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Mathematical Model in Urban Passenger Transport

Starting from the idea that the traveller wishing to move between two zones of town chooses the variant of minimum path, the author draws up a mathematical model with the aid of which the structure of transport demand of the active population is determined.

The solving of the problems linked with public transport requires the knowing of the existing transport demand and the envisaging of its future structure, an adequate planning of the transport network being made on this basis.

To establish the movement of persons to and from various zones, the Bucharest Transport Enterprise made a poll on the transport of the population. The data of the poll have established the present transport demand n_{ij} [6]. By using a mathematical model of forecasting the transport demand, the transport demand is determined at the level of the years 1975, 1980 (by knowing from the soundings n_{ij} at the level of 1970) [1], [2], [3], [6].

Finally, by using a mathematical model for the distribution of the transport demand on the minimum path, the loads on all sections of the town graph are determined [4], [5].

A clear image of the distribution of demand in the public transport network is thus obtained.

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NOTATIONS

Of interest in our present study are only the movements home-place of work and vice versa.

We assimilate each zone of the town to one point; thus, the size a_i, e_i is considered [6], concentrated in each point. Consequently, the (continuous)

zonal structure of the town turns into a (discontinuous) punctiform structure.

For the analysis of the transport system to have a high precision, a transformation must be considered which should reduce a town to a big number of points. In the first stage, a division was made into 33 zones (by comparison: the town of Paris was divided into 90 zones).

The graph of the town is thus obtained, in which the sizes will be defined:

l_{ij} — the length between the gravity centres of zones i and j;
l_{ij}^1 — the length taken on the present transport network;
l_{ij}^2 — the length taken on a straight line;
t_{ij} — the necessary time for making the trip between zone i and zone j;
k_{ij} — the cost of the trip between zone i and zone j.

To analyse the way of choosing the route between zone i and zone j by the traveller, we shall tackle the sizes which might characterize the arces of the graph.

In Bucharest, the price variations are small, so that the price of the trip k_{ij} will not influence a very big number of travellers in choosing the route.

There is a direct link between length l_{ij} and time t_{ij} in case of the length taken in straight line covered by the same means of transport (the case of the underground). In this case, the size l_{ij}^2 may be used in the analysis.

In the event of the existance of several types of transport between two zones, at different speed, the length l_{ij}^2 can no longer influence the traveller in choosing the route. It does not count for him that the length between the two zones is smaller on a route if the time affected to the trip on that route is longer. That is why, in this case, the analysis is made with the aid of size t_{ij}.

2. MATHEMATICAL MODEL FOR FORECASTING THE TRANSPORT DEMAND

The town of Bucharest being continuously developing, a permanent increase is taking place in the number of work places (e_i) and of dwellings (a_i). This is leading to an increase in the number of passengers participating in the inter-zonal traffic. For a better knowledge of the future transport demand, it is necessary to use a mathematical model for forecasting the transport demand of the type of *the preferential equilibrium model.* Besides the structure data of the town, this model considers the inhabitants' preferences as regards the proximity of the dwelling to the place of work and the harmonization of these wishes with the real possibilities offered by the town.

The relations existing between the parameters of the mathematical model are defined in [6].

The transport demand n_{ij} of passengers moving from zone i to zone j is:

$$n_{ij} = f(\alpha, \beta, \gamma, \lambda, \quad a_i, e_j, N) \tag{1}$$

In the first stage, for the known values of n_{ij}, a_i, e_j, N the coefficients of global structure α, β, γ of the town are determined.

Functions (1) being linear in α and β for fixed λ we obtain the equations of the straights lines

$$\frac{\alpha}{a_{ij}} + \frac{\beta}{b_{ij}} - 1 = 0 \quad i, j = 1, 2, ..., m \tag{2}$$

If there is a value of λ for which the straight lines (2) converge to a point V belonging to the area, then the town population can be characterized as global through the values α, β of point V and the model may be considered as acceptable. The tests made in several towns of the world have shown that the family of straight lines approximates a fascicle and thus the model is applicable.

The optimum value of λ is determined from the condition that the sizes of the traffic n_{ij}, calculated with this value should best approximate the values n_{ij} in the sounding.

In the next stage, with the aid of relation (1), the model may serve as an instruments of prognosis for the determination of n_{ij} at the level of the 1975—1980 period, this time α, β, γ, λ, a_i e_i and N being known.

3. MATHEMATICAL MODEL FOR DISTRIBUTING THE TRANSPORT DEMAND ON MINIMUM ROADS.

3.1. Model. Calculation Time.

a. To establish the major directions of the transport demand in the town of Bucharest, the length in a straight line (l^2_{ij}) was considered. We consider that zone i has zones l and k as neighbours, zone k has i, j as neighbours and zone l has i and j as neighbours.

