M. STOICA
The Academy of Economic Studies
The Department of Economic Cybernetics

Some Applications of the Markov Chains for Optimizing the Work Processes

Modern mathematics has still unknown possibilities to reflect the reality however complex this would be. In this line, the theory of Markov chains and processes can be useful in attempts of modelling human activities.

In work processes currently met with in practice (mechanical processing, parts making, device adjusting, intervention) the man — machine — environment system assumes at different times 1, 2, ... , t, $t+1$,... a number of possible states $S_1^{(t)}$, $S_2^{(t)}$, ..., $S_n^{(t)}$, which can be plotted by means of a transition graph (Fig. 1). Taking into account some practical observations, we can assume the hypothesis that the transition from state $S_i^{(t)}$ to state $S_j^{(t)}$ depends solely on these two states. Consider $p_i^{(t)}$ to be the probability for the system to assume at time t state $S_i^{(t)}$ and $p_{ij}^{(t)}$ the probability for the system to pass from state $S_i^{(t)}$ to state $S_j^{(t)}$. The work process can be thought out as a Markov process containing relationship:

$$p_j^{(t+1)} = \sum_{i=0}^{n} p_i(t)\, p_{ij}^{(t)} \qquad j = 1,\ 2,\ ...,\ n \tag{1}$$

Consider p(t) to be the column vector of the probabilities for the vector to assume a certain state at time t. Consider P(t) to be the square matrix, made up by elements $p_{ij}^{(t)}$.

Then Chapman-Kolmogorov's relationship becomes:

$$p(t+1) = p(t)P(t) \tag{2}$$

The transition matrix $P(t)$ is a stochastic matrix, since its elements meet relationships:

$$p_{ij}^{(t)} \geqslant 0 \qquad i,\ j = 1,\ 2,\ ...,\ n \tag{3}$$

$$\sum_{j=1}^{n} p_{ij}^{(t)} \geqslant = 1 \qquad i = 1,\ 2,\ ...,\ n \tag{4}$$

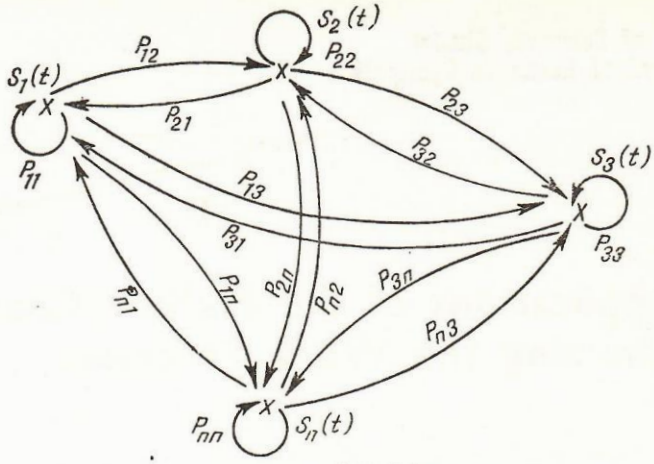


Fig. 1 A work process plotted by means of a transition graph

The components of the state vector meet relationships:

$$p_j^{(t)} \geqslant 0 \qquad j = 1, 2, \ldots, n \tag{5}$$

$$\sum_{j=1}^{n} p_j^{(t)} = 1 \tag{6}$$

The original state of the work process is yielded by vector $p(0)$ which is assumed to be known. If the transition probabilities $p_{ij}^{(t)}$ change at different times 1, 2,..., t, we say that the process is non-stationary, whereas if they are constant in time p_{ij} the process is stationary.

For a stationary process relationships (1) and (2) become:

$$p_j^{(t)} = \sum_{i=0}^{n} p_i^{(t)} P_{ij} \qquad j = 1, 2, \ldots, n \tag{7}$$

$$p(t+1) = p(t)\mathbf{P} \tag{8}$$

We assume the hypothesis that the states of the work process make up an indecomposable system. Consequently, for two events whatever of system S_j and S_k, there is a number of events distinct from the previous ones, so that:

$$p_{j j_1} p_{j_1 j_2} \cdots p_{j_n k} \neq 0$$

By applying relationship (8) on assuming $t = 1, 2, \ldots, m$, we get:

$$p(1) = p(0)P;\ p(2) = p(1)P = p(0)P^2,\ \ldots,\ p(m) = p(0)P^m \tag{9}$$

The behaviour of these relationships for $m \to \infty$, termed the ergodic problem, is of particular interest. From this point of view there are three types of work processes which are considered to be Markov processes:

1) *Complete ergodic processes* are termed "stable probability processes" and "permanent states". In this case we have:

$$\lim_{m\to\infty}[p(m)] = \lim_{m\to\infty}[p(0)\,P^m] = p_0 \lim_{m\to\infty} P^m = p_0\widetilde{P} = p \tag{10}$$

Under a fully ergodic process, matrix $\widetilde{P}$ has identical lines. Since vector p_0 is a stochastic vector, it follows that p is one of the identical lines of matrix $\widetilde{P}$. The final state of the system, after an infinite time, does no longer depend on the original state vector p_0. Hence, irrespective of the state where the work process starts, the final state after an infinite time will be the same.

2) *Incomplete ergodic processes.* In this case matrix $\widetilde{P}$ has not identical lines. Consequently, the final state depends on the original state of the work process.

3) *Non-ergodic processes.* For such processes there is no $\lim_{m\to\infty} P^m$

We are particularly interested in complete ergodic processes. In order to compute the elements of matrix $\widetilde{P}$ and of limit vector p we can apply a simple algebraic method. Taking into account that $\lim_{m\to\infty} p(m) = p$, system (8) becomes:

$$p = pP \tag{11}$$

Writing E for the unit matrix we get D = dynamic matrix (the difference between matrix P and matrix E),

$$pP - p = p(P - E) = 0 \tag{12}$$

Taking into account D, relationship (12) becomes:

$$pD = 0 \tag{13}$$

Thus, we find a system of n homogeneous equations that admits uncommon solutions and contains an indetermination. It is known that such a solution assumes the form:

$$p_i = \lambda\Delta_i \tag{14}$$

where
Δ_i is the characteristic determinant with $n - 1$ lines and $n - 1$ columns whereas λ is a parameter to be found.

If we attach equation:

$$p_1 + p_2 + \ldots + p_n = 1 \tag{15}$$

to the system of equations (13), we get a system of equations in which parameter λ is to be found. This system has always a sole solution. In order to solve this system of equations, we may use either classical procedures or simulation procedures through the Monte Carlo method.

2. APPLYING THE MONTE CARLO SIMULATION IN ORDER TO FIND STATE PROBABILITIES

For a comparative illustration of the classical and Monte Carlo procedures, a numerical example for a case reduced to seven states is given. The

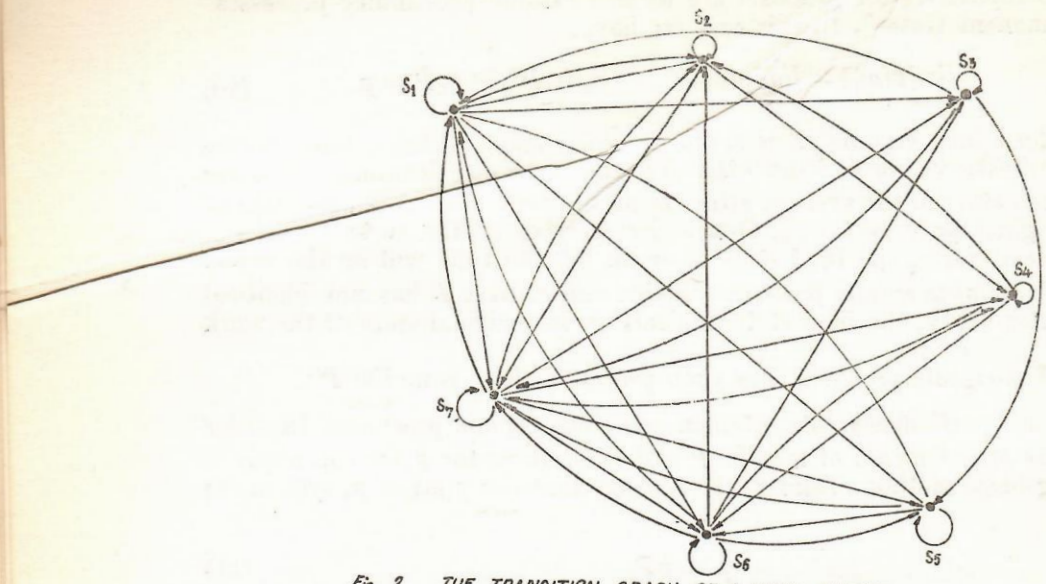


Fig. 2 THE TRANSITION GRAPH OF A WORK PROCESS

transition matrix is pictured in Table I, whereas the transition graph is illustrated in Fig. 2. The seven states represent various positions of the worker during the work process. In some other circumstances these states may represent distinct levels of the work rates power consumption, etc. The dynamic matrix D is pictured in Table II. Leaving aside the equation yielded by the last line of matrix D and substituting it by a relationship of type (15), we get the following system of equations written in matrix form:

$$\begin{vmatrix} -0.95 & 0.10 & 0.10 & 0.10 & 0.10 & 0.10 & 0.45 \\ 0.30 & -0.90 & 0.00 & 0.40 & 0.00 & 0.00 & 0.20 \\ 0.10 & 0.00 & -0.40 & 0.00 & 0.20 & 0.00 & 0.10 \\ 0.20 & 0.20 & 0.00 & -0.70 & 0.00 & 0.20 & 0.20 \\ 0.10 & 0.00 & 0.00 & 0.00 & -0.40 & 0.20 & 0.10 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & -0.30 & 0.05 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{vmatrix} \begin{vmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{vmatrix} = \begin{vmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 1.00 \end{vmatrix}$$

Solving this system through current methods we get:

$$P = \begin{vmatrix} 0.210 \\ 0.072 \\ 0.110 \\ 0.104 \\ 0.146 \\ 0.250 \\ 0.108 \end{vmatrix}$$

In order to make use of the simulation method, the cumulated occurrences from state S_1 to state S_k ($k = 1,2,...,7$) have been computed on the strength of Table I. The results have been entered in Table III. Table IV contains the results obtained through 1,000 simulation cycles. 1,000 random numbers have been generated by means of subroutine RANDU and an asterisk has marked the state having the closest occurrence to the generated number. The absolute and relative occurrences of the seven states have finally been computed.

We notice that a new vector of the final state has been obtained:

$$p^* = \begin{vmatrix} 0.207 \\ 0.068 \\ 0.112 \\ 0.107 \\ 0.150 \\ 0.251 \\ 0.105 \end{vmatrix}$$

By comparing the components of vector p^* with the components of the strict vector p, we notice that there are no significant differences, which renders the method valid.

3. D.R.-TYPE WORK PROCESSES AND R.D.-TYPE WORK PROCESSES

During the work process there occur at certain times t some random events causing the transition of the system from state $S_i(t)$ to state $S'_j(t)$. According to Companys there are two types of processes:

— A decision-random event process (D.R.), where the worker's decision occurs before the event;

— A random event-decision process (R.D.), where the random event occurs before the worker's decision.

For a D.R. process we can draw up a graph $\Gamma_D(t)$ illustrating the transition of the system from state $S_i(t)$ to state $S'_j(t)$ when the worker has acted in agreement with decision D (Fig. 3). Decision D pertains to the set of the worker's possible decisions.

A cost $c_{ij}^{(t)}$ is associated to each arc (i, j) of this graph. At time t' ($t < t' < t + 1$) a random event occurs, bringing the system from state $S'_j(t)$ back to one of the states $S_k^{(t)}$ in agreement with a graph $\Gamma_H^{(t)}$ with a probability $p_{jk}^{(t)}$ and a cost $C_{jk}^{(t)}$.

The interval from time t to time $t + 1$ is termed "stage t." We write $\bar{f}(T - t,\ T)$ for the optimal mathematical expectation with respect to the set of stages $t, t + 1, \ldots, T - 1, T$, when the system assumes state $S_i^{(t)}$ at time t. On the strength of Bellman's optimality principle we write:

$$\bar{f}_i(T - t,\ T) = min\{c_{ij}^{(t)} + \Sigma p_{ij}^{(t)}[C_{jk}^{(t)} + \bar{f}_k(T - t + 1,\ T)]\}$$
$$S'_j{}^{(t)} \in \Gamma_D^{(t)} S_i^{(t)}; \qquad S_k^{(t)} \in \Gamma_H^{(t)} S'_j{}^{(t)} \tag{16}$$
$$i = 1, 2, \ldots, n; \qquad j = 1, 2, \ldots, T$$

In an R.D. process, the random event occurs before the worker's decision and the system passes from state $S'_j{}^{(t)}$ to state $S_k^{(t)}(S_k^{(t)} \in \Gamma_H^{(t)} S'_j{}^{(t)})$ with

