ABBAS MIRAKHOR
Associate Professor of Economics
University of Alabama

Application of the Optimal Control Theory to Economic Analysis*)

"The base of the whole physiology must be the physiology of permanence".
Darlington

The application of methods which have been developed in optimal control theory to problems in economics has gained an immense popularity in the last few years. Thus far, however, this has been limited only to a narrow class of problems in economics. The main thrust of this development has been in the theory of economic policy. Within this range of problems there have been two distinct subareas where optimal control has been applied: (A) Theory of optimal growth, and (b) Theory of stablization policy. Based on the present progress made one can confidently say the future trends are, first, that there will be more refinements of models used in (A) and (B), and second, that the spectrum of economic problems to which optimal control can be applied will be broadened. In this paper first the developments in (A) and (B) will be very briefly reviewed and suggestions will be made as to further immediate contributions that the optimal control can make in these areas. Secondly, few examples of further problem areas in economics which should prove fertile and natural fields in application of optimal control will be given. Finally, some long range contributions which the optimal control can make to the economic science will be discussed. Before proceeding with the discussion of these ideas, it is helpful for the sake of clarity of discussion, that some of the terminology and basic concepts used in economics, and in this paper, be discussed.

The scope of the "standard" economics, that is the science practiced by the majority of those in the profession, deals with the allocation of given

---

* An earlier version of this paper was delivered at the Third Southeastern Symposium on Systems Theory, Atlanta, Georgia, April 5—6, 1971.

means toward optimal satisfaction of given ends. For the most part, economics uses "logical-deductive" methods and derives generalizations from fundamental axioms and assumptions. The theoretical rules, concepts, and models in economics are based on a few basic premises which are assumed to be self evident by simple introspection. Summarily, these are that man is rational, he prefers more of a commodity to less, he pursues the greatest economic gain with minimum effort. It is also assumed that the commodities consumed by individuals possess a common characteristic called utility, i.e., they are capable of giving satisfaction, and that this utility, although it increases with additional consumption, does so at a decreasing rate. The economists have found it convenient to divide their science into two distinct areas: (1) macroeconomics, which deals with the economy or its major components as a whole, and (2) microeconomics, which deals with the economic behavior of the most elementary units of the economy such as consumer or producer. Further, both these areas are broken down into (a) descriptive macro or microeconomics in which the economist simply describes the behavior of the components under consideration, and (b) prescriptive macro or microeconomics in which the economist gives prescription for the economic behavior of the components under consideration. For instance, prescription given for the purpose of public policy (referred to as normative macroeconomics) or prescription given to the firms or individual consumers (referred to as normative microeconomics) are examples of prescriptive economics. Generally, the economists have been preoccupied with the problems of macroeconomics. They have been interested in microeconomics only insofar as it provides a foundation for descriptive and prescriptive macroeconomics. The methods of optimal control have been applied in the areas of normative macro and microeconomics [the selected references attached present major works in these areas]. It will, however, be pointed out in the last part of the paper that optimal control is potentially capable of serving a very useful purpose in the areas of descriptive, as well as prescriptive, economics.

I. **Application to the Theory of Economic Policy** [41]. In general, the economic policy is concerned with the analysis and choice in situation related to a set of alternative feasible governmental action (control variable). Traditionally, the theory of economic policy has been divided into two different theories: (A) theory of optimal growth in which the basic decision process is an optimal choice among alternative feasible time paths in transforming an economy from a given initial state to a desired final state at the end of a specified planning horizon. In effect one is interested in choosing a set of optimal control variables which determine the time paths of state variables, and (B) the theory of stabilization policy in which the role of government is solely that of dampening fluctuations in the economy and restoring steady-state behavior in the growth path of national income. Until very recently these two were dealt with separately in the economic literature and have distinct histories of development. Although it is not the intention here to review the immense body of literature in these areas, it is helpful to mention very briefly these developments so that the problem areas may be distinguished.

A. *Application to the Theory of Optimal Growth* [2, 63, 69]. In general, economists have assumed that since the ultimate aim of every consumer is to maximize the utility derived from consumption, it follows that an optimal

growth path of the economy should be concerned with maximization of a given function (or functional) which is composed of the utilities of all members in the society. Such a function is called the social welfare function· It is further argued that to have the commodities which consumers can purchase a society must have the capital goods required to produce them. To have capital goods a society must in turn save a portion of its present income stream.

