E. ŢIGĂNESCU

Modelling Economic Growth at the Level of a Branch

The management of the national economic system implies the economic-mathematical modelling of its structure and of the relations among its elements.

The scientific substantiation of decisions concerning the development of the economy in general and on each branch requires the use of economic-mathematical modelling and of modern computing technique in planning. It is well known that the input-output model is one of the most frequently used methods for this purpose, a fact which has been emphasized also by the ever more intense theoretical and applicative preoccupations existing in our country.

This study proposes to tackle the problem of the use of an input-output dynamic model of the inter-branch balance in planning the economic growth of a branch.

1. GENERAL PRESENTATION OF THE DYNAMIC MODEL

The statical input-output models are based on a series of hypotheses limiting the possibilities of their utilization in future planning. Besides, the statistical models do not include the factors of which action varies in time and, consequently, the solution is valid only for the very near period. Obviously, the evolution of the economy in time implies structural transformations (the extension of the productive capacities, a change in the technical coefficients, etc.) which should be taken into conside-

ration in drawing up the long term production programmes. The possibility of fulfilling the production targets for the plan period depends, from the point of view of the technical-material potential, first and foremost, on the production capacity of the fixed assets of each branch and, consequently, on the volume of investments made with a view to extending the production capacities. The interdependence between the production targets of the branches of the national economy and the production capacities necessary in each branch for the fulfilment of the requested level of production can be expressed with the aid of the input-ouput dynamic model. On the basis of the dynamic model, the behaviour of the system from the point of view of the relations among branches is analysed, both as regards the satisfaction of the requirements of current production and the development of the production capacities in the coming periods. The extension of the production capacity in a branch is reflected on all connected branches and, consequently, the dynamic model must reflect the effect of the investments on the total production and the distribution of the development fund in keeping with the planned growth of production and the final consumption. To attain this purpose, that part is separated from the final demand of each branch, which is designed for raising the production capacities in all branches of the national economy. This element is treated as an endogenous variable of the model and represents its dynamic factor. It should be mentioned that the investments appear in the model as a demand for products of a branch and as a result of the direct and indirect connections among branches. The effect of the extension of the capacity of the equipment in a certain branch will be reflected on all branches. The level of the production of branch $[x_i(t)]$ in each year of the time horizon ($t = 1, 2, ..., T$) will be expressed in keeping with the intermediary consumption of all branches $[\Sigma a_{ij} X_j(t)]$, with the part destined for consumption $[C_i(t)]$ * and in keeping with the development fund $[\Delta F_i(T)]$. The production functions describing the relation among the afore-mentioned variables are of the tye of the equations of distribution of production in the statistical model of the balance of the relations among branches, namely:

$$X_i(t) = \sum_{j=i}^{n} a_{ij} X_j(t) + C_i(t) + \Delta F_i(t) \quad i = 1, 2, ..., n; \quad t = 1, 2, ... T.$$

If the final demand is considered to be determined exogenously, then, the above relation becomes:

$$X_i(t) - \sum_{j=1}^{n} a_{ij} X_j(t) - \Delta F_i(t) = C_i(t) \quad i = 1, 2, ..., n; \quad t = 1, 2, ... T. \qquad (1.1)$$

The solution of the dynamic model is composed of vectors $X(1)$, $X(2)$, ..., $X(T)$ the component parts of which describe the trajectory of the development of the production of each branch.

---

* The indicator $C_i(t)$ comprises the non-productive, social and personal consumption, exports and the growth of reserves.

In case that the time variable of the production functions (1.1) is considered continuous, the solution is obtained by solving a system of differential equations and, in case that time is considered as a discrete variable, a system of equations with finished differences will result. In both cases, the relation between the increase of the level of production of a branch and the raising of the production capacity of all branches should be achieved through the variable $\Delta F_i(t)$. This relation can be established if the indicator "fixed assets necessary for the fabrication of a unit of the production of each branch" is introduced. The indicator S_{ij}, including the total volume of fixed assets and the standardized average stock of circulating means resulting from branch i and used in branch j, is used in the generalized dynamic models. S_{ij} thus represents the part of the production of branch i used in branch j as fixed assets or as an average stock of circulating means.

The structure of the necessary funds for production *), according to the branches of source and effective use, is characterized by matrix $\mathbf{S}$ or by the matrix of the coefficients of the consumptions of funds necessary for production (Φ):

$$\mathbf{S} = \begin{Vmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \dots & \dots & \dots & \dots \\ S_{n1} & S_{n2} & \dots & S_{nn} \end{Vmatrix}; \quad \Phi = \begin{Vmatrix} f_{11} & f_{12} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2n} \\ \dots & \dots & \dots & \dots \\ f_{n1} & f_{n2} & \dots & f_{nn} \end{Vmatrix}$$

$$\text{where: } \left[f_{ij} = \frac{S_{ij}}{X_j} \right]; \qquad i, j = 1, 2, \dots, n. \tag{1.2}$$

The coefficients f_{ij} are usually calculated on the basis of the data of a year-taken as basis and are considered as constant throughout a period for which the plan is drawn up.

As a dynamic model drawn up by W. Leontief **) was used in the applicative study, we shall present, in brief, the corresponding mathematical model.

As W. Leontief pointed out, at the basis of his dynamic model was the idea of the characterization of the structure of the economy in each year of the period for which the long-range planning is made through a system of linear equations.

The current flows of each year are covered from the production of the same year while the fixed assets accumulated in the course of a year are destined for raising the production capacity in the following year. In this

*) We mention that the indicator Funds necessary for production does not include the current consumptions. The circulating funds are included in the indicators x_{ij}, thus, in the matrix of direct expenses A.

**) W. Leontief, The Dynamic Inverse, Fourth International Conference on Input-Output Techniques, Geneva, January 8—12, 1968

way, a relation is established between the levels of production in two successive years. In contradistinction to the statical model, the component parts of the vector of the final consumption $C(t)$ will not contain that part of production which is destined for investments. The structure of the national economy is characterized with the aid of the matrix of technical coefficients A_t and of the matrix of the coefficients of fixed assets Φ_t*).

