1}\right]^3 < \left[\frac{a-1-e}{b-1-e}\right]^3$. In this case, for Φ to admit a maximum, the condition is sufficient:

$$\frac{p(1-a+e)}{(1-p)(1-b)} < \left[\frac{a-1}{b-1}\right]^3$$

Both conditions are obvious because the term on the left is negative and that on the right is positive.

Admitting that $\frac{\partial^2 \Phi}{\partial k} < 0$, we shall find the value of k from the equation resulting by annuling the first derivate:

$$\frac{\partial \Phi}{\partial k} = eF_1 \left[\frac{p(1 - a + e)}{[a - (1 + ke)]^2} + \frac{(1 - p)(b - 1)}{[b - (1 + ke)]^2} \right] = 0 \qquad (12)$$

Introducing $c = \frac{(1 - p)(b - 1)}{p(1 - a + e)}$ and making the necessary developments, we come to the general equation of k:

$$k^2(e^2 + ce^2) - k[2e(b - 1) + 2e(a - 1)c] + [(b - 1)^2 + (a - 1)^2 c] = 0 \qquad (12 \text{ bis})$$

We shall also present a digit example: $e = 0.2$; $a = 1.21$; $b = 1.22$ and $p = 0.9515$. $c = -1.121$ is deduced, the equation being obtained:

$$-0.00484\ k^2 + 0.006164\ k - 0.001036 = 0$$

with the solution $k = 0.1993$

Thus, the rate maximizing Φ, in the conditions of the above-mentioned up-to-dating coefficients, is of about 20%.

This model eludes the problem of time horizon because it is built for the hypothesis $t \to \infty$, but, it raises another — no less difficult — one, namely that of determining three parameters (a, b and p), all of them having, to some extent, an optional character.

It is noteworthy that, *in certain conditions, the model with a finite horizon leads to results similar to those obtained by the model with an infinite horizon.* We shall formulate the optimum criterion on the basis of expression (3), giving up the up-to-dating coefficient a (which is implied unitarily):

$$\sum_{t=1}^{n} B_t = (1 - k)eF_1 \sum_{t=1}^{u} (1 + ke)^{t-1} = F_1 \frac{1 - k}{k} [(1 + ke)^n - 1] = max \qquad (13)$$

Supposing as before that $e = 0.2$, we shall calculate the values of k maximizing ΣB_t for 12—14 years, with subdivisions as part of the year 12. The results are the following:

Length of the optimization period	Optimum value of k
12 years	0.11
12 years and 6 months	0.16
12 years and 9 months	0.18
12 years and 10 months	0.19
12 years and 11 months	0.20
13 years	0.21
14 years	0.28

As can be seen, for a period of neraly 13 years, the value of k is equal to that obtained on the basis of the previous model (we have applied to year subdivisions for obtaining results close to those offered by the previous model).

This analysis shows that the *infinite time horizon with an up-to-dating factor must not be absolutely counterpoised to the finite horizon without such a factor.*

The models of both types reflect — in different forms — the problem of the socially accepted limit of the accumulation rate. In our opinion, there are no decisive reasons for giving preference to one or another of these possible solutions. In the work "The Correlation between Accumulation and Consumption" [1]) I have chosen the model with a finite time horizon, on the one hand, because it raises the problem in a more conveninent form for planning (with operates with delimited periods) and, on the other hand, because the investigation of (the technical-material and social-political) factors conditioning the optimization period seemed to me more promising than the attempt to substantiate the up-to-dating coefficients. Although in an incipient phase, the studies made so far have revealed a number of decisive factors of the optimization period of the accumulation rate, such as: the structure of economy and its degree of adaptability to the change in the final use of the national income the reproduction cycle of the technical-material basis of the economy, the socially acceptable limit of the accumulation rate in the respective period.

I shall refer, in brief, to the problem of reversibility of the regression functions, which concerns, to the highest extent, the theory and practice of modelling.

I shall examine this problem in the case of linear functions since the overwhelming majority of the functions used in econometric models can be linearized. The polynomial functions are transformed considering $z^i =$. At the same time, by logarithmization an important class of functions becomes linear or linearizable.

As is known, the application of the method of the smallest squares over the distances between the given points (the statistical series) and the adjusting curve by parallels to axes do not ensure the obtaining of reversible functions. For illustration, we shall refer to the simplest example-analysed by other authors as well-that of the straight line.

