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Use of Econometric Calculus in Studying the Efficiency of Fertilizers Applied for Vegetables (III)

"An accumulation of deeds is not a science, as well as, a heap of stones is not a house".

Henri Poincaré

III. BEST COMBINATION AND SUBSTITUTION OF FERTILIZERS FOR GREENHOUSE TOMATOES.

The fertilization of soil in vegetable greenhouses presents many particularities in comparison with its application in field vegetable growing. The ensurance of the content in nourishing substances corresponding to the biological requirements of plants is being achieved in the form of basic and soil fertilizations in the vegetation period.

In the vegetable greenhouses which have large possibilities of directing the meteorological elements, a satisfactory supply with nourishing elements can be achieved provided that a lack of balance is avoided between the environment agents in the air and soil. A physiological or biochemical lack of balance with direct implications as regards the decrease of production may appear both in the excessive and insufficient use of mineral fertilizers.

That is why, the permanent control of fertilization and the establishment of corresponding doses of fertilizers through the optimization method represent sure premises for achieving a best nutrition policy and for applying a judicious programme of fertilizers on stages.

Setting out from these considerations, the present study is trying to demonstrate, by econometric computation methods, the influence of various doses of fertilizers, of their combination and of the possibilities of substitu-

tion in a greenhouse tomato culture, the economic optimum being indicated in the end.

Table 1 presents the doses of fertilizers in various variants, as they had been experimented at the Popeşti-Sere State Agricultural Enterprise, in the first production cycle (January-June), on 100 tons of manure per hectare.

Table 1

The influence of the doses of fertilizers on tomato production

Variants	In kg/ha N	In kg/ha P	In kg/ha K	Tomato production kg/ha
	non-fertilized			
1 witness				70.500
2	100	200	300	79.700
3	200	200	300	76.200
4	300	200	300	72.000
5	400	200	300	73.200
6	500	200	300	74.700
7	600	200	300	81.500
8	700	200	300	81.500
9	300	50	300	80.700
10	300	100	300	82.200
11	300	200	300	82.200
12	300	300	300	84.200
13	300	400	300	79.700
14	300	500	300	77.700
15	300	200	100	78.200
16	300	200	200	76.700
17	300	200	300	81.500
18	300	200	400	79.500
19	300	200	500	79.700
20	300	200	600	81.200
21	300	200	700	81.000

By applying the multiple correlation method, the simplex method, the regression equation was obtained:

$$Y = 7.3985 - 0.0531\ x_1 + 0.1182\ x_2 - 0.0319\ x_3$$

in connection with which the notations were used:

$$y = \text{tomato crop};\quad x_1 = \text{N};\quad x_2 = \text{P};\quad x_3 = \text{K}.$$

The contribution of each element to the variation of the tomato crop is determined with the aid of the determination partial coefficients:

$dyx_1 \cdot x_2x_3 = 8.03\%$ (the influence of nitrogen);
$dyx_2 \cdot x_1x_3 = 1.66\%$ (the influence of phosphorus);
$dyx_3 \cdot x_1x_2 = 16.56\%$ (the influence of potassium);
26.25%

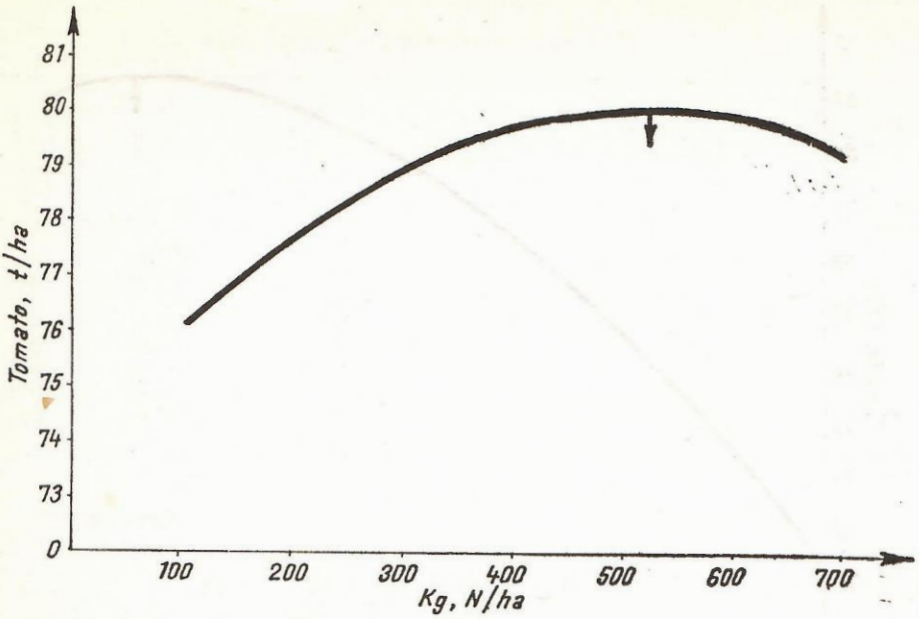


Fig. 1 Variation of tomato production in keeping with nitrogen quantity modification