As has already been underlined, the fixed assets produced in the year t are supposed to be installed and commissioned in the year $t + 1$. Consequently, the level of production in the year t must cover, on the one hand, the current expenses and the final demand and, on the other hand, the necessary investments of the following period. The direct connection between the production levels of all branches in two successive periods can be described by the following system of equations:

$$X(t) - A_t X(t) - \Phi_{t+1}[X(t+1) - X(t)] = C(t) \tag{1.3}$$

Relation (1.3) is obtained by setting out from the static model which can be written as follows:

$$X(t) - A_t X(t) = C(t) + \Delta F(t)$$

where $\Delta F(t)$ is a column vector the component parts of which represent the part of the production of each branch destined for the development fund. As this vector can be written in the form:

$$\Delta F(t) = \Phi_{t+1}[X(t+1) - X(t)]$$

relation (1.3) results immediately. In defining the constant elements of the equation system (1.3) it was considered that the expenses devoted to maintaining the fixed assets at the level of the basic period have been included in the matrix of technical coefficients A_t.

The third term on the left side of relation (1.3) achieves the connection between the development fund and the increase of production capacity from $X(t)$ to $X(t+1)$. The matrix of coefficients of the fixed assets used in establishing the necessary investments refers to the year $t + 1$ e when the fixed assets will be commissioned. Relation (1.3) becomes, after some transformations:

$$(I - A_t + \Phi_{t+1})\, X(t) - \Phi_{t+1} X(t+1) = C(t).$$

If we note $G_t = (I - A_t + \Phi_{t+1})$ it can be written in a simpler form:

$$G_t X(t) - \Phi_{t+1} X(t+1) = C(t) \tag{1.4}$$

---

*) For simplification, it is considered that elements f_{ij} of matrix refer only to the fixed assets.

If the last year of the period is noted by T and it is considered that $X(T+1)=0$, then the equation system corresponding to each year of the period will have the form *:

$$G_T X(T) = C(T) \qquad (t = T)$$

$$G_{T-1} X(T-1) - \Phi_T X(T) = C(T-1) \qquad (t = T-1)$$

$$G_{T-2} X(T-2) - \Phi_{T-1} X(T-1) = C(T-2) \qquad (t = T-2)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$G_2 X(2) - \Phi_3 X(3) = C(2) \qquad (t = 2)$$

$$G_1 X(1) - \Phi_2 X(2) = C_1(1) \qquad (t = 1)$$

As the solutions obtained for two successive years are interdependent, it results that the production level which must be attained in the last year of the plan period will determine the level of the production of the preceeding years and inversely. Consequently, the T equation systems are interdependent and they can be written in the matrix form, as follows:

$$\begin{bmatrix} G_1 - \Phi_2 & & & & & \\ & G_2 & & & & \\ \cdots\cdots & \cdots\cdots\cdots\cdots & & & & \\ & G_{T-3} - \Phi_{T-2} & & & & \\ & & G_{T-2} - \Phi_{T-1} & & & \\ & & & G_{T-1} - \Phi_T & & \\ & & & & G_T & \end{bmatrix} \begin{bmatrix} X(1) \\ X(2) \\ \cdots\cdots \\ X(T-3) \\ X(T-2) \\ X(T-1) \\ X(T) \end{bmatrix} = \begin{bmatrix} C(1) \\ C(2) \\ \cdots\cdots \\ C(T-3) \\ C(T-2) \\ C(T-1) \\ C(T) \end{bmatrix} \qquad (1.5)$$

The structure of the matrix on the left side of relation (1.5) allows the solving of the equation system by substitution because the solution to the last system is obtained immediately:

$$X(T) = G_T^{-1} \cdot C(T).$$

By successive substitutions beginning from the last equation system to the first, the solution to the dynamic model (1.5) is obtained, having the form:

*) It must be underlined that this terminal condition is adequate only for describing the properties of the dynamic model and for this reason, in practical applications it is replaced by another one.

$$\begin{bmatrix} X(1) \\ \cdots\cdots \\ X(T-3) \\ X(T-2) \\ X(T-1) \\ X(T) \end{bmatrix} = \left[\begin{array}{c|ccc} G_1^{-1} & R_1 \ldots R_{T-3} G_{T-2}^{-1} & R_1 \ldots R_{T-3} R_{T-2} G_{T-1}^{-1} & R_1 \ldots R_{T-3} R_{T-2} R_{T-1} G_T^{-1} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ & G_{T-3} G_{T-2}^{-1} & R_{T-3} R_{T-2} G_{T-1}^{-1} & R_{T-3} R_{T-2} R_{T-1} G_T^{-1} \\ & G_{T-}^{-1} & R_{T-2} G_{T-1}^{-1} & R_{T-2} R_{T-1} G_T^{-1} \\ & & G_{T-1}^{-1} & R_{T-1} G_T^{-1} \\ & & & G_T^{-1} \end{array}\right] \begin{bmatrix} C(1) \\ \cdots\cdots \\ C(T-3) \\ C(T-2) \\ C(T-1) \\ C(T) \end{bmatrix} \tag{1.6}$$

where $R_T = G_T^{-1}.\ \Phi_{t+1}$

The matrix on the right of relation (1.6) which will be noted by **D** for simplification is inverse to the corresponding matrix of relation (1.5) and contains T lines and T columns. Each column appears in the form of a triangular matrix composed of square submatrices of the order n. Each line of the last "column" represents the direct and indirect expenses of the production of each branch in the year $t = 1, 2, \ldots T$ necessary for the production of a final consumption unit in the year T. The total expenses of the year T determine part of the expenses of the preceeding years and therefore they are included in the total expenses of the preceeding years. The elements of matrix G_T^{-1} represent the coefficients of the total expenses (including deliveries for investments of the last year T and they will ensure the fabrication in each branch, of a consumption unit $C_t(T)$. The first term $R_1 \ldots R_{T-3}R_{T-2}R_{T-1}G_T^1$ shows the level of production in each branch in the year $t=1$, with a view to ensuring a consumption unit for the year T. Thus, the solution to the dynamic model allows also the separation in time of the requirements.