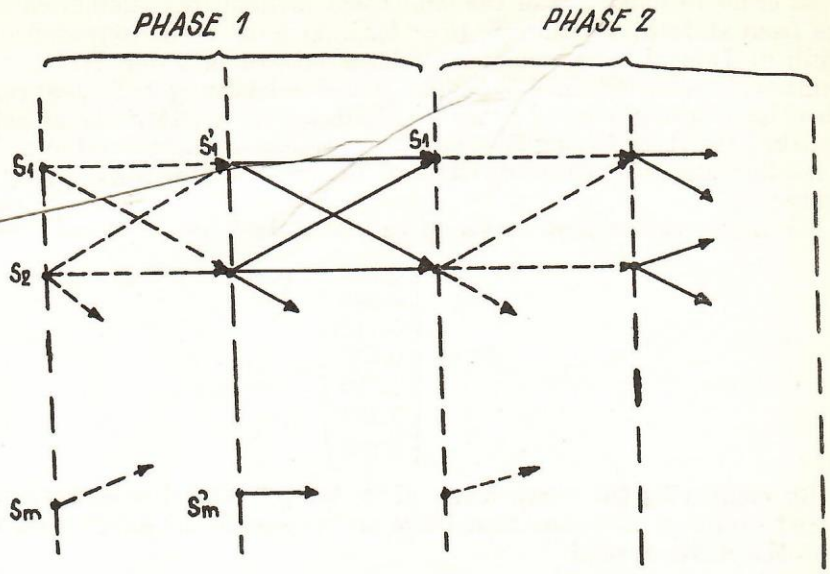


Fig. 3 : THE D.R. TYPE PROCESS (the $\Gamma_D(t)$ graph)

a probability $p_{jk}^{(t)}$ and a cost $c_{jk}^{*(t)}$ (Fig. 4). At time t' $(t < t' < t+1)$, by the worker's decision the system comes back to one of the states, $S_k'^{(t)}(S_k'^{(t)} \in \Gamma_D^{(t)} S_k^{(t)})$ while the cost is $C_{kh}^{*(t)}$. Writing $\bar{g}_j\ (T-t,\ T)$ the mathematical expectation (17) for the set of stages $t,\ t+1,\ \ldots,\ T-1,\ T$, when the system assumes state, $S_h'^{(t)}$ we get :

$$\bar{g}_j(T-t,T) = \sum \{p_{jk}^{(t)} C_{ik}^{*}(t) + min\,[C_k^{*}(t) + \bar{g}_h(T-t+1,T)]\} \qquad (17)$$

$$S_j^{(t)} \in \Gamma_H^{(t)} S_h'^{(t)}; \qquad S_h'^{(t)} \in \Gamma_D^{(t)} S_k^{(t)}$$

$$i = 1,\ 2,\ \ldots,\ n; \qquad t = 1,\ 2,\ \ldots,\ T$$

For (manual or automatic) computation it is more convenient to reduce any work process to the circumstance of first taking the decision and afterwards occurring the random event.
We do so because the number of decisions to be analyzed is far larger with R.D.-type processes than with D.R.-type processes. By simulating at a venture some sequences of decisions, this difference in the computation volume is no longer significant, because the number of cycles under the Monte Carlo method, which enables the same accuracy, grows up relatively very slightly when the number of analyzed variants substantially increases.

4. SIMULATING SOME D.R. PROCESSES THROUGH THE MONTE CARLO METHOD.

There are instances that no particular accuracy of the result is necessary, especially for current production needs. In such cases the Monte Carlo method

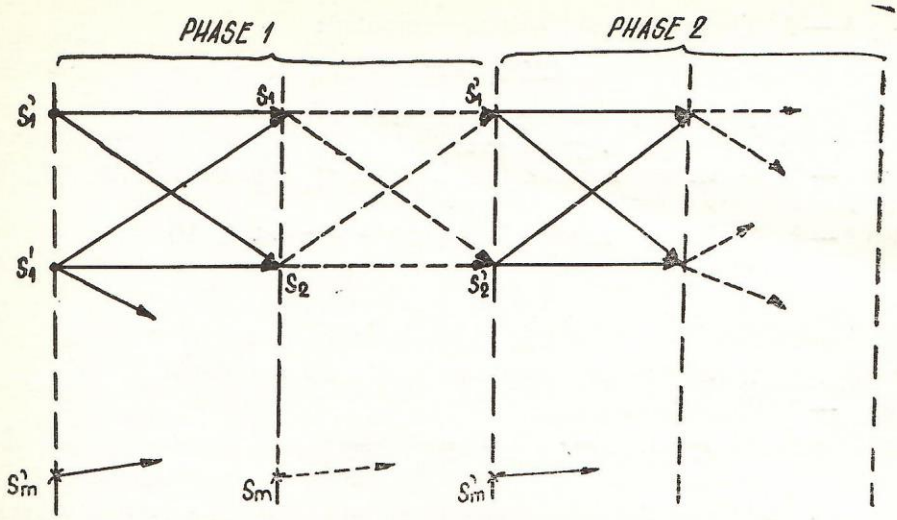


Fig. 4: THE R.D. TYPE PROCESS (the $\Gamma_H(d)$ graph)

is a great help. To describe the simulation process we shall use the following notations:

Consider S_{ν_0} to be the original state of the system. In agreement with the graph in Fig. 3, the worker can bring the system to one of the states S'_{h1}, S'_{h_2} ,..., S'_{h_m}. For instance, the performer decides the transition of the system into state S'_{j0}. This decision is written $D^{(j0)}_{\nu_0}$. The cost corresponding to this decision, at stage 1, is $c^0_{\nu_0 j_0}$. For state S'_{j0} there is the probability $p_{j_0\nu_1}$ for the system to go into state S_{ν_1} at a cost $C^0_{j_0\nu_1}$.

At the subsequent stage the system starts from state S_{ν_1}. The worker makes the decision $D^{(j_1)}_{\nu_1}$ of bring it from state S_{ν_1} to state S'_{j_1}; the corresponding cost is $c_{\nu_1 j_1}$.

From state S'_{j_1} the system goes into state S_{ν_2} (the probability for this is $p^1_{j_1\nu_1}$ and the cost is $C^1_{j_1\nu_2}$). In general from state S'_{ν_i} it is brought to state S_{j_i} at a cost $c^i_{\nu_i j_i}$, in agreement with decision $D^{(ji)}_{\nu_i}$. From state S_{j_i} it is brought to state $S'_{\nu_{i+1}}$ (with a probability $p^i_{j_i\nu_{i+1}}$ and a cost $C_{j_i\nu^i_{i+1}}$). The computation algorithm can in short be described as follows:

Step 0 — 1. A random number $\nu_0 \in \{1, 2, ..., n\}$ is generated, and state S_{ν_0} is considered.

Step 0 — 2. A random number $j_0 \in \{1, 2, ..., r_{\nu_0}\}$ is generated, and arc (S_{ν_0}, S_{j_s}) and the corresponding cost $c^0_{\nu_0 j_0}$ are considered.

Step 0 — 3. A random number $\mu_0 \in (0,1)$ is generated and the double inequality

$$\sum_{k=1}^{\nu_1 - 1} p^0_{j_0 k} < \mu_0 \leqslant \sum_{k=1}^{\nu_1} p^0_{j_0 k} \tag{18}$$

is tested. Thereupon arc (S'_{j_0}, S_{ν_1}) and the associated cost $C^0_{j_0\nu_1}$ are considered

Step 0 — 4. The cumulated cost is computed:

$$\mathrm{CUM}_1 = c^0_{\nu_0 j_0} + C^0_{j_0 \nu_1} \tag{19}$$

The algorithm is resumed from step 0—2.

For an i current stage we get:

Step i — 2. A random number $j_i \in \{1, 2, ..., r\}$ is generated. Arc (S_{ν_i}, S'_{j_i}) and the associated cost $c^i_{\nu_i j_i}$ are considered.

Step i — 3. A random number $\mu_i \in (0,1)$ is generated. If:

$$\sum_{k=1}^{\nu_{i-1}} p^i_{j_i k} < \mu_i \leqslant \sum_{k=1}^{\nu_i} p^i_{j_i k} \tag{20}$$

arc $(S_{j_i}, S_{\nu_{i+1}})$ and the associated cost $C^i_{j_i \nu_{i+1}}$ are considered.

Step i — 4. We compute:

$$\mathrm{CUM}_i = \mathrm{CUM}_{i-1} + \mathrm{C}^i_{\nu_i j_i} + C^i_{j_i \nu i+1} \tag{21}$$

The computation of the cycle comes to an end after a number of T stages. On this occasion, a simulation cycle has come to an end. The simulation cycles are L times repeated, so as to secure the due precision (with a certain probability). In practice, even when the convergence of the algorithm cannot be proved, simulation is useful because, assuming the number L to be 0.1 per cent of the number of variants and 100 at least, the results obtained enable the worker to choose certain sequences of decisions leading to values close to the maximum of function objective CUM.