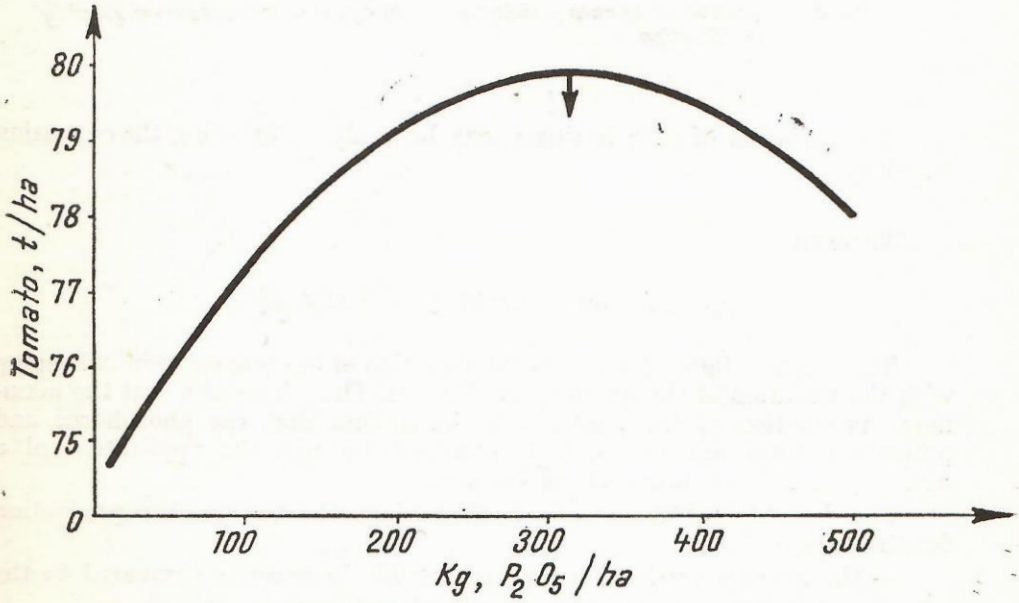


Fig. 2 Variation of tomato production in keeping with phosphor quantity modification

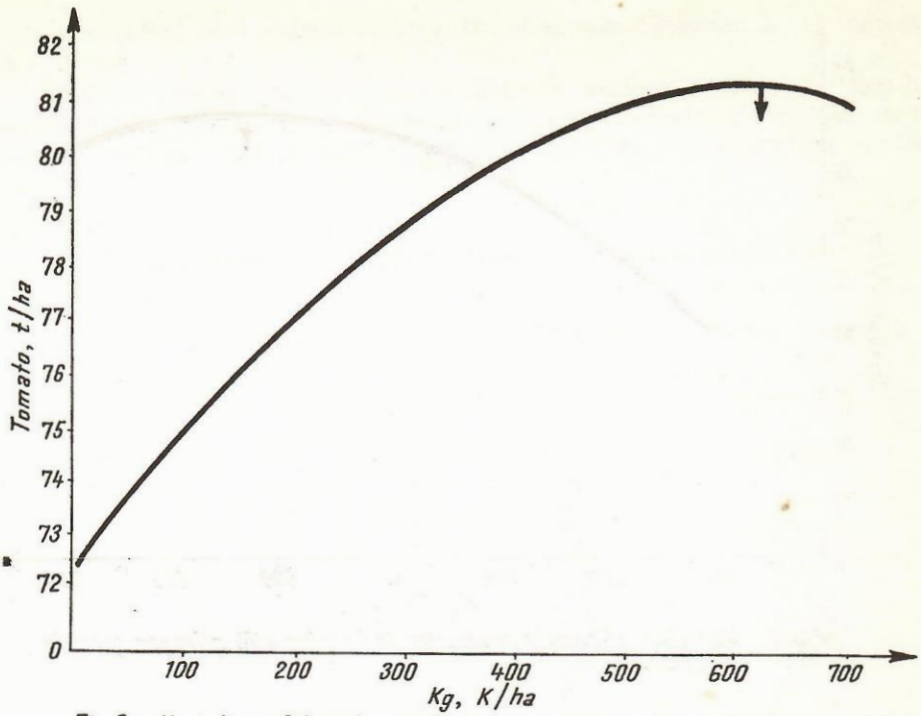


Fig. 3 Variation of tomato production in keeping with potassium quantity modification

The variation of each fertilizer may be analysed by using the regression equation.

Nitrogen

$$y_{x1} = 7.3939 + 0.2418\ x_1 - 0.0234\ x_1^2$$

The graph in figure 1 presents the variation of the tomato crop in keeping with the variation of the quantity of nitrogen. Thus, it results that the maximum production of fruit, 80 185 kg/ha in case that the phosphorus and potassium doses are constant, is obtained through the application of a quantity of 517 kg. nitrogen per hectare.

The derived indicators were determined on the basis of the production function (table 2):

— the average production (the production increase as compared to the witness related to the quantity of applied fertilizer);

— the marginal product (the production increase as compared to the previous level related to the increase of applied fertilizer).