The road from zone i to zone j may be chosen in two variants:

$$ilj \text{ — with the value } l^2_{il} + l^2_{l,j}$$

$$\lceil ikj \text{ — with the value } l^2_{ik} + l^2_{kj}$$

The passenger wishing to move between zone i and zone j chooses the route of the trip according to the criterion of the minimum road between them.

Suppose that the route satisfying this condition is ikj made up of the sections ik and kj.

The total number n_{ij} of the persons travelling between zone i and zone j was previously determined (the preferential equilibrium model).

Determining all the minimum roads among all the zones of the town and considering that all passengers are travelling on minimum roads, the loads of all sections are obtained in the end, according to the logical scheme (fig. 1).

— all the minimum roads are determined in the first stage (I) and the results are registered on a band B.

— In the second stage (II), the data of the matrix of the transport demand n_{ij} will be distributed on the minimum roads on band B.

The loads of all sections are thus obtained. Supposing that several minimum roads between two zones i and j were obtained in the first stage, these

roads may be analysed and the decision may be taken on which of the minimum roads the distribution of passengers n_{ij} should be made, changing the data of the band B by the decisions adopted (the third stage).

Phases I, II or I, III, II (as the case may be) will be carried out with the data of the matrix n_{ij} for the years 1970, 1975 and 1980, the three matrices of the sections load being determined.

Among the algorithms used for determining the minimum roads the Bellman-Kalaba algorithm was used because it allows the ordination of the calculations and programming on electronic computers [5].

After we build a graph G the arces of which have the lengths l_{ij}^2 we attach to it a matrix C of elements C_{ij} necessary for determining the minimum roads among all the peaks of the graph and the peak x_m. An artifice must be made for determining the minimum roads from all the peaks of the graph to another fixed peak X_{m-1}.

To this purpose, a re-arrangement of the initial matrix C_{ij} is made.

$$\begin{bmatrix} C_{1,1} & C_{1,2\ldots} & C_{1,m-1} & C_{1,m} \\ C_{2,1} & C_{2,2\ldots} & C_{2,m-1} & C_{2,m} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ C_{m-1,1} & C_{m-1,2} & C_{m-1,m-1} & C_{m-1,m} \\ C_{m,1} & C_{m,2} & C_{m,m-1} & C_{m,m} \end{bmatrix} \tag{3}$$

so that the last line should not be m but $m - 1$.

$$\begin{bmatrix} C_{m,1} & C_{m,2\ldots} & C_{m,m-1} & C_{m,m} \\ C_{1,1} & C_{1,2\ldots} & C_{1,m-1} & C_{1,m} \\ C_{2,1} & C_{2,2\ldots} & C_{2,m-1} & C_{2,m} \\ & & & \\ \cdot & & & \\ & & & \\ C_{m-1} & C_{m-1,2\ldots} & C_{m-1,m-1} & C_{m-1,m} \end{bmatrix} \tag{4}$$

and thus the last column should not be X_m but X_{m-1}

$$\begin{bmatrix} C_{m,m} & C_{m,1} & C_{m,2\ldots} & C_{m,m-1} \\ C_{1,m} & C_{1,1} & C_{1,2\ldots} & C_{1,m-1} \\ C_{2,m} & C_{2,1} & C_{2,2\ldots} & C_{2,m-1} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ C_{m-1,m} & C_{m-1,1} & C_{m-1,2\ldots} & C_{m-1,m-1} \end{bmatrix} \tag{5}$$