The lack of a change in the technological structure of the economy is expressed by the fact that matrices A_t and Φt are considered constant in each year t of the period for which the long-range programme is drawn up, that is $A_t = A$ and $\Phi_t = \Phi$. Setting out from this hypothesis, it can be demonstrated that the series R_{T-1}; $R_{T-t} \ldots R_{T-2}\, R_{T-1}$ turn into the following series: R, $R^2, \ldots, R^t$. Consequently, the elements of each column ofthe inverse matrix **D** will be calculated on the basis of relations:

$$G^{-1},\ RG^{-1},\ R^2G^{-1},\ \ldots,\ R^tG^{-1},\ \ldots,\ R^{T-1}G^{-1}.$$

For values sufficiently high of t, the elements of matrix R_t tend to zero, which means that the production of the year $t(t = T,\ T-1, \ldots, 1)$ meant to cover the final demand of the final year T becomes ever smaller.

Thus, the dynamic model will have the form:

$$(I - A + \Phi) \cdot X(t) - \Phi \cdot X(t+1) = C(t) \tag{1.7}$$

or

$$G \cdot X(t) - \Phi \cdot X(t+1) = C(t) \tag{1.8}$$

To eliminate the variable $X(t+1)$ from the equation system (1.8) corresponding to the year T, it will be considered that $X(T+1) = \alpha \cdot X(T)$, where α represents the growth rate index. This condition corresponding to the last year of the plan period will change the equation system (1.8) into *:

$$G \cdot X(T) - \Phi \cdot [\alpha X(T)] = C(T). \tag{1.9}$$

Hence it follows that in model (1.5), instead of matrix **G**, matrix $\overline{\mathbf{G}} = (G - \alpha \cdot \Phi)$ will appear, which is obtained from relation (1.9) after the classification of terms, namely:

$$(G - \alpha\Phi) \cdot X(T) = C(T)$$

*) The use of the arbitrary hypothesis $X(T+1) = 0$ is thus avoided.

The solution to this system of n equations will be obtained immediately from relation:

$$X(T) = (G - \alpha\Phi)^{-1} \cdot C(T) \quad (1.10)$$

By successive substitutions in the equation system (1.8) corresponding to each year preceding T, the component parts of vectors $X(t)$, $(t = 1, 2, ..., T-1)$ will be obtained.

Thus for $t = T - 1$ the equation system (1.8) will be:

$$G \cdot X(T-1) - \Phi \cdot X(T) = C(T-1)$$

The component parts of vector $X(T)$ being known (see relation 1.10) the above mentioned system becomes:

$$G \cdot X(T-1) - \Phi \cdot \bar{G}^{-1} C(T) = C(T-1)$$

Hence it follows immediately:

$$X(T-1) = \bar{G}^{-1} \cdot C(T-1) + G^{-1} \cdot \Phi \cdot \bar{G}^{-1} \cdot C(T) =$$

$$= G^{-1} \cdot C(T-1) + R \cdot \bar{G}^{-1} \cdot C(T)$$

(because $G^{-1} \cdot \Phi = R$).

Continuing this method of successive substitution, we come to the following general equation:

$$X(t) = G^{-1} \cdot C(t) + RG^{-1} \cdot C(t+1) + R^2 \cdot G^{-1} C(t+2) + ... +$$

$$+ R^{T-t} \bar{G}^{-1} C(T) \quad (t = 1, 2, ..., t-1)$$

or (1.11)

$$X(t) = G^{-1} \cdot C(t) + RG^{-1} C(t+1) + R^2 G^{-1} C(t+2) + ... +$$

$$+ R^{T-t} G^{-1} C(T) + \alpha R_{-1}^{T-t+1} [I - A - (1-\alpha)\Phi]^{-1} C(T).$$

The matrices appearing on the right side of relation (1.11) represent elements of the inverse matrix (**D**) of the dynamic model.

The dynamic model described above contains nT linear equations with $2nT$ unknown quantities and therefore it is necessary to compare the various possible solutions in view of choosing that solution which is workable from the point of view of the limits imposed by the technological structure of the economy and, at the same time, corresponding to the targets of economic policy.

The maximum growth rate permitted by the technological structure and the resources of the economy should be taken into consideration in the planning of economic development. The analysis of the potential of economic growth, taking into consideration the home production, may be started, setting out from the total level of production $X(t)$.

Thus, if we note by σ the rate of production increase, then between the level of production in the year t and $t + 1$, the following relation will exist:

$$X(t + 1) = (1 + \sigma) \cdot X(t) \tag{1.12}$$

The problem is raised of maximizing the rate of the total production increase by observing the technological restrictions imposed by the dynamic model. The maximization of the increase rate $(1 + \sigma)$ is obtained when the final consumption is zero and hence, relation (1.8) becomes:

$$G \cdot X(t) - \Phi \cdot X(t + 1) = 0$$

or

$$X(t) = R \cdot X(t + 1)$$

Replacing $X(t + 1)$ by the value given by relation (1.12), we obtain

$$X(t) = R \cdot (1 + \sigma) \cdot X(t)$$

or

$$\left(R - \frac{1}{1 + \sigma} \cdot I\right) \cdot X(t) = 0.$$

The solving of this equation system for the maximum rate of increase is equivalent to the calculation of the maximum characteristic value λ of matrix $\mathbf{R}$ which is obtained from the characteristic equation:

$$(R - \lambda I)\, X = 0.$$

Based on the characteristic vector X corresponding to this equation, the structure of the fixed assets ensuring a maximum rate of increase is established. The relation between the maximum growth rate and the maximum characteristic value of matrix $\mathbf{R}$ is established by relation:

$$\sigma = \frac{1 - \lambda}{\lambda}$$

2. DRAWING UP THE DEVELOPMENT PROGRAMME FOR THE MACHINE-BUILDING AND METAL PROCESSING BRANCH

The input-ontput technique is also adequate for the analysis of the economic processes which are taking place in various branches of the national economy: industry, constructions, agriculture, etc. The mathematical model and the methodological principles at the basis of the drawing up of these input-output balances are, generally, similar to those used within the balance of the national economy of the relations among branches, naturally with certain changes determined by their extent. These balances refer to the subsystems of the national economy and therefore, they can by drawn up irrespective of the balance of the national economy, the relations between

the component parts and the whole going to be achieved through the input-output dimensions. The advantage of drawing up such balances lies in the fact that decisions may be taken on the basis of the examination of several variants concerning the production programme, investments, etc. of the examined branch, which contributes to raising the level of planning and analysis.