Table 2

Applications of the marginal calculus to N

N/ha kg	Tomato crop kg/ha	Increase with fixed base	Average production	Increase with a chain base	Marginal product
x	y	Δy_0	$\Delta y_0/x$	Δy_{u-1}	$\Delta y_{u-1}/x_{n-1}$
0	73 939	—	—	—	—
100	76 123	2 184	21.84	2 184	21.84
200	77 839	3 900	19.30	1 716	8.58
300	79 087	5 848	17.16	1 248	4.16
400	79 867	5 928	14.82	780	1.95
500	80 179	6 240	12.48	312	0.624
600	80 023	6 084	10.14	— 156	— 0.26
700	79 399	5 460	7.80	— 624	— 0.89

Phosphorus

$$\bar{y}_{x2} = 7.4562 + 0.3476\ x_2 - 0.0561\ x_2^2.$$

The graph in Figure 2 presents the variation of tomato production in keeping with the variation of phosphorus. The maximum fruit production 79 947 kg/ha is obtained with 310 kg phosphorus per hectare. The derived indicators can be seen in table 3.

Table 3

Applications of the marginal calculus to P

kg P/ha	Tomato crop kg/ha	Increase with fixed base	Average production	Increase with a chain base	Marginal product
x	y	Δy_0	$\Delta y_0/x$	Δy_{u-1}	$\Delta y_{u-1}/x_{n-1}$
0	74 562	—	—	—	—
100	77 477	2 915	29.15	2 915	29.15
200	79 270	4 708	23.54	1 793	8.96
300	79 941	5 379	17.93	671	2.24
400	79 490	4 928	42.32	— 451	— 1.23
500	77 917	3 355	6.71	—1 573	— 3.15

Potassium

$$\bar{y}_{x3} = 7.24286 + 0.2866x_3 - 0.0232\ x_3^2.$$

The quantity of potassium leading to the highest level of fruit production: 81 337 k/ha is obtained with 618 kg potassium per hectare (Fig. 3). The applications of the marginal calculus are presented in table 4.

Table 4

Applications of the marginal calculus to K

K/ha kg	Tomato crop kg/ha	Increase with fixed base	Average production	Increase with a chain base	Marginal product
x	y	Δy_0	$\Delta y_0/x$	Δy_{u-1}	$\Delta y_{n-1}/x_{n-1}$
0	72 486	—	—	—	—
100	75 120	2 634	26.34	2 634	26.34
200	77 290	4 804	24.02	2 170	10.85
300	78 996	6 510	21.70	1 706	5.69
400	80 338	7 752	19.38	1 242	3.11
500	81 016	8 530	17.06	778	1.56
600	81 330	8 844	14.74	314	0.52
700	81 180	8 694	12.42	— 150	— 0.21

Recapitulating the effect of the three fertilizers (in kg/ha) with various doses on the production increase, table 5 results.

Table 5

Simultaneous influence of N, P *and* K *on the tomato production increase* (in kg/ha)

Quantity of fertilizers	N	P	K	Total
100	2 184	2 915	2 634	7 733
200	3 900	4 708	4 804	13 412
300	5 148	5 379	6 510	17 037
400	5 928	4 928	7 752	18 608
500	6 240	3 355	8 530	18 125
600	6 084	660	8 844	15 588
700	5 460	—	8 694	14 154

It results from the data of table 5 that various increases are obtained in the application of the same quantities of fertilizers, but of different sorts.

In continuation, the combined action of two fertilizers is presented, with the respective production increase in (kg/ha), (tables 6,7 and 8.)

Table 6

Action of the variation of N *and* P *on the tomato production*

Nitrogen / Phosphorus	0	100	200	300	400	500	600
0	63 407	65 591	67 307	68 555	69 335	69 647	69 491
100	66 322	68 506	70 222	71 470	72 250	72 562	72 406
200	68 115	70 209	72 015	73 263	74 043	74 355	74 199
300	68 786	70 970	72 686	72 934	74 714	75 026	74 870
400	68 335	70 519	72 235	73 483	74 263	74 575	74 419

Table 7

Action of the variation of N *and* K *on the tomato production*

Nitrogen / Potassium	0	100	200	300	400	500	600
0	63 407	65 591	67 307	68 555	69 335	69 647	69 491
100	66 041	68 225	69 941	71 189	71 969	72 281	72 125
200	68 211	70 395	72 111	73 359	74 139	74 451	74 295
300	69 917	72 101	73 817	75 065	75 845	76 157	76 001
400	71 159	73 343	75 059	76 307	77 087	77 399	77 243
500	71 937	74 121	75 837	77 085	77 865	78 177	78 021
600	72 251	74 435	76 151	77 399	78 179	78 491	78 335
700	72 101	74 285	76 001	77 249	78 029	78 341	78 185

Table 8

Action of the variation of K *and* P *on the tomato production*

Potassium / Phosphorus	0	100	200	300	400	500	600	700
0	63 407	66 041	68 211	69 017	71 159	71 937	72 251	72 101
100	66 322	68 956	71 126	72 832	74 074	74 852	75 166	75 016
200	68 115	70 749	72 919	74 625	75 867	76 645	76 959	76 809
300	68 786	71 420	73 590	75 296	76 538	77 316	77 630	77 480
400	68 335	70 969	73 139	74 846	76 087	76 865	77 179	77 029