5. SIMULATING SOME R.D. PROCESSES THROUGH THE MONTE CARLO METHOD

The system is originally in state S'_{ν_0}, and it can be brought to one of states S_{h1}, S_{h2}, ..., $S_{hs_{\nu_0}}$. The probability for going from state S'_{ν_0} into state S_{j_0} is $p^0_{\nu_0 j_0}$ and the associated cost is $c^0_{\nu_0 j_0}$. From state S_{j_0} the worker operates on the system in order to bring it to state S'_{ν_1}, in agreement with decision $D^{(\nu_i)}_{j_0}$ (the corresponding cost is $C^0_{j_0 \nu_1}$). In general, from state S'_{ν_i} it goes into state S_{j_i} with a probability $p^i_{\nu_i j_i}$, and the cost is $c^i_{\nu_i j_i}$ From state S_{j_i} it goes into state $S'_{\nu_{i+1}}$ and the associated cost is $C^i_{j_i \nu_{i+1}}$. The computation algorithm is:

Step 0 — 1. We generate $\nu_0 \in \{1, 2, ..., m\}$, and consider state S'_{ν_0}.

Step 0 — 2. A random number $\alpha_0 \in (0,1)$ is generated. If:

$$\sum_{k=1}^{j_0 - 1} p^0_{\nu 0 k} < \alpha_0 \leqslant \sum_{k=1}^{j_0} p^0_{\nu 0 k} \tag{22}$$

We consider arc (S'_{ν_0}, S_{j_0}) and the associated cost $C\nu^0_{0 j_0}$.

Step 0 — 3. We generate $\nu_1 \in \{1, 2, ..., s_{j_0}\}$ and consider arc (S_{j_0}, S'_{ν_1}) and cost $C^0_{j 0 \nu_1}$.

Step 0 — 4. We compute the cumulated cost according to relationship (19).

The same as in D.H. processes, the algorithm can be generalized for stage i.

The algorithm is resumed from step 0 — 2 until T stages are achieved.

D.R. and R.D. processes simulation does not require the use of the stationary regime hypothesis.

There is also the possibility of alternating the D.R. and R.D. processes, for instance at first stage a D.R. process, at second stage an R.D. process, at third stage an R.D. process, at fourth stage a D.R. process, a.s.o. Consequently $D_1R_1R_2D_2R_3D_3D_4R_4$, etc. Such a succession can be generated with the aid of some N random numbers by making use of the following rule:

D.R. is chosen if $N \in \left(0, \frac{1}{2}\right]$, and R.D. is chosen if $N \in \left(\frac{1}{2}, 1\right]$.

6. APPLYING MULTIPLE MARKOV CHAINS

The worker's decision dynamic process can be illustrated by means of a multiple Markov chain. For a state S_i, m_i possible strategies are considered. Let r be the rank of a strategy, that is $r \in \{1, 2, ..., m_i\}$. Consider m_i to be stochastic vectors of the form:

$$[p_i^{(r)}] = [p_{i1}^{(r)}, p_{i2}^{(r)}, ..., p_{in}^{(r)}] \tag{23}$$

We associate to vector $p_i^{(r)}$ a resulting vector $R_i^{(r)}$ (labour productivity, quality of work performed, profit, etc.) of the form:

$$[R_i^{(r)}] = [R_{i1}^{(r)}, R_{i2}^{(r)}, ..., R_{in}^{(r)}] \tag{24}$$

or a cost vector:

$$[\Gamma_i^{(r)}] = [\Gamma_{i1}^{(r)}, \Gamma_{i2}^{(r)}, ..., \Gamma_{in}^{(r)}] \tag{25}$$

The worker's decision consists in choosing n stochastic vectors $p_1^{(r_1)}$, $p_2^{(r_2)}$, ..., $p_n^{(r_n)}$. On the strength of the hypothesis that the system assumes state S_i and the worker has made use of the strategy of rank r, the mathematical expectation $\bar{q}_i^{(r)}$ of the result is:

$$\bar{q}_i^{(r)} = \sum_{j=1}^{n} p_{ij}^{(r)} R_{ij}^{(r)} \qquad i = 1, 2, ..., n \tag{26}$$