The author proposes to clear up the problems raised by the applications of the dynamic model in economic analysis and the substantion of the production plan of the machine-building and metal processing branch.

2.1. *Particularities of the mathematical model of the balance of the relations among branches*

The model used in this study has the following particularities, from the methodological point of view:

a) the branches were formed through the aggregation of similar products (pure branches);

b) production and consumption are expressed in the producer's prices;

c) the total level of production of each branch is expressed by the gross output;

d) the competitive import is included on branches together with the home production consumptions;

e) the capital repairs of the fixed assets in the productive sphere are included in the costs.

To complete with figures the input — output table data were collected both as regards the distribution of production and the structure of the costs of each branch of the analysed system, the statistical balance of the relations among branches for 1970 being obtained.

It should be underlined from the very beginning that the data used in the study do not refer to the whole branch, but, only to an economic subsystem which should have the main characteristics of the original. The few enterprises forming the system are not representative for the whole branch and hence, nor the coefficients of matrices **A** and Φ. The author does not intend to estimate the parameters of the model by selective registering, but, only to simulate on the basis of data the stages of the input-output analysis.

Quadrant I contains 20 branches* classified on branches forming the basic activity of the system (the first 12) and branches representing the auxiliary activity in keeping with the characteristic of the machine-building and metal processing industry. Thus, the matrix of the coefficients of direct expenses will have the following structure:

$$\mathbf{A} = \left\| \begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array} \right\| .$$

Submatrix A_{11} is a square matrix of the order 12 and characterizes the intermediary flows among the branches in which the basic activity of the system was separated; submatrices A_{12} and A_{21} characterize numerically

*) It should be mentioned that these are branches taking into consideration the classification used in drawing up the balance of the relations among branches.

the current flows between the branches of basic activity and those of auxiliary activity, submatrix A_{22} refers to the flows among the branches of auxiliary activity. For simplifying the calculations, the auxiliary branches have been aggregated, a single branch being formed (branch 13).

As the dynamic model operates simultaneously with the matrices of the coefficients of direct expenses (A) and the matrix of the coefficients of fixed assets (Φ), it is necessary for the latter to be of the same order. To satisfy this requirement, the branch 14 was formed, being obtained through the aggregation of the branches supplying fixed assets which were not included among the first 13 branches of the balance. For the statical model, this branch has a "fictitious" character because the corresponding flows are null. Branch 14 justifies its role only within the dynamic model, determining a demand for fixed assets coming from branch 14.

Quadrant II of the balance reveals the way of distribution of production for consumption outside the 14 branches included in the model. The component parts of the final consumption are: deliveries for production and exploitation, deliveries of products and services for export, deliveries for investments, deliveries for unproductive consumption; the difference between the raw and finished production*.

Finally, quadrant III is made up of a matrix of material expenses resulting from branches other than those appearing in quadrant I and from import (for simplification, they were aggregated in a single line); the liquidation of the fixed assets; the newly created value on component elements.

Taking into consideration that the competitive imports are registered in quadrant I, the mathematical model balance equations corresponding to the analysed system has the form:

$$\left\| \begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array} \right\| \cdot \left(\begin{array}{c} Z_1 \\ \hline Z_2 \end{array} \right) + \left\| \begin{array}{c|c} \Gamma_{11} & \Gamma_{12} \\ \hline \Gamma_{21} & \Gamma_{22} \end{array} \right\| \cdot \left(\begin{array}{c} Z_1 \\ \hline Z_2 \end{array} \right) + \left(\begin{array}{c} Y_1 \\ \hline Y_2 \end{array} \right) = \left(\begin{array}{c} Z_1 \\ \hline Z_2 \end{array} \right) \quad (2.1)$$

or

$$A \cdot Z + \Gamma \cdot Z + Y = Z$$

where Z is the vector of gross production, $\Gamma = (r_{ij})$ is the matrix of the import coefficients and Y — the vector of final consumption.** We mention that the variant has been adopted, where:

$$a_{ij} = \frac{X_{ij}}{Z_j} \text{ and } r_{ij} = \frac{N_{ij}}{Z_j} \quad (i, j, = 1, 2, ..., 13)$$

where N_{ij} represents the total value of import from the products which can be assimilated to branch i and used in branches j ($j = 1, 2, ..., 13$).

---

*) This indicator comprises: the variation of finished products stocks; the variation of the balance for forwarded products, work and services carried out, invoiced and not cashed; the variation of the balance of the unfinished production.

**) In the statical model $Y = C + \Delta F$

The solving of the equation system (2.1) is obtained on the basis of the known relation:

$$Z = [I - (A + \Gamma)]^{-1} \cdot Y.$$

As the system is made up of sub-branches of the machine-building and metal processing industry, most of the deliveries for the final consumption are alloted to the production and investments of the other branches of the national economy, hence, these demands result from the production programme of the repsective branches. Through the final consumption vector, a relation is formed between the analysed system and the rest of the national economy, to the effect that the functioning parameters of the system must be adjusted so as to ensure the achievement of the ordered dimension Y and thus, the proper functioning of the national economy. Another element through which the analysed system is interconnected with the rest of the national economy appears in quadrant III: expenses of raw materials, auxiliaries, etc. from the other branches of the national economy. One may say that this indicator is an order size for the rest of the national economy, introducing in the model a resource demand from the analysed system to the rest of the national economy.

Having all the elements necessary for calculation, one may proceed to drawing up the plan input-output take, by using the following relations:

$H_{pe} = (A + \Gamma)\widetilde{Z}_{pe}$	for calculating the intermediary consumptions;
$C_{pe} = c\widetilde{Z}_{pe}$	for calculating the expenses resulting from outside the system;
$a_{pe} = a_c\widetilde{Z}_{pe}$	for calculating the amortization;
$V_{pe} = V_c\widetilde{Z}_{pe}$	for calculating the elements of the newly created value.