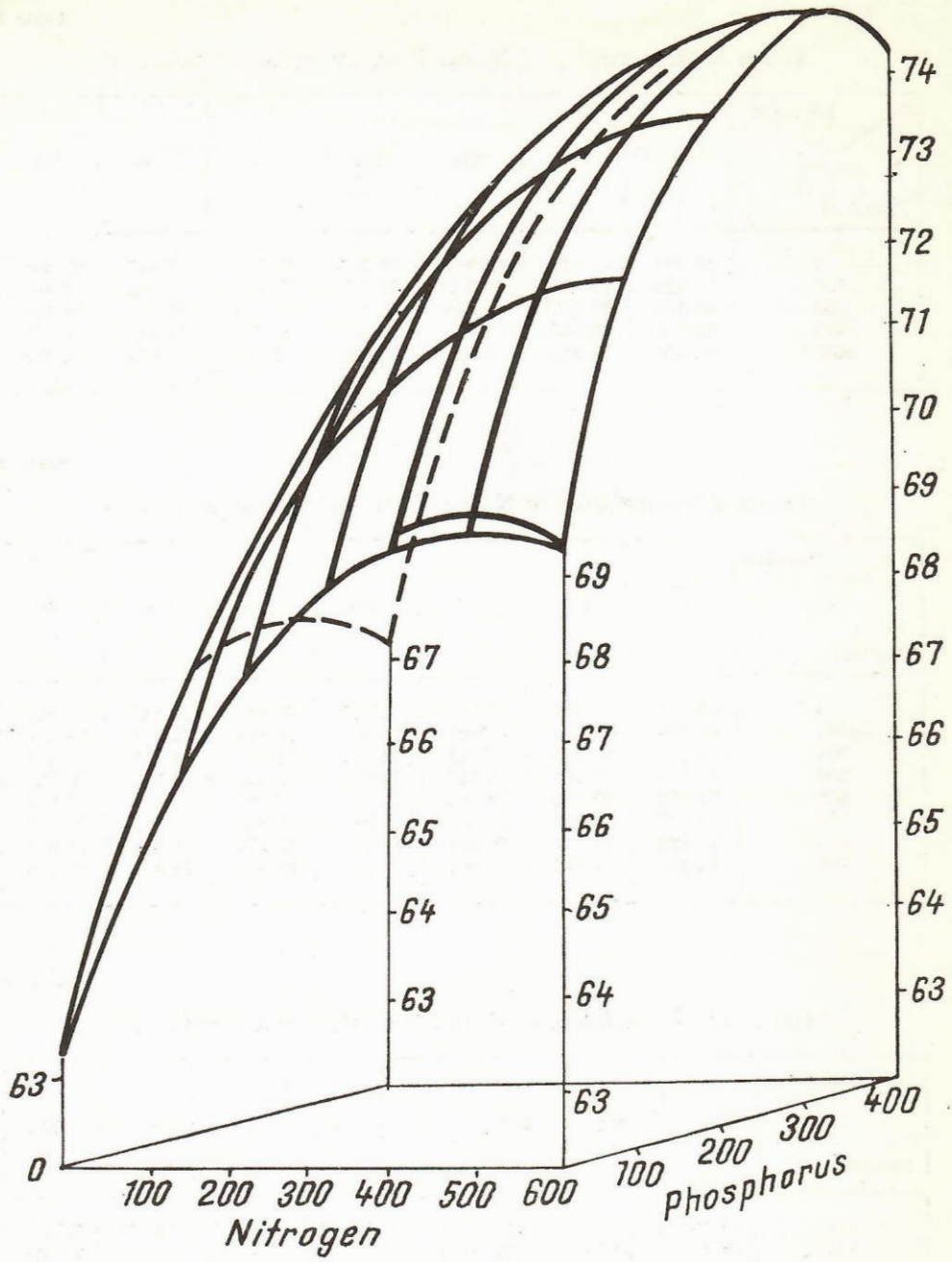


Fig. 4 N P production surface