Coefficients a_0 and a_1 in the function $x_2 = a_0 + a_1 x_1$ are determined by:

$$a_1 = \frac{n\Sigma x_1 x_2 - \Sigma x_1 \Sigma x_2}{n\Sigma x_1^2 - (\Sigma x_1)^2}$$

$$a_0 = \frac{\Sigma x_2 - a_1 \Sigma x_1}{n} = \bar{x}_2 - a_1 \bar{x}_1$$

where $\bar{x}_1$ and $\bar{x}_2$ represent the simple arithmetic means of the series of respective data.

Coefficients a_0' and a_2 in the function $x_1 = a_0' + a_2 x_2$ are determined by:

$$a_2 = \frac{n\Sigma x_1 x_2 - \Sigma x_1 \Sigma x_2}{n\Sigma x_2^2 - (\Sigma x_2)^2}. \tag{14}$$

$$a_0' = \frac{\Sigma x_1 - a_2 \Sigma x_2}{n} = \bar{x}_1 - a_2 \bar{x}_2. \tag{14 bis}$$

[1]) published in our journal no. 2/1972

The reversibility of the two functions would imply that $a_1 a_2 = 1$, relation which is verified only when:

$$\frac{[n\Sigma x_1 x_2 - \Sigma x_1 \Sigma x_2]^2}{[n\Sigma x_2^2 - (\Sigma x_2)^2][n\Sigma x_1^2 - (\Sigma x_1)^2]!} = 1. \tag{15}$$

thus, only in case that the correlation coefficient is unitary. If $a_1 a_2 = 1$, the condition is also observed:

$$a_0' = -\frac{a_0}{a_1}, \tag{16}$$

Consequently, reversibility, is ensured in the ideal case when all the values of the empiric series are placed on the adjusting straight line.

The fact that the adjusting functions are not reversible can generate difficulties in the modelling of economic-social processes, characterized by multiple inverse connections.

The reversibility of the adjusting functions is ensured by using the method of orthogonal regression, which can be expressed by:

$$\Sigma(D^h)^2 = \frac{\sum_{h=1}^{m}\left(a_0 + \sum_{s=1}^{n} a_s x_s^h\right)^2}{\Sigma^2 a_s^2} = min. \tag{17}$$

where D^h represents the distance from the co-ordinate point $(x_1^h \ldots x_u^h)$ to the hyperplane $\Sigma a_s x_s + a_0 = 0$.

We shall apply the method to function $x_2 = a_0 + a_1 x_1$

$$\Sigma D^2 = \frac{\Sigma(x_2 - a_0 - a_1 x_1)^2}{1 + a_1^2} = min. \tag{18}$$

Annulling the partial derivates, the following equations result:

$$\frac{-2\Sigma(x_2 - a_0 - a_1 x_1)}{1 + a_1^2} = 0. \tag{19}$$

$$\frac{-(1 + a_1^2)\,2\Sigma(x_2 - a_0 - a_1 x_1)\,x_1 - 2a_1\Sigma(x_2 - a_0 - a_1 x_1)^2}{(1 + a_1^2)^2} = 0. \tag{20}$$

From equation (19) is deduced:

$$a_0 = \frac{\Sigma x_2 - a_1 \Sigma x_1}{n}. \tag{21}$$

Equation (20), after making the necessary transformations, becomes:

$$\Sigma[a_0(-x_1 + a_1^2 x_1 - 2a_1 x_2)^n + (x_1 x_2 - a_1 x_1^2 - a_1^2 x_1 x_2 + a_1 x_2^2)] + \\ + n a_1 a_0^2 = 0. \tag{22}$$

Replacing in (22) the expressions of a_0 and a_0^2 in (21), we finally obtain:

$$c_1^2[\Sigma x_1 \Sigma x_2 - n\Sigma x_1 x_2] + a_1[(\Sigma x_1)^2 -$$
$$- (\Sigma x_2)^2 - n(\Sigma x_1^2 - \Sigma x_2^2)] - [\Sigma x_1 \Sigma x_2 - n\Sigma x_1 x_2] = 0. \quad (23)$$

Introducing the notations:

$$A = \Sigma x_1 \Sigma x_2 - n\Sigma x_1 x_2$$
$$B = (\Sigma x_1)^2 - (\Sigma x_2)^2 - n(\Sigma x_1^2 - \Sigma x_2^2)$$

equation (23) can be written as follows:

$$Aa_1^2 + Ba_1 - A = 0$$

with the roots

$$a_1 = \frac{-B \pm \sqrt{B^2 + 4A^2}}{2A}. \quad (24)$$

Correspondingly, starting out from equation (21), the free term a_0 is also determined.