The notations used in the above relations have the following significance:

H — is the matrix of intermediary consumptions;

c and ca — are line vectors the component parts of which are equal to the coefficients of the consumptions outside the system and, respectively, to the liquidation per leu gross production;

V_c — is a matrix containing the coefficients of the unitary consumption of salaries, the profit per leu gross production, etc.

$\widetilde{Z}_{p1}$ — is the diagonal matrix of the planned gross production.

To draw up the production programme for the 1971—1975 period, data are necessary in connection with the fixed assets existing in each branch, classified according to their source and according to the evolution in time of consumption $C(t)$.

The component parts of vectors $C(t)$ mostly result from the production plan of the other branches and from estimating the demand for export (the deliveries for unproductive consumption having a smaller share).

The mathematical model we shall further use for long-range planning and for the dynamic analysis of the relations among the sub-branches of the

machine-building and metal processing industry is based on a number of hypotheses, among which we mention:

1) It is considered that the structure of the technological relations among the branches of the system, characterized by matrices **A** and Φ remains unchanged for the whole plan period.

2) The coefficients of the fixed assets * $f_{ij} \frac{F_{ij}}{Z_j}$ $(i, j, = 1, 2, \ldots, 14)$ were calculated on the basis of the maximum capacities of the fixed assets, so that the level of utilization of the fixed assets the same as in the basic year will be planned in the future.

3) The relation between the contribution of the fixed assets and of labour to the increase of production is considered as constant.

The knowing of the hypotheses on which the model is based is useful in the interpretation and validation of the solution resulting by solving it.

Before passing on to the calculation of the production programme, the exactness and stability of the coefficients of matrices **A** and Φ were tested. The methods which can be used to this purpose differ in keeping with the data at our disposal. The method we could apply to the data at our disposal consisted in the calculation of the raw production $Z_i(t)$ for values of t which preceede the basic year (in the case examined $to = 1970$) and their comparison to the effective values. Resulting from this comparison was the estimation of the exactness and stability of the technical coefficients and hence, a maximum limit for the time horizon in which these coefficients yield satisfactory results. The results obtained on the whole and on each branch were represented graphically, the fact being noted that the values resulting through calculation for the 1968—1971 period do not significantly differ from the real ones and thus, the conclusion is drawn that matrices **A** and Φ correctly estimate the structure of the analysed system. We mention that n view of ensuring the comparability of data, all indicators have been estimated in the prices of 1970, based on corresponding price indices.

Two of these graphs have been chosen, typical of the examined phenomenon:

- the dynamics of the production of the branch "Energetic machinery and equipment" (Fig. 1).
- the dynamics of the production of the branch "Machinery and technological equipment for industry, constructions and trade" (Fig. 2)

Data estimated through the "dynamized" static model were represented in both graphs, in which the values $Z^s(t)$ were obtained as follows:

$$Z^s(t) = B[C(t) + \Delta F(t)]$$

where the vector of the final consumption also includes the share of production designed for the development fund $[\Delta F(t)]$.

The first conclusion which can be drawn is that in the branches producing means of labour, the estimations made through the dynamic model

*) Indicators F_{ij} represent the volume of fixed assets resulting from branch i and used in branch j. The coefficients f_{ij} form matrix Φ.

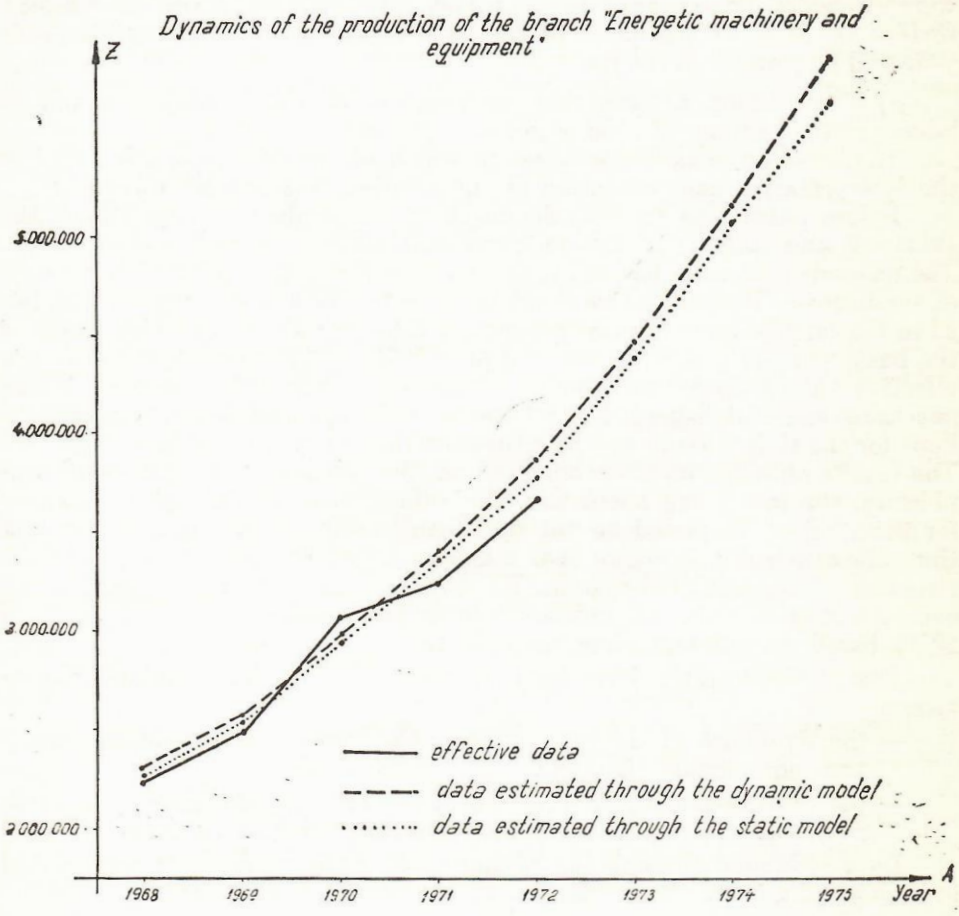
Dynamics of the production of the branch "Energetic machinery and equipment"
Z
5.000.000
4.000.000
3.000.000
2.000.000
effective data
data estimated through the dynamic model
data estimated through the static model
1968
1969
1970
1971
1972
1973
1974
1975
Year
t

Figure 1

surpass those of the static model. The discordance between the estimated data and the real ones in 1972 (Fig. 2) may be determined by the decrease of material consumptions in the respective branch owing to technological changes and the decrease of effective consumptions, or, by the incomplete use of the production capacities planned to be commissioned in 1972.