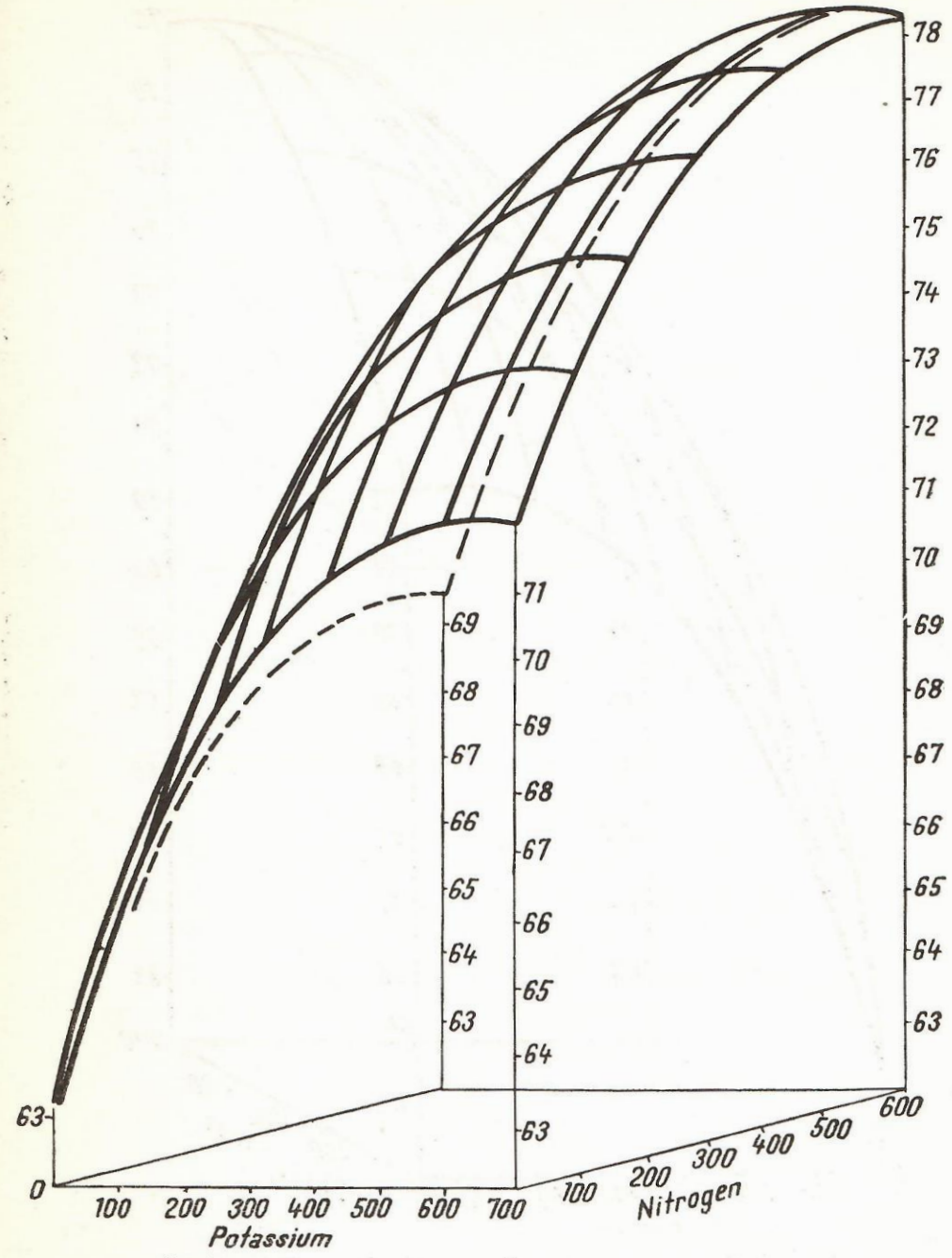


Fig. 5 K N production surface