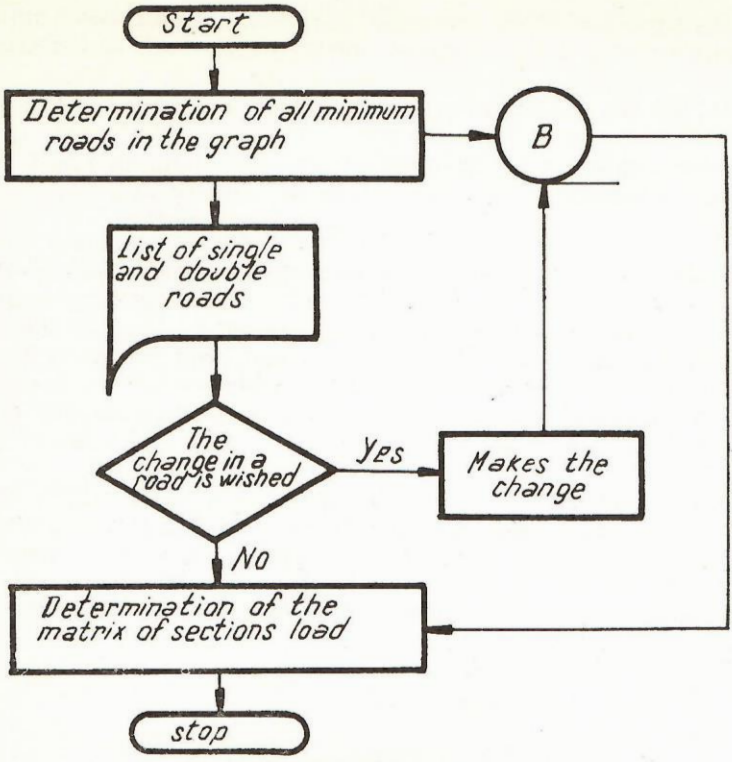


Figure 1. The system logical scheme of the model for the distribution of the transport demand on the minimum roads.

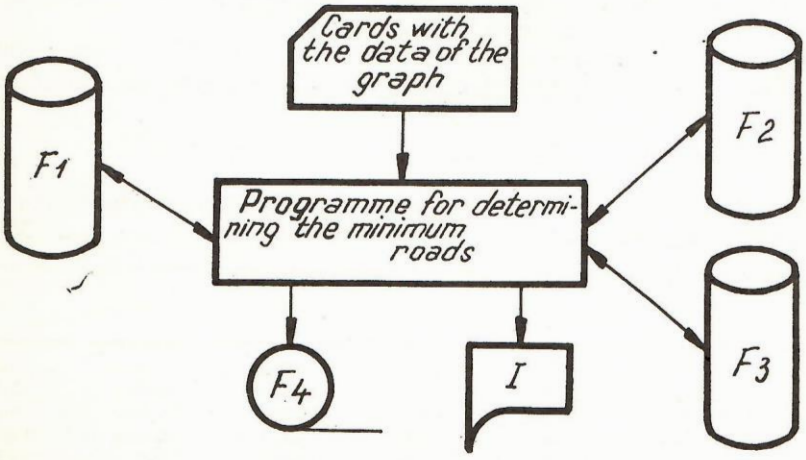


Figure 2

With this matrix C thus arranged (5) the minimum roads from all peaks of the graph to peak X_{m-1} are determined, by repeating the Bellman-Kalaba algorithm.

Finally, all the minimum roads among all the peaks of the graph are obtained.

Thus, for a graph with m peaks, if $(m-1)$ minimum roads at a fixed peak are calculated each time, they will be determined in total.

$$m\ (m-1) \text{ minimum roads.}$$

b. For determining all the minimum roads in a graph with 33 peaks, the programme which follows identically the algorithm operating with two size matrices, holds in the storage about 50 k bits out of the 128 k available of the IBM 360/40 computer (where k=1024) and lasts for about 15 minutes. Taking into consideration the maximum possible size of the computer, the fact is determined that operating with matrices of two sizes, the problem of the determination of roads may be raised in this way and solved for a graph with maximum 80—90 peaks.

In the event that this problem is solved for a graph with a number of peaks bigger than the above-mentioned one, we change the matrix into a number of lines, operating in the working store only with a single line at a given moment.

This programme is operating with four files (fig. 2), namely:

F_1 — the file of initial data

F_2 — the file of the re-arranged matrix

F_3 — the file of minimum roads

F_4 — the file of results

The performances of this programme are:

— the maximum size of the graph: 900 peaks;

— the calculation time of all minimum roads for a graph with 33 peaks: 38 minutes.

The calculation of all roads at a peak has required 70 seconds, during which seven data, working store-magnetic disk transfer and a working store-magnetic tape transfer have taken place.

In conclusion, they operate with the first variant, for a graph with up to 90 peaks and for a graph with $90 \leqslant m \leqslant 900$ with variants using the four files.

3.2. Application of the Model

The initial data of the 1st phase were the lengths in straight line among all zones l^2_{ij} which formed the matrix C of elements C_{ij} $(i, j = 1, ..., 33)$.

	1	2	3	4	5	6
1	0.0	2.700	1000.000	4.050	1000.000	3.675
2	2.700	0.0	1.500	2.250	1000.000	3.450
3	1000.000	1.500	0.000	2.700	1000.000	4.500
4	4.050	2.250	2.700	0.0	2.400	2.175
5	1000.000	1000.000	1000.000	2.400	0.0	2.250
6	3.675	3.450	4.500	2.175	2.250	0.0

The results of phase I were the minimum roads and a list of cases with several minimum roads between two peaks.

PL	2	SO	1	VAL	2.70	TOTP	2	2	1	
PL	3	SO	1	VAL	4.20	TOTP	3	3	2	1
PL	4	SO	1	VAL	4.05	TOTP	2	4	1	
PL	5	SO	1	VAL	5.92	TOTP	3	5	6	1
PL	6	SO	1	VAL	3.67	TOTP	2	6	1	

The initial data of phase II were
— the minimum roads;
— the matrix of the transport demand n_{ij}.