According to relation (1.6), it results that the production programme for the five years, under the hypothesis that Z (1976) = 0, is obtained by making the following calculation:

$$\begin{bmatrix} Z_{1971} \\ Z_{1972} \\ Z_{1973} \\ Z_{1974} \\ Z_{1975} \end{bmatrix} = \begin{bmatrix} G^{-1} & RG^{-1} & R^2G^{-1} & R^3G^{-1} & R^4G^{-1} \\ 0 & G^{-1} & RG^{-1} & R^2G^{-1} & R^3G^{-1} \\ 0 & 0 & G^{-1} & RG^{-1} & R^2G^{-1} \\ 0 & 0 & 0 & G^{-1} & RG^{-1} \\ 0 & 0 & 0 & 0 & G^{-1} \end{bmatrix} \cdot \begin{bmatrix} C_{1971} \\ C_{1972} \\ C_{1973} \\ C_{1974} \\ C_{1975} \end{bmatrix} .$$

The above inverse matrix has five lines and five columns, each column being made up of submatrices of the order 14. The production necessary for the fabrication of a consumption unit in 1975, since the production expenses of that year are ensured from the production of the preceeding years, appears distributed in time, in the last column. Submatrix R^4G^{-1} shows how high the production level must be in each of the 14 branches in 1971 for ensuring the fabrication of a consumption unit in 1975.

By the graphic representation of the coefficients of the inverse matrix, a curve is obtained illustrating both the distribution in time of the production meant to cover the final demand and the intensity and direction of the production connections among branches.

In Fig. 3 one may follow the distribution in time of the production of the branches "Cast and forged parts of crude iron and steel"; "Energetic machinery and equipment" and "Machinery and equipment for metal processing, treatment and protection" meant to ensure an increase by one million in the final consumption of the branch "Machinery and technological equipment for industry, constructions and trade" in 1975. The point in question being about objects of labour, the demand for the products of the branch "Cast and forged parts of pig iron and steel" is the maximum in the last year (T = 1975) when the consumption is effectively taking place and is ever smaller in the preceding years. In exchange, the demand for products of the branch "machinery and equipment for metal processing, treatment and protection", for example, attains its maximum level in 1974, the hypothesis being thus confirmed that the log period of delay of the investments is of at least one year.

The first conclusion resulting from the analysis of the solution obtained is that the model does not lead to correct estimations of production if only a small number of future effects is taken into consideration. The conclusion is confirmed by the fact that as we are approaching the last year, the relation decreases between the rate of increase of production and the final consumption, a level of production lower than the final consumption being obtained for some branches.

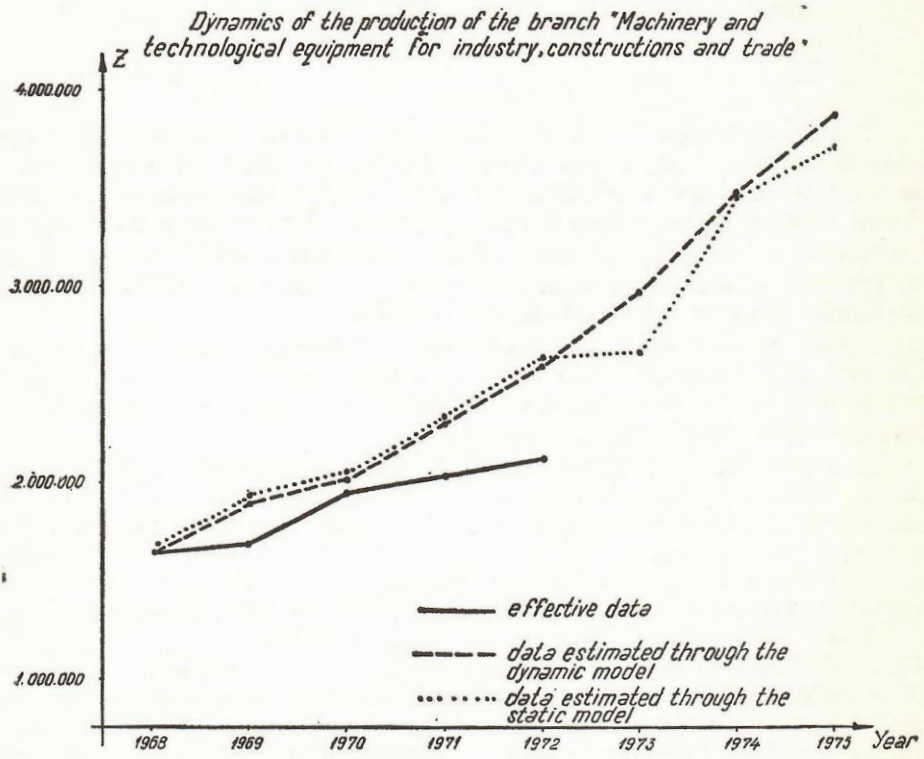
Dynamics of the production of the branch "Machinery and technological equipment for industry, constructions and trade"
Z
4.000.000
3.000.000
2.000.000
1.000.000
1968
1969
1970
1971
1972
1973
1974
1975
Year
effective data
data estimated through the dynamic model
data estimated through the static model