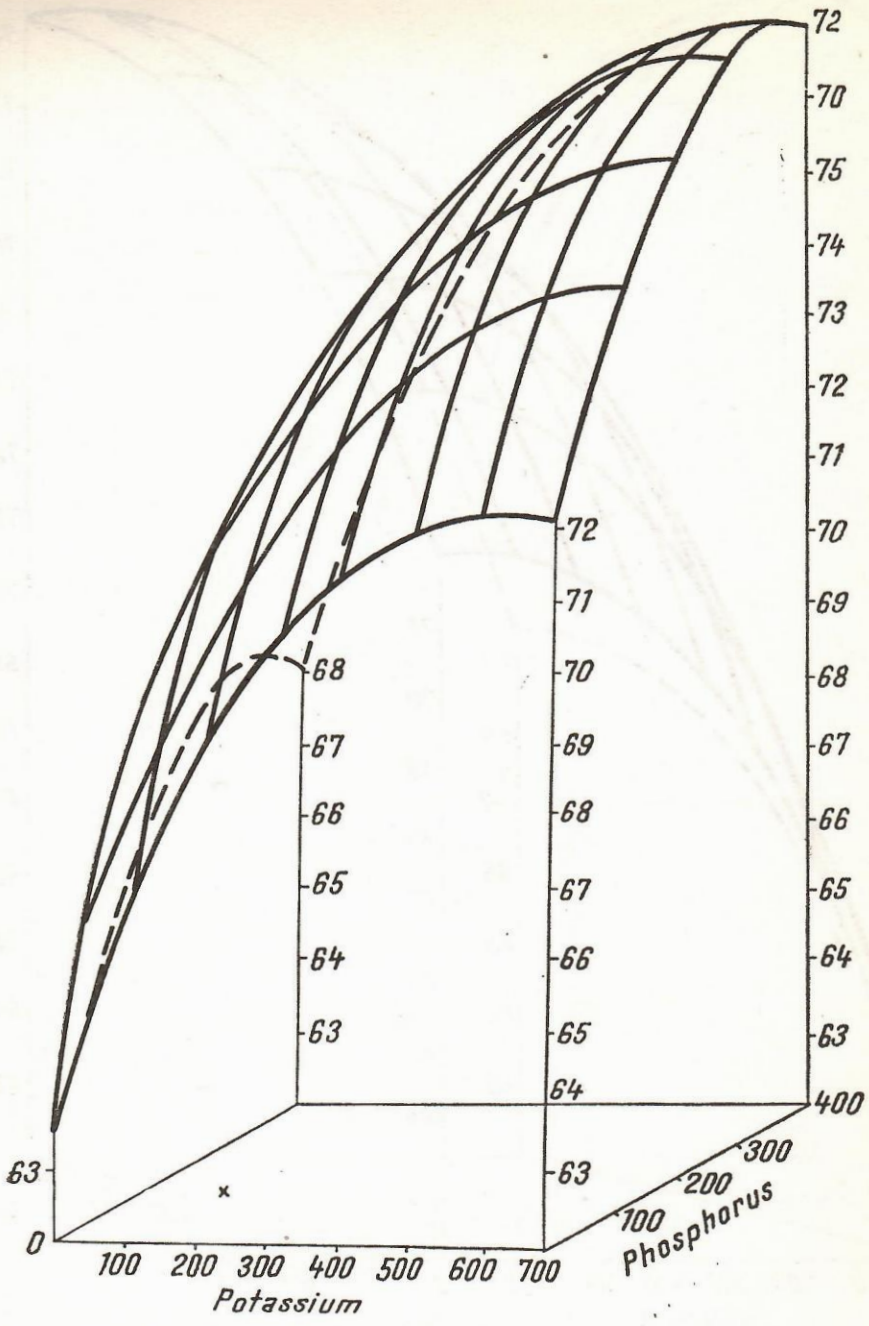
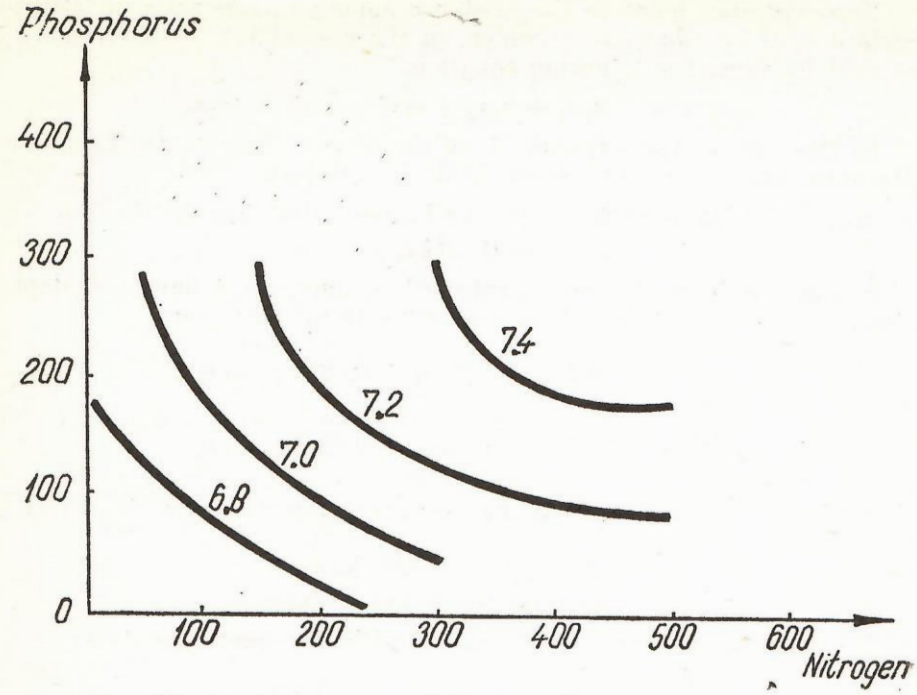