Consider $\bar{v}_i(T - t, T)$ to be the mathematical expectation of the total amount of income per t stages, from $T - t$ to T. We have:

$$\bar{v}_i(T - t, T) = max[\bar{q}_i^{(r)} + \sum_{j=1}^{n} p_{ij}^{(r)} v_j(T - t + 1, T)] \tag{27}$$

$$r = 1, 2, ..., m; \qquad t = 1, 2, ..., T$$

We can simplify our notations as follows:

$$\bar{v}_i(T, T) = \bar{v}_{0i} \qquad i = 1, 2, ..., n \tag{28}$$

$$\bar{v}_i(t) = \bar{v}_i(T - t, T) \qquad t = 0, 1, 2, ..., T \tag{29}$$

$$i = 1, 2, ..., n$$

where: $\bar{v}_i(T, T)$ *is the income for time* T *in state* i;

$\bar{v}_i(t)$ *is the income from time* t *up to the final time* T *(at the final time* there is state i).

In order to find the optimal strategy we compute by turns:

$$\bar{v}_i(1) = max[\bar{q}_i^{(r)} + \sum_{j=1}^{n} p_{ij}^{(r)} \bar{v}_{0j}]$$

$$\bar{v}_i(2) = max[\bar{q}_i^{(r)} + \sum_{j=1}^{n} p_{ij}^{(r)} \bar{v}_j(1)]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (30)$$

$$\bar{v}_i(T) = max[\bar{q}_i^{(r)} + \sum_{j=1}^{n} p_{ij}^{(r)} \bar{v}_j(T-1)]$$

$$t = 1, 2, \ldots, T \qquad r = 1, 2, \ldots, m$$

If the worker has become familiar with the work process, described by a complete ergodic Markov chain, a matrix of the form P with identical lines of elements $p_h (h = 1, 2, \ldots, N)$ is obtained; this matrix represents the long term state probabilities. Thus, we get a system of N equations with $N + 1$ unknowns [2]. In order to solve the system, one of the unknowns is assumed to be zero.

If the chain is not completely ergodic, the system of equations has $(2N)$ equations with $(2N)$ unknowns, and the matrix is singular. In order to solve the system we assume an unknown to be zero also in this case.

With such a mathematical model, an approach through systems of equations becomes prohibitive even if using the quickest electronic computers. A more elegant optimization method was offered by Howard by means of successive iterations; the calculus comes to an end when there is no more increase in the income [2].

The simulation method can also be used in order to achieve long-term strategies. To obtain an early solution we assume the state probabilities to be uniform, that is $p_i = \frac{1}{n}$ (for the n states). By simulating L times the process for T stages and choosing the function objective CUM with the most favourable strategy, we can compute the number of occurrences a_i of strategy S_i. We get:

$$p_i \cong \frac{a_i}{L} \qquad (31)$$

Generating further, the original state is no more obtained from a uniform distribution but from distribution:

$$\begin{Bmatrix} i \\ \frac{a_i}{L} \end{Bmatrix}$$

With a complete ergodic process, after repeating these computations a few times we achieve sequences approximately equal:

$$\frac{a_i}{L} \approx \frac{a_i^{\bullet}}{L^{\bullet}} \approx \frac{a_i^{\bullet\bullet}}{L^{\bullet\bullet}} \approx \ldots \tag{32}$$

With a non-ergodic process the variations will be significant.

For practical problems, the simulation method allows certain sub-optimal strategies which prove to be adequate when applied.

Table I

Elements of the transition matrix (Stochastic matrix)

States	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0.05	0.10	0.10	0.10	0.10	0.10	0.45
S_2	0.30	0.10	0	0.40	0	0	0.20
S_3	0.10	0	0.60	0	0.20	0	0.10
S_4	0.20	0.20	0	0.30	0	0.20	0,10
S_5	0.10	0	0	0	0.06	0.20	0.10
S_6	0.05	0.05	0.05	0.05	0.05	0.70	0.05
S_7	0.50	0.10	0.10	0.10	0.10	0.05	0.05

Table II

$$D = P - E$$

−0.95	0.10	0.10	0.10	0.10	0.10	0.45
0.30	−0.90	0	0.40	0	0	0.20
0.10	0	−0.40	0	0.20	0	0.10
0.20	0.20	0	−0.70	0	0.20	0.10
0.10	0	0	0	−0.40	0.20	0.10
0.05	0.05	0.05	0.05	0.05	−0.30	0.05
0.50	0.10	0.10	0.10	0.10	0.05	−0.95