Figure 2

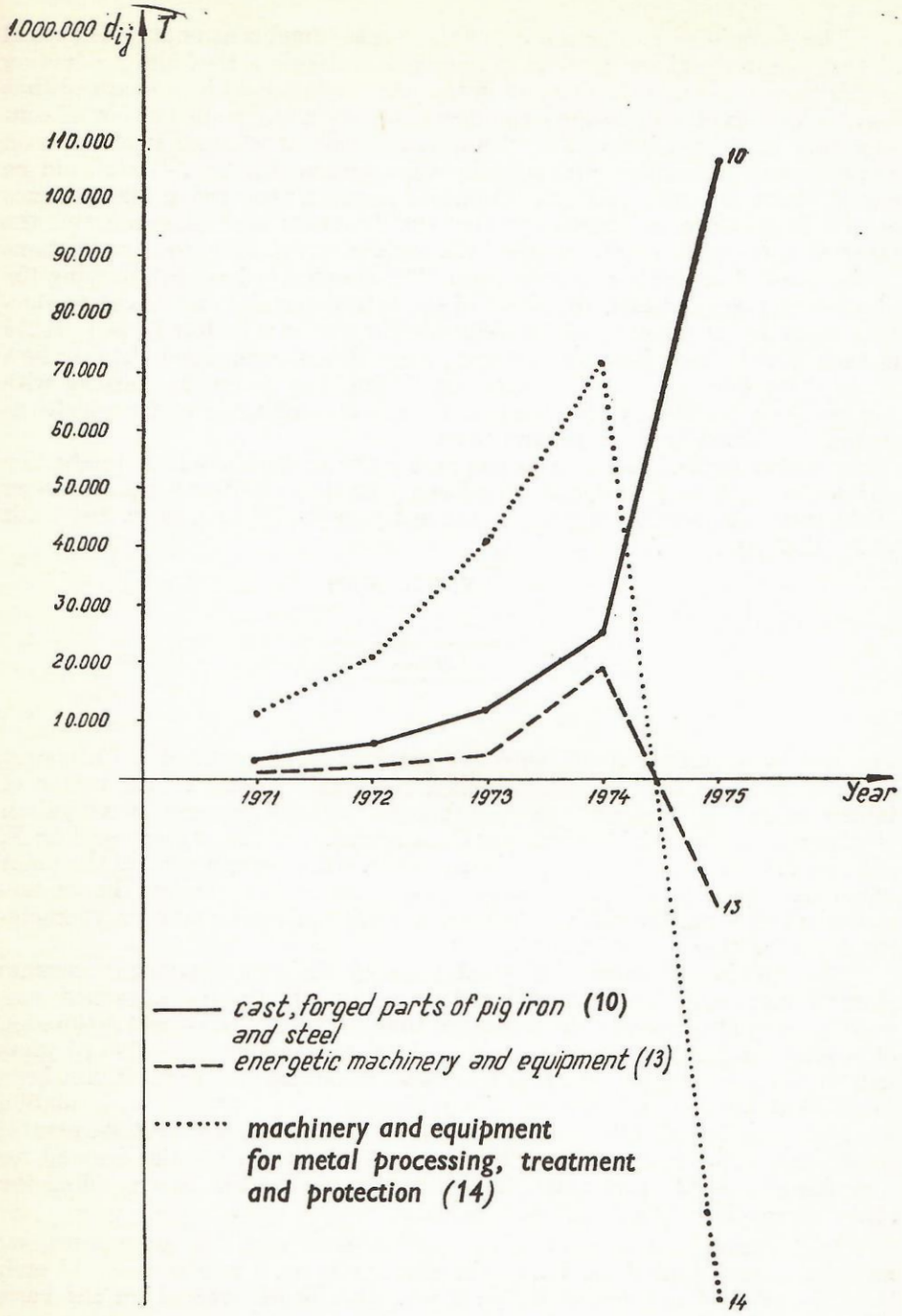
1.000.000 d_{ij} T
110.000
100.000
90.000
80.000
70.000
60.000
50.000
40.000
30.000
20.000
10.000
1971
1972
1973
1974
1975
Year
10
13
14
cast, forged parts of pig iron and steel (10)
energetic machinery and equipment (13)
machinery and equipment for metal processing, treatment and protection (14)

Figure 3

The decrease of production below the level of final consumption is typical of the means of labour producing branches and shows that the production of this branch exceeds the demand in the year T. It should be underlined that the stock of fixed assets shows an excess only from the point of view of consumption in the year $T = 1975$. To obtain a perfect estimation of the level of production, the effect produced by consumption (C_t) for $t \to \infty$ should be considered in the calculation. To the extent to which t is growing, the influence of the next years on current production decreases and consequently, the problem is raised to determine the time horizon for which correct estimations of the level of production are obtained. The simplest way of establishing the number of years in which the effect of the future demand on current production is significant might consist in referring the elements on line $i (i = 1, 2,...14$ of each year to their sum. In this way, a coefficient is obtained showing how much the production level increases by adding new terms. In keeping with the precision wished by the planner, a limit is established under which the increase is considered as insignificant.

Another method consists in comparing the coefficient of the production increase of each year as against the basic year (to = 1970) to the coefficient of the final consumption increase in the last year [Y(T)] as against the basic year, that is:

$$r_i(t) = \frac{\dfrac{X_i(t) - X_i(0)}{X_i(0)}}{\dfrac{Y_i(r) - Y_i(0)}{Y_i(0)}}$$

The above calculated coefficients are graphically represented in a system of cartesian axes in which the abscissa represents time. For the means of labour producing branches $r_i(t)$ may surpass level one because the extension of the production capacities stresses these branches in the years preceding T, the demand for these products decreases when the economy reaches the point of equilibrium. (Fig. 4). The dynamics of demand in the branches the production of which is mainly destined for covering the final consumption is characterized by another curve (Fig. 5).

The number of years comprised between the first significant increase above a minimum level (γ) chosen by the planner (in the examined case $\gamma = 0.1$) and T represents the necessary time horizon for a correct estimation of production. In the first case, the time horizon is made up of $(T - t_1)$ years and in the second case, of $(T - t_p)$ years. Analysing the data, it has been found that for certain branches it is necessary to include in the calculation a bigger number of future influences for obtaining correct estimations of production. Even in case that a five-year plan is drawn up, the demand for a period of 10—12 years must be included in the model, thus avoiding the effect of the hypothesis $Z(T + 1) = 0$.