Fig. 6 K P production surface



Graphic reprezentation of isoquanta

Fig. 7

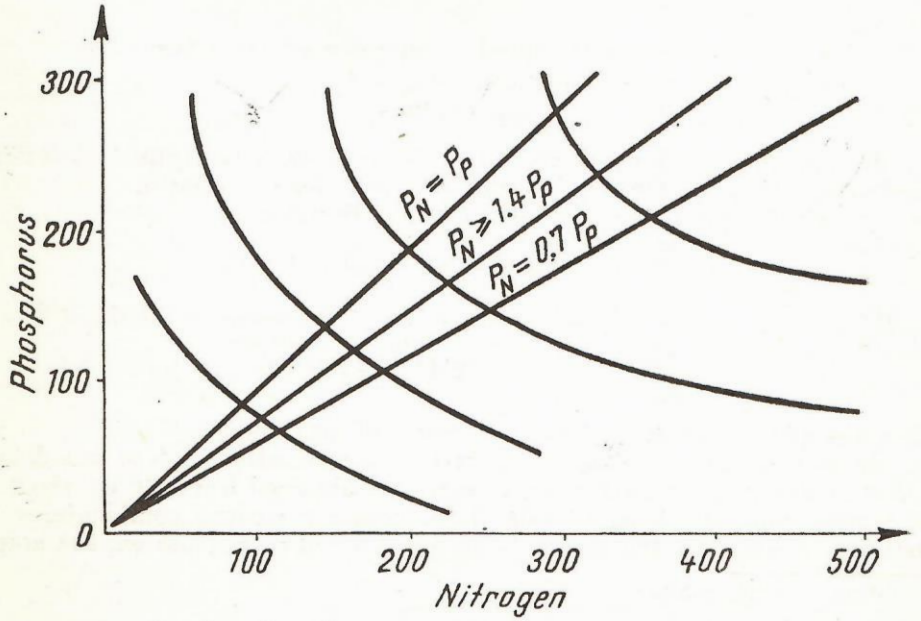


Fig. 8 Graphic representation of isoclines

Recent studies point to the existence among various sorts of fertilizers of relations of interdependence which, in the case of two fertilizers may be modelled by using the following equation:

$$y = a + b_1x_1 + b_2x_2 + c_1a_1^2 + c_2a_2^2 + dx_1x_2$$

In the case of the experiment at the Popești-Sere State Agricultural Enterprise, the calculations made lead to equation:

$$\bar{y}_{x_1x_2} = 7.2395 + 0.1867\,x_1 + 0.2567\,x_2 - 0.0186\,a_1^2 - 0.0375x_2^2 + \\ + 0.0211x_1x_2$$

Setting out from the afore-mentioned relation, the following system of normal conditions is obtained by successive transformations:

$$\frac{\partial y}{\partial x_1} = 0.1867 - 0.0186\,x_1 + 0.0211\,x_2 = 0$$

$$\frac{\partial y}{\partial x_2} = 0.2567 - 0.0373\,x_2 + 0.0211\,x_1 = 0.$$

Solving them, the doses giving the maximum tomato crops are found, as follows:

nitrogen = 498 kg/ha
phosphorus = 350 kg/ha

The economic optimum is determined with the equation system:

$$\frac{Px_1}{Pp} = 0.1867 - 0.0186\,x_1 + 0.0211\,x_2;$$

$$\frac{Px_2}{P_p} = 0.2567 - 0.0373\,x_2 + 0.0211\,x_1;$$

where:

P_p — represents the production price of tomatoes
Px_1 — the cost of nitrogen in a. s. *
Px_2 — the cost of phosphorus in a. s.

In our case, by solving them, the solutions to the normal equation system are found: 421 kg/ha for nitrogen and 300 kg/ha for phosphorus.

For checking we shall also calculate the Hessian:

$$\mathbf{H}\begin{vmatrix} \frac{\partial_y^2}{\partial x_1^2} & \frac{\partial_y^2}{\partial x_1 \partial x_2} \\ \frac{\partial_y^2}{\partial x_2 \partial x_1} & \frac{\partial_y^2}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} -0.0186 & 0.0211 \\ 0.0211 & -0.0373 \end{vmatrix} = -0.0011 < 0$$

thus, the established critical point is really of maximum.

In view of a more complete analysis, the production surface was determined, comprising the total production results obtained from the intersection of various harvest plans. Certain differences for various combinations of fertilizers result from the graphs in figures 4,5 and 6. Further on, the graph

* a.s. = active substance

in Fig. 7 with the isoproducts (isoquanta) was drawn up, showing the possible combinations among fertilizers in the case of a constant production. They are graphically represented as intersection points between the surface and the production plan.

By using the equation of the isoproduct or isoquanta, the production increase which may be obtained with various combinations of fertilizers can be determined:

$$x_1 = \frac{-b_1}{2b_2} + \sqrt{\left(\frac{b_1}{2b_2}\right) - \frac{a + c_1x_1 + c_2x_2^2 - y}{b_2}}$$

and

$$x_2 = \frac{c_1}{2c_2} \pm \sqrt{\left(\frac{c_1}{2c_2}\right) - \frac{a + b_1x_1 + b_2x_1^2 - y}{c_2}}$$

which is being illustrated through the data in table 9.

Table 9

Combination N, P *for obtaining a constant crop increase*

6,8 t/ha		7,0 t/ha		7,2 t/ha		7,4 t/ha	
N	P	N	P	N	P	N	P
100	78	100	175				
200	19	200	89	200	197		
		300	44	300	122		
				400	88	400	194
				500	76	500	172
				600	82	600	182
				700	108	700	238

It results from those shown above that the dynamics of the two fertilizers is inversely proportional and the technical efficiency of fertilizers is variable and unproportional. By applying the quantity of 200 kg nitrogen per hectare and variable quantities of phosphorus, the following efficiencies are obtained as results from table 10.

Table 10

Technical Efficiency of Fertilizers

Quantity of tomatoes tons/ha	Harvest increase t/ha	Quantity of phosphorus kg/ha	Production increase per kg P	Marginal product
	x	y	$\frac{x}{y}$	$\Delta x_{n-i}/\Delta y_{n-1}$
63.4	—	—		
70.2	6.8	19	357.9	357.9
70.4	7.0	89	78.6	28.5
70.6	7.2	197	36.5	1.8

It is also noted that for smaller quantities, the increase per fertilizer unit is more economical and above certain limits, the increase of the fertilizer dose is absolutely improper. Thus, to obtain 7.2 t/ha of tomato increase in case of the administration in the soil of 300 kg phosphorus, 156 kg nitrogen are to be applied. Increasing the quantity of phosphate fertilizers per 400 kg/ha this entails an increase of the quantity of nitrogen from 156 kg/ha to 185 kg/ha.