One of the two straight lines is the adjusting straight line and the other is perpendicular on it in the point with the abscissa $\frac{\Sigma x_1}{n}$. To decide, the two correlation relations are calculated and the straight line with the positive relation is chosen.

We shall illustrate the above-mentioned analyses with the aid of a practical example.

The index series of labour productivity (the national income per worker) and of the efficiency of the productive fixed assets (the national income per assets unit) for the preceeding 12 years.

Table 1

Indices compared to the level of $t = -13$

t	Labour Productivity ($\bar{w}$)	Net efficiency of the productive fixed assets ($\bar{s}$)
−12	1.131	1.173
−11	1.189	1.258
−10	1.274	1.303
− 9	1.307	1.298
− 8	1.524	1.389
− 7	1.534	1.295
− 6	1.733	1.339
− 5	1.876	1.421
− 4	2.013	1.459
− 3	2.228	1.550
− 2	2.330	1.637
− 1	2.440	1.722

Establishing on the basis of these data, a function between $\tilde{w}$ and $\tilde{e}$, and, supposing that the deduced function remains valid in the next period — the indices of the national income are to be determined for $t = 0, 1, 2, ..., 15$, knowing:

— the index of the number of workers, equal to $\tilde{L}_t = 1.03^t \cdot 1.785$ and the indices of the volume of productive fixed assets, equal to

$$\tilde{F}_t = 1.07^t \cdot 2.500$$

All indices are expressed compared to the level of $t = -13$.

We shall first solve the problem by the traditional methods.

Variant I. $\tilde{w}$ is determined in relation to $\tilde{e}$, based on the simplest function, but adequate to prognosis calculations:

$$\tilde{w}_t = \frac{\tilde{e}_t^{2.25123}}{1 \cdot 27738}. \tag{25}$$

Setting out from the equality

$$\tilde{w}_t = \tilde{e}_t \tilde{f}_t \tag{26}$$

where $\tilde{f}_t$ represents the index of the endowment of manpower with productive fixed assets, that is $\tilde{f}_t = \frac{\tilde{F}_t}{\tilde{L}_t}$, we easily come to the expression of $\tilde{w}_t$ in keeping with $\tilde{f}_t$:

$$\tilde{w}_t = \sqrt[1.25423]{1.27738\, \tilde{f}_t^{2.25423}}. \tag{27}$$

As $\tilde{f}_t$ is directly calculated (from the initial data), the index of the national income ($\tilde{V}_t$) is determined by:

$$\tilde{V}_t = \tilde{w}_t \tilde{L}_t. \tag{28}$$

Variant II. We shall use the inverse relation. Applying the traditional methods to the series of indices in table nr. 1, we come to the expression

$$\tilde{e}_t = 1.142\ \tilde{w}_t^{0.39582}. \tag{29}$$

Taking into account the equality (26), we establish:

$$\tilde{e}_t = \sqrt[060418]{1.142\ \tilde{f}_t^{0.39582}}. \tag{30}$$

Consequently, the index of the national income is calculated on the basis of relation:

$$\tilde{V}_t = \tilde{e}\tilde{F}_t. \tag{31}$$

From the economic point of view, no preference can be given to any of expressions (28) and (31), as both are perfectly admissible. However, they yield sensibly different results (see table no. 2)

Table 2

t	$\tilde{V}_t$, variant I	$\tilde{V}_t$, variant II
0	3.975	3.884
1	4.385	4.260
2	4.949	4.674
3	5.334	5.127
4	5.883	5.625
5	6.489	6.170
6	7.157	6.769
7	7.894	7.426
8	8.706	8.147
9	9.603	8.937
10	10.591	9.805
11	11.682	10.756
12	12.885	11.799
13	14.212	12.945
14	15.675	14.201
15	17.289	15.579

The differences result from the lack of reversibility of the two regression functions used: they would have been even bigger had there not existed a close correlation between the indices $\tilde{w}$ and $\tilde{e}$ (the correlation coefficient for the corresponding series of decimal logarithms — on the basis of which the functions were calculated-amounts to 0.94462). It can be easily imagined how much these differences would have amplified if $\tilde{F}_t$ had been determined in relation to the development of the national income — a method frequently used in the modelling of economic growth.