In a second variant used in calculating the production programme, we have considered that the index of the average increase rate is $\alpha = 1.14$ and, then the level of production for each year can be determined on the basis of relation (1.11).

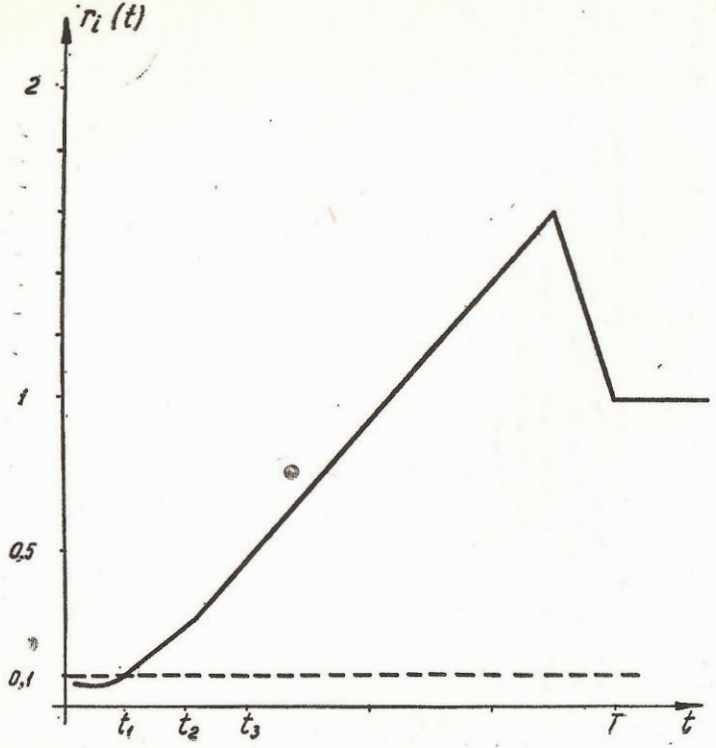

Figure 4

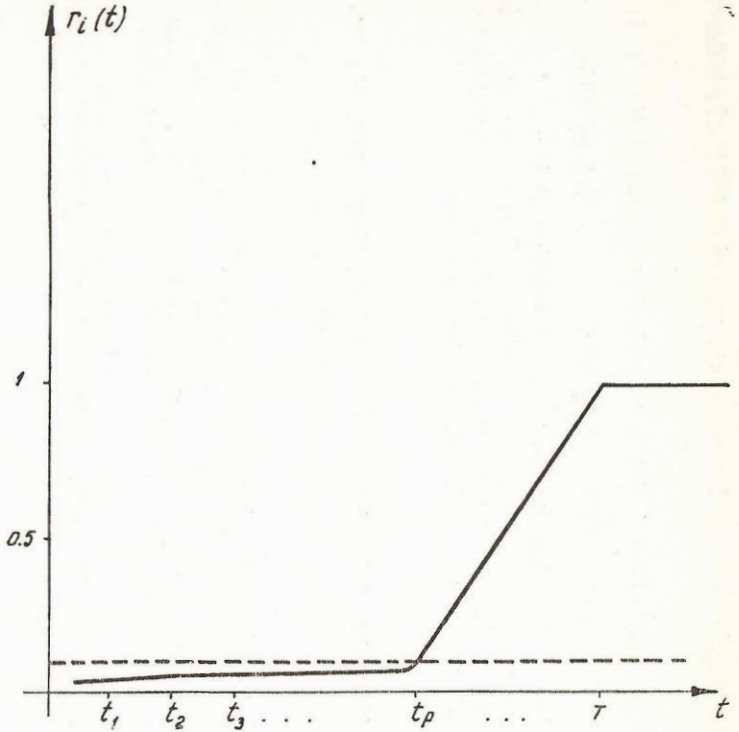

Figure 5

For example, for $t = 1$ (represents the year 1971) the calculation relation is:

$$Z(1) = G^{-1} \cdot C(1) + RG^{-1} \cdot C(2) + R^2 \cdot G^{-1}C(3) + R^3G^{-1}C(4) +$$
$$+ R^4G^{-1} \cdot C(5) + 1.14\, R^5[I - A - 0.14\, \Phi]^{-1}C(5).$$

In view of choosing the most correct solution, it is useful to compare the solutions corresponding to different values of $C(t)$ to the real values of $Z(t)$ for the period prior to the basic year. The solutions obtained by using several variants of dynamic models can also be compared.

LE MODELAGE DE LA CROISSANCE ECONOMIQUE AU NIVEAU D'UNE BRANCHE

La direction du système économique national suppose l'étude des relations qualitatives et quantitatives ayant lieu entre ses éléments et impose la transposition en langage mathématique de sa structure et des relations qui existent entre les éléments de ce système.

Les modèles input-output sont facilement adaptables à l'analyse des processus économiques qui se produisent dans les différentes sous-branches de l'économie nationale: industrie, bâtiments, agriculture, etc.

L'auteur se propose, dans le présent article, d'élucider les problèmes posés par l'application d'un modèle dynamique discret du genre input-output à l'élaboration du programme de croissance économique de l'industrie des mécaniques et de celle de l'usinage des métaux.

МОДЕЛИРОВАНИЕ ЭКОНОМИЧЕСКОГО РОСТА НА УРОВНЕ ОТРАСЛИ

Руководство национальной экономической системой предполагает изучение качественных и количественных отношений, существующих между ее элементами, и требует выражения на математическом языке ее структуры и отношений между элементами этой системы.

Модели «input — output» могут быть легко приспособлены и для анализа экономических процессов, имеющих место в рамках различных подсистем народного хозяйства — промышленности, строительства, сельского хозяйства и т.д.

В настоящей статье автор ставит перед собой задачу выяснить вопросы, вытекающие из применения дискретной динамической модели типа «input — output» при разработке программы экономического роста машиностроительной и металлообрабатывающей промышленности.