Of particular interest are also the isoclines (fig. 8) namely the geometrical place of the points situated on the growing curves of the isoquanta with a certain relation of prices, equal to $\left(\frac{P_2}{P_1}\right)$ or $\left(\frac{P_1}{P_2}\right)$. The following expressions are used for computation:

$$x_1 = \frac{\left(b_1 \frac{P_2}{P_1} - c_1\right) - \left(2c_2 + \frac{P_2}{P_1}\right) x_2}{1 + 2b_2 \frac{P_2}{P_1}}$$

and

$$x_2 = \frac{\left(c_1 \frac{P_1}{P_2} - b_1\right) + \left(2b_2 + \frac{P_1}{P_2}\right) x_1}{1 + 2c_2 \frac{P_1}{P_2}}.$$

Each isocline is calculated for a certain price ratio between the two fertilizers. The tangency points of the isoclines with the isoquanta curves show the best combination of the two fertilizers.

For replacing, in certain limits and in the given technical conditions, some resources by others, their substitution is applied to. The marginal substitution rate shows how much the utilization of a fertilizer in the combination must increase for maintaining production at the same level, in case that the quantity of the other fertilizer is reduced.

For the present analysis, we set out from establishing the partial differentials of the production function:

$$y = a + b_1 x_1 + b_2 x_1^2 + c_1 x_2 + c_1 x_2^2$$

equalizing them afterwards to the price ratio:

$$\frac{\partial y}{\partial x_1} = b_1 + 2b_2 x_1 = \frac{P_1}{Py}$$

$$\frac{\partial y}{\partial x_2} = c_1 + 2c_2 x_2 = \frac{P_2}{Py}$$

from where the equation of the substitution marginal rate becomes:

$$\frac{\partial x_1}{\partial x_2} = \frac{c_1 + 2c_2 x_2}{b_1 + 2b_2 x_1} = \frac{P_2}{P_1}$$

Concretely, the production function is the following:

$$\bar{y}_{x_1 x_2} = 6.3407 + 0.2418\, x_1 - 0.0234\, x_1^2 + 0.3476\, x_2 + 0.0561\, x_2^2$$

and the equation of the substitution marginal rate becomes:

$$\frac{x_1}{x_2} = \frac{0.3476 - 2 \cdot 0.0561\ x_2}{0.2418 - 2 \cdot 0.0234\ x_1} = \frac{P_1}{P_2}.$$

Finally

$$x_1 = \frac{0.2418 - \dfrac{0.3476 - 0.1122\ x_2}{1.4}}{0.0468}$$

and respectively:

$$x_2 = \frac{0.3476 - 1.4\ (0.2418 - 0.0468\ x_1)}{0.1122}$$

where, 1.4 calculated represents the price ratio between N and P.

On the basis of these calculations, table 11 was drawn up comprising the economic limits of replacing the nitrogen fertilizers (x_1) by the phosphate ones (x_2) and inversely, as well as their substitution relations.

Table 11

Substitution Relations		
x_1	x_2	$\frac{x_2}{x_1}$
100	67	0.67
157	100	0.64
200	125	0.62
300	183	0.61
329	200	0.61
400	242	0.60
500	300	0.60
600	358	0.59
661	400	0.60

In comparison with the ever increasing quantity of fertilizers used in greenhouse vegetable cultures, the application of computing methods for fertilizer combinations and the substitution possibilities become extremely useful for raising the economic efficiency of production expenses and for increasing efficiency per surface unit.

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UTILISATION DU CALCUL ECONOMETRIQUE DANS L'ETUDE DE L'EFFICACITE DES ENGRAIS APPLIQUES AUX LEGUMES (III)

Mélange optime et substitution des engrais pour les tomates de serre

S'occupant de l'application du calcul économétrique à la résolution des problèmes de la production, les auteurs cherchent à déterminer l'influence de différentes doses d'engrais et de leur administration combinée en diverses proportions, ainsi que de la possibilité de substituer un engrais à un autre. En même temps, ils s'occupent du résultat, en tant que volume, de la production de légumes, et déterminent l'optime économique.

ПРИМЕНЕНИЕ ЭКОНОМЕТРИЧЕСКОГО РАСЧЕТА ПРИ ИЗУЧЕНИИ ЭФФЕКТИВНОСТИ ВНЕСЕНИЯ УДОБРЕНИЙ В ОВОЩЕВОДСТВЕ (III)

Оптимальное комбинирование и замена одних видов удобрений другими для парниковых культур помидоров

В рамках заботы о применении эконометрического расчета для разрешения задач производства авторы настоящей статьи занимаются вопросом определения влияния, оказываемого различными дозами удобрений и их комбинированным внесением в различных пропорциях, а также возможностями замены одного вида удобрений другим на объем продукции овощей. Одновременно определяется экономический оптимум.