	1	2	3	4	5	6...
1	11 348	3 501	970	560	455	513
2	1 215	2 533	4 991	1 134	200	448
3	1 427	1 843	2 907	3 024	858	329
4	1 157	1 817	1 990	8 231	3 741	968
5	1 103	1 665	1 814	4 883	14 072	1 096
6	2 919	3 032	1 011	1 117	1 103	5 764

The results of phase II were
— the matrix of the sections load

	1	2	3	4	5	6...
1	0	7 028	0	1 421	0	1 496
2	6 522	0	10 048	3 895	0	1 510
3	0	12 660	0	5 912	0	1 412
4	1 678	4 749	9 136	0	7 024	1 537
5	0	0	0	11 245	0	2 199
6	4 807	5 383	5 094	3 921	1 558	0

— the matrix of the values of minimum roads

	1	2	3	4	5	6...
1	0.0	2.700	4.200	4.550	5.925	3.675
2	2.700	0.0	1.500	2.250	4.650	3.450
3	4.200	1.500	0.0	2.700	5.100	4.500
4	4.050	2.250	2.700	0.0	2.400	2.175
5	5.925	4.650	5.100	2.400	0.0	2.250
6	3.675	3.450	4.500	2.175	2.250	0.0

The programme of minimum roads has shown the existence of several minimum roads between two zones. 28 such cases have been revealed:
26 roads — with the characteristic of two equal minimum roads;

2 roads — with the characteristic of three equal minimum roads. The values n_{ij} corresponding to these minimum roads being very low, band B was not changed so that phase III was no longer necessary.

Figure 3 presents a section of the structure of the transport demand of the active population in 1970, obtained on the basis of the application of the

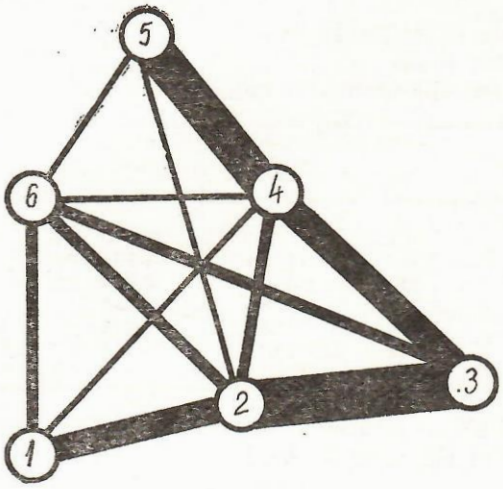


Figure 3

mathematical model for the distribution of the transport demand on minimum roads, in which the size of the transport demand is proportional to the width of the connection lines among the zones of the town of Bucharest.

In conclusion, the particular load of some sections puts before town-planners the problem of finding new means of public transport which should ensure a civilized service for passengers.

The building of apartment houses in the vecinity of the big factories, which should be distributed with priority to the employees in the area will be of real advantage with a view to reducing the volume of transport in the future. Taking into consideration the fact that in the past, dwellings were not distributed at all according to the employees' proximity to their places of work, it is advisable to find solutions — eventually at the level of the Bucharest People's Council in view of the stimulation of dwelling exchanges.

Another requirement is the establishment of more reasonable routes for some means of public transport (bus, troleybus), the ensurance of wider and well-built roads as well as their disaffection for other means of transport (which should be directed on parallel roads). These are only some of the many conclusions drawn from the study of public transport flows and which will undoubtedly find their complete solution in the next few years.

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MODELE MATHEMATIQUE DANS LE TRANSPORT URBAIN DES PERSONNES

Vu l'accroissement actuel de la population dans les grandes villes, il est nécessaire d'assurer des conditions optimes pour le transport en commun des habitants. C'est ce qui crée une série de difficultés dues, tant à une demande toujours plus grande de transport des habitants entre les diverses zones de la ville, qu'à l'augmentation du nombre des autos.

Dans cet article l'auteur présente l'application d'un modèle mathématique pour déterminer la demande de transport au niveau des années 1975—1980, dans la ville de Bucarest.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ, ПРИМЕНЯЕМАЯ В ГОРОДСКОМ ПАССАЖИРСКОМ ТРАНСПОРТЕ

На нынешнем этапе рост населения в больших городах требует и обеспечения оптимальных условий для коммунального транспорта населения. Это создает ряд трудностей как в результате все большей потребности в перевозках населения между различными зонами города, так и вследствие роста парка автомашин.

В настоящей статье автор представляет применение математической модели для определения потребности в перевозках на уровне 1975—1980 гг. в городе Бухаресте.