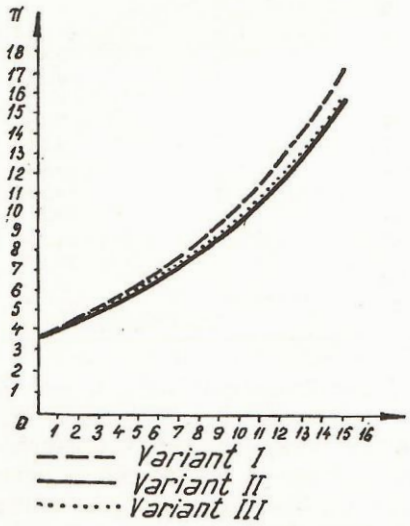

Fig. 1

Variant III of calculus V_t, made with the aid of orthogonal regression, leads to

$$\widetilde{w}_t = \frac{\widetilde{e}_t^{2.48518}}{1.37947}. \tag{32}$$

$$\widetilde{w}_t = \sqrt[1.48518]{1.37947\, \widetilde{f}_t^{2.48518}}. \tag{33}$$

The results obtained are presented in table nr. 3. The data of variant III are comprised between those of variants I and II, but, very close to the second variant (see Figure 1).

TRENDS OF DEVELOPMENT OF THE MODELS

M. Altăr underlined that "... it would be interesting to analyse the solutions to the model in which the share of investments alloted to each of the five branches in the total investments and the share of manpower affected to each branch of the total manpower would be considered as control parameters. In this case, however, a weakening of the isoperimetric conditions should be achieved, as regards the compulsory correlations among branches, espe-

Table 3

t	$\widetilde{V}_t$, variant III
0	3.895
1	4.276
2	4.695
3	5.153
4	5.657
5	6.211
6	6.818
7	7.485
8	8.216
9	9.020
10	9.902
11	10.869
12	11.933
13	13.101
14	14.381
15	15.788

cially by turning some of them from equalities into inequalities" (Probleme Economice, Bucharest no. 3, 1972, page 88). In connection with the extension of the models presented in the work, engineer N. Costache also pointed out that "the introduction of restrictions of manpower, of energy and similar other factors might further be studied. Another interesting extension would be the introduction of the element of exchange with other economic systems, thus the part of foreign trade. The second kind of extension, I think, refers

to the disaggregation of the model" (Probleme Economice, Bucharest, no. 3, 1972, page 89). Professor R. Moldovan also referred to the necessity of taking the foreign economic factors into consideration.

The multisectorial model I have presented in the work: "Correlation between Accumulation and Consumption", because *it is worth improving* in the first place — should be developed in two stages. *Firstly*, I think *the introduction of changes without increasing the number of branches would be necessary. To the extent to which it will thus yield satisfactory results, a sectorial disaggregation could be also achieved.* In the following, I shall dwell on the first group of problems.

1. The more precise expression of the delaying factor in the allocation of investments

In the work, the index of the productive fixed assets is represented in keeping with the volume of productive investments by relation:

$$\widetilde{F}_{(t)} = \widetilde{F}_{(t-3)} - 0.140616 + 0.07084[\widetilde{I}_{p(t-3)} + \widetilde{I}_{p(t-2)} + \widetilde{I}_{p(t-1)}]. \qquad (34)$$

The determination of different coefficients increases the precision of the function and, I shall come back to this question again. I wanted to underline, first ane foremost, the fact that under certain conditions — the coefficient alone may ensure a good approximation of the process. To demonstrate this, let us examine the relation:

$$a(\widetilde{I}_{p(t-3)} + \widetilde{I}_{p(t-2)} + \widetilde{I}_{p(t-1)}) = a_3\widetilde{I}_{p(t-3)} + a_2\widetilde{I}_{p(t-2)} + \\ + a_1\widetilde{I}_{p(t-1)}. \qquad (35)$$

from where we deduce:

$$a = \frac{a_3\widetilde{I}_{p(t-3)} + a_2\widetilde{I}_{p(t-2)} + a_1\widetilde{I}_{p(t-1)}}{\widetilde{I}_{p(t-3)} + \widetilde{I}_{p(t-2)} + \widetilde{I}_{p(t-1)}}.$$

If the volume of productive investments develops exponentially, that is $\widetilde{I}_{p(t)} = \widetilde{I}_p^t$, where $\widetilde{I}_p$ is the average annual index, the relation becomes

$$a = a_3\frac{1}{1 + \widetilde{I}_p + \widetilde{I}_p^2} + a_2\frac{\widetilde{I}_p}{1 + \widetilde{I}_p + \widetilde{I}_p^2} + a_1\frac{\widetilde{I}_p^2}{1 + \widetilde{I}_p + \widetilde{I}_p^2}. \qquad 36)$$

Consequently, regardless of t, the whole sum is constant, although a_3, a_2 and a_1 may differ. Thus, it results that *in the case of a dynamics close to the exponential curve, the use of a single coefficient is justified even if the shares, in which the investments of various years turn into fixed assets, are not identical.* Or, the series of data on the basis of which function (34) was determined satisfied this requirement, to a certain extent.