Table III

Cumulated occurrences

States	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0.05	0.15	0.25	0.35	0.45	0.45	1.00
S_2	0.30	0.40	0.40	0.80	0.80	0.80	1.00
S_3	0.10	0.10	0.70	0.70	0.90	0.90	1.00
S_4	0.20	0.40	0.40	0.70	0.70	0.90	1.00
S_5	0.10	0.10	0.10	0.10	0.70	0.90	1.00
S_6	0.05	0.10	0.15	0.20	0.25	0.95	1.00
S_7	0.50	0.60	0.70	0.80	0.90	0.95	1.00

Table IV

Auxiliary table for performing the simulation calculus

States Simulation cycles	The random numbers	S_1	S_2	S_3	S_4	S_5	S_6	S_7
1	0.051	x						
2	0.125							x
3	0.942		x					
. . .	. . .			x				
999	0.271				x			
1000	0.737							
Total amount of circumstances		207	68	112	107	150	251	105
Relative occurences		0.207	0.068	0.112	0.107	0.150	0.251	0.105

REFERENCES

1. ONICESCU O. *Calculul probabilităţilor.* (Computation of Probabilities), Editura tehnică Bucharest, 1956.
2. KAUFMANN A.: *Metode şi modele ale cercetării operaţionale.* (Methods and Models of Operational Research); Editura ştiinţifică, Bucharest, 1967.
3. KEMENY J.G. and SNELL J.L.: *Finite Markov Chains*, Van Norstrand, New York, 1959.
4. BELLMAN R.E., DREYFUS S.E.: *Programarea dinamică aplicată.* (Applied Dynamic Programming), Editura tehnică, Bucharest, 1967.
5. STOICA M.: *Some Applications of the Markov Chains for Optimizing the Work Processes*; The IV-th Conference on Probability Theory, Braşov, România, September 12—18, 1971.
6. ZIDĂROIU C.: *Strategii optime pentru lanţuri cu legături complete reglabile.* (Optimal Strategies for Chains with Complete Adjustable Connections). Studii şi cercetări matematice, No. 1, Bucharest, 1972.

APPLICATIONS DES CHAINES MARKOV A L'OPTIMISATION DES PROCESSUS DU TRAVAIL

Au cas des processus du travail rencontrés dans les entreprises, le systéme homme-machine-milieu passe à des moments divers par une succession d'états possibles, ce qui peut être répresenté au moyen d'un graphe de passage. Une grande partie des processus du travail peut être décrite à l'aide des chaînes Markov, tandis q'une classe plus grande encore rentre dans les processus Markov. Trois cas furent analysés, fonction du comportement du processus du travail aprés un trés grand nombre de phases: les processus *complètement ergodiques* (ayant des propriétés de convergence et d'independance par rapport à l'état initial), les processus *incomplètement ergodiques* (ou la convergence est assurée, mais l'état initial n'est pas indifférent) et les processus *non-ergodiques* (dénués d'une propriété de convergence).

Pour le calcul des probabilités d'état, au cas des problémes de grandes dimensions, on peut employer les méthodes de simulation numérique.

De même, au cas de certains problémes de choix des stratégies optimes ou au moins sous-optimes du type "événement perturbateur-décision", "décision-événement perturbateur„ ou alternativement aléatoire, ce sont les procédés de simulation par la méthode Monte-Carlo qui s'avèrent être utiles, du fait que le nombre des cycles de simulation augmente d'une façon linéaire fonction des dimensions du probléme.

ПРИМЕНЕНИЕ ЦЕПЕЙ МАРКОВА ПРИ ОПТИМИЗАЦИИ ТРУДОВЫХ ПРОЦЕССОВ

В случае трудовых процессов, часто встречающихся на предприятиях, система «человек — машина — среда» проходит в различное время через ряд последовательных возможных состояний, что можно представить при помощи переходного графа. Значительная часть трудовых процессов может быть описана при помощи цепей Маркова, а более крупный класс входит в процессы Маркова. В зависимости от поведения трудового процесса после очень большого числа фаз были рассмотрены три случая: *полностью эргодические* процессы (со свойством сходимости и независимости от первоначального состояния), *неполностью эргодические* процессы (при которых обеспечена сходимость, но первоначальное состояние не является безразличным) и *неэргодические* процессы (не обладающие свойством сходимости).

Для расчета вероятностей состояния в задачах больших размеров можно применять методы численного симулирования.

Также и в задачах выбора оптимальных или по меньшей мере субоптимальных стратегий типа «нарушающее событие — решение», «решение — нарушающее событие» или случайной альтернативы методы симулирования по Монте-Карло подтверждают свою полезность, так как число циклов симулирования возрастает линейно в зависимости от размеров задачи.