As the use of different coefficients ensures better results, *I* have recalculated the function of the productive fixed assets, also introducing the influence of the investments of the current year:

$$\widetilde{F}_{(t)} = \widetilde{F}_{(t-3)} - 0.13952 + 0.087997\,\widetilde{I}_{p(t-3)} + 0.075813\,\widetilde{I}_{p(t-2)} + \\ + 0.029985\,\widetilde{I}_{p(t-1)} + 0.020884\,\widetilde{I}_{p(t)}. \qquad (37)$$

It is worth mentioning that the free term is practically the same in the two formulas of the function (—0.14). As regards their degree of precision, the differences are small.

Of course, the introduction of the influence of investments of the current year — which depends also on the level of the national income of this year— complicates the problem to a certain extent. In exchange, the economic process is rendered more faithfully, because — as shown by practice — commissionings also depend on current investments.

2. Improving the determination of the production function of industry

As part of the multisectorial model for industry and other non-agricultural branches, I have resorted to bi-factorial functions of the Cobb-Douglas type. Dr. Tiberiu Schatteles drew attention to the fact that, in the case of industry, the sum of the flexibilities of the two production factors is superunitary, whereas — according to calculations made by him as regards the economy in the 1959—1970 period — the sum is subunitary. The difference also derives from the fact that the provisions of the present five-year plan period (characterized by superior efficiency indicators), but, only in part were introduced in the series of data used in the work. The difference is explained, to a great extent, by the different construction of the functions; a trend of the technical progress was also taken into consideration in the calculations of Mr. Schatteles. A simpler function was used in the model presented in the work ($\widetilde{V} = \widetilde{F}^{\alpha} \widetilde{L}^{\beta}$) without such a factor; the giving up of the separate expression of this factor has made the influence of technical progress to be expressed in the sum of flexibilities α and β, a fact which has already been stressed in the work (pages 127—128). As a matter of fact, determining the sum of flexibilities α and β for 10-year mobile series (as Mr. Schatteles has proceeded in his own calculations), under the conditions in which the trend of technical progress is not appearing, we note that this sum is super-unitary also for the period prior to the present five-year period (see table no. 4)

Table 4

Period	Sum $+\alpha+\beta$ in industry
1951—1960	2.308157
1952—1961	1.748135
1953—1962	1.594757
1954—1963	1.555525
1955—1964	1.591769
1956—1965	1.663488
1957—1966	1.706461
1958—1967	1.890346
1959—1968	1.820074
1960—1969	1.788490
1961—1970	1.621829
1962—1971	1.646506
1963—1972	1.697244
1964—1973	1.741968
1965—1974	1.633588
1966—1975	2.564980

That is why, we maintain our opinion that — in the case of time series — the sum of the flexibilities of L and F reflects not only the influence of the increased dimensions of the economic activity, but also the effect of the technical progress.

Of course, in the form used in the work, the production function remains much too general, having an important shortcoming: it does not express directly the implications on the dynamics of the national income of the correlation between the two main factors of production — manpower and the fixed assets.

That is why, it would be interesting to introduce this correlation in the formula of the production function. As it is known, the relation $w = fe$ is formed between labour productivity (w), the equipment of manpower with fixed assets $\left(f = \frac{F}{L}\right)$ and (e) the efficiency of fixed assets. Admitting that the flexibility of manpower is, from the economic point of view, similar to labour productivity, the dependence $\beta = \varphi(\alpha, f)$. f might be taken into consideration — in the case of bi-sectorial production functions. Of course, the level of the equipment of manpower with fixed assets positively influences the general efficiency of the production factors only to a certain point, namely, that which corresponds to the maximum use of technical progress in the respective stage. An excessively big f starts reflecting the sub-utilization of the productive fixed assets (a reduced coefficient of exchanges, the super-dimensioning of the production capacities, an inadequate correlation between buildings and the active means of labour, etc); this must also be reflected in the formula.

3. Development of the production function of agriculture

As the number of people employed in agriculture has decreased, a manofactorial production function was used for this branch, taking into consideration only the development of the productive fixed assets. Undoubtedly, this has considerably simplified the problem — at the same time — limiting the possibilities of investigating the dynamics of the national revenue created in agriculture. That is why, it seems necessary to me, to further pass on to a multifactorial production function.

The inclusion of manpower would be unjustified, because the tendency of the decrease of the number of people employed in agriculture is maintained. In exchange, other factors might be attracted, which have really exerted and continue to exert an important influence on the efficiency of this branch, such as the consumption of chemical fertilizers and the proportion between zootechnical and vegetable productions.

4. Inclusion of the foreign economic relations in the model

Such a development of the model is obviously necessary, especially if we take the fact into consideration that the Romanian economy will ever more intensely participate in the world economic circuit. The solution adopted in the form presented in the work — the taking into consideration of the budget differences for foreign trade activity as part of the "écart" between the produced and used national income — cannot be fully satisfactory.

The explicite inclusion of foreign economic exchanges in the model raises two essential problems: a) the delimitation of the positions of the balance of foreign payments expressed in lei currency and b) the establishment of their

relations with the internal economic indicators, especially with the national income. At the level of aggregation of the examined multisectorial model, we can confine ourselves to the following main positions: G_1 — total exports; G_2 — total imports; G_3 — returns and G_4 — payments from non-commercial operations; G_5 — rate and interest cashing for previously granted credits and from credits received in the respective year; G_6 — rate and interest payments for previously received credits and credits granted in the respective year; R — currency reserves. All positions — I repeat — are expressed in lei — currency. The equilibrium of the balance of payments implies the equality:

$$G_1 - G_2 + G_3 - G_4 + G_5 - G_6 + R = 0 \tag{38}$$

As the balance of non-commercial operations and of the credit relations, including the currency reserves, is finally expressed in an adequate flow of goods, it can be treated unitarily

$$G = G_3 - G_4 + G_5 - G_6 + R$$

having a negative or positive sign. Thus:

$$G_1 - G_2 \pm G = 0 \tag{39}$$

The used national income is influenced by the proportion G (increased, if the sign is + and, diminished if the sign is—). To express this, the lei currency are turned into internal lei, based on the recurrence circulation specific of export (C_1) and of import (C_2) which can be applied — for simplification- — directly to the balance G; if the latter is positive, GC_2 is added to the created national income and if it is negative GC_1 is subtracted. Of course, these relations must be adapted to the specific of the model in the discussed work, based on the use of indices.

More difficult is the determination of G, which is partly pre-determined by the extent to which economy is engaged in the international division of labour and by the preceeding evolution of the credit relations, but, which to a certain extent represents an optional parameter, as it also depends on the orientation of the subsequent development of economy and of the credit relations; a satisfactory solution to the problem would require the drawing up of a distinct submodel.

The separate expression of the influence of the foreign economic relations necessitates, of course, a change in the coefficient — used in the work — for turning the created national income into used national income.

CONCERNANT LE MODELAGE DE LA CORRELATION FONDAMENTALE DE L'UTILISATION DU REVENU NATIONAL

L'auteur exprime ses opinions concernant les problèmes soulevés par les participants au colloque "Modèles de la corrélation entre les fonds d'accumulation et les fonds de consommation pour la prévision du développement de l'économie roumaine". Ces problèmes ont été assemblés et concentrés dans son livre: « Corrélation entre l'accumulation et la consommation » publié en 1971 par l'Editura politică.

L'auteur groupe ses idées autour des problèmes suivants: le critère optime macroéconomique dans le socialisme; types de modèles de prévision; normatifs et explorateurs; séries de données nécessaire pour déterminer les équations du modèle; méthodes pour estimer les paramètres du modèle; les directions pour le développement des modèles de prévision de la corrélation entre l'accumulation et la consommation.

О МОДЕЛИРОВАНИИ ОСНОВНОГО СООТНОШЕНИЯ ИСПОЛЬЗОВАНИЯ НАЦИОНАЛЬНОГО ДОХОДА

Автор высказывает свое мнение по вопросам, которые были поставлены участниками дискуссии «Модели увязки фондов накопления и потребления для прогнозирования развития румынского народного хозяйства», изложенное в его книге «Соотношение между накоплением и потреблением», опубликованной в 1971 г. Политическим издательством.

Автор группирует свои идеи по следующим вопросам: оптимальный макоэкономический критерий при социализме; тип моделей прогнозирования; нормативы или предварительные величины; серии данных, необходимых для определения уравнений модели, методы оценки параметров модели; направления развития моделей прогнозирования соотношения между накоплением и потреблением.