Beginning in the early 1960's a whole class of published research began to appear in the area of application of control theory to determination of optimal growth path of national income. All these studies have been classified as Ramsey economics [23], after Frank Plumpton Ramsey, a Cambridge logician and philosopher. Ramsey's greatest contribution to economic theory, according to many economists, is his 1928 article [64] in which he dealt with the question of how much should a society optimally save [57]. The optimal rate of saving was to be chosen so as to maximize a given functional, i.e., an integral of a time stream of consumption-generated utility. He formulated and solved the problem as a variational problem using the classical Euler-Lagrange method of solution. Ramsey's approach mathematically and economically had many inadequacies (for detailed discussion consult 23]), one of which was the one generally associated with classical variational problem, that is, it cannot handle all classes of problems, e.g., when the restriction is imposed that the control variable should belong to a closed set. For these inadequacies, Ramsey's paper did not receive much attention until very recently. With the development of Bellman's dynamic programming and Pontryagin's maximum principle, Ramsey's paper was resurrected since now with these powerful techniques of optimal control, Ramsey's problem could be reformulated and solved [69]. Much of the effort thus far has been in theoretical optimal growth problems and these attempts have yet produced any significant work of operational usefulness.

B. *Application to Stabilization Policy* [67, 41]. Based on the assumptions of economic theory stated above, the early economists thought that since everyone in the society is rational and is in pursuit of his owns self-interest that the best policy for the government is to keep out of the way of workings of the economic system. That quided by a motive of self interest while pursuing their economic goals, individuals at the same time are collectively achieving the best results for the society as a whole. Thus it was argued that the economic system should be allowed to operate with a minimum of governmental interference, i.e., the concept of Laissez-faire capitalism. The Laissez-faire capitalism under prevailing assumptions represented a self-regulating system in which the invisible hand of competition (control variable) would ensure equilibrium within the system. The realities, however, proved different. The capitalistic economies operating on the basis of Laissez-faire concept were plagued with chronic and at times violent fluctuations in income and employment, with booms and depressions, with recessions and recoveries. The Laissez-faire system proved to be much like the system controlling the temperature in the human body which is a self-regulating and self-controlling system. It works beautifully but only within given limits. Exposed to too great a heat input, the body produces higher metabolism which produces greater heat, which in turn creates positive feedback leading to generation of more heat

and eventual destruction of the system. The same results follow when a body is exposed to too low a heat input. J.M. Keynes (and Marx before him) saw precisely this sort of behavior in a capitalistic economy operating as a self-regulating system. Keynesian economics argued that the capitalistic economy cannot guarantee a happy and stable situation in which everyone is employed—that such an economy is constantly walking a tight rope between inflation and recession, between high degrees of unemployment and hyperemployment. Thus, argued the Keynesian economics, the government economic policy (control variable) must operate in such a way so as to guarantee stability in the system. Although the Keynesian prescriptions were in effect calling for the analysis of the economic system as a control system with government economic policies as the control, these ideas were not put in the precise terminology of control engineering until Arnold Tutsin, an English electrical engineer, showed in 1955 [77] that in its simplest form the Keynesian system is analogous to a partially self-exciting generator and that many variations of Keynesian theory can be formulated as direct analogs of electrical systems. Tutsin then suggested the use of Nyquist diagrams and Laplace transforms in determination of proper amount of feedback to be used in the control of such systems. Tutsin's work as well as those of A.W. Phillips [59, 60, 61] generated an intense interest in application of control system theory to problem of stability of economic systems. Thus far a major attempt in this area has been in application of control theory to economic systems which are linear or linearized in nature. These works have had limited actual application since more often than not the nonlinearities in economic system are of greater magnitude than can be handled by attempts at linearization. Under these circumstances a predicted stability in a linearized system proves to be unstable when disturbances, no matter how moderate, appear in the system.

What is of much more fundamental importance to most stabilization models thus far set up in economics is the fact that recent works in control system theory have emphasized that the stability alone does not guarantee a suitable and optimum design, thus requiring that an admissible control should in some way have an optimizing characteristic. With few exceptions [41, 67, 65], major works in economic system stability analysis suffer from a lack of some kind of optimality characteristics in control system design.

In general, control models developed in the area of theory of economic policy suffer from a lack of operational feasibility. The works done so far are inadequate in that they cannot easily be applied to actual problems of policy, since the major portion of them are based on continuous-deterministic models. The continuous nature of formulations become extremely troublesome when one attempts to apply these models. Some of the sources of difficulties are that, first, all economic data are gathered in form of discrete time series and one can only approximate the continuous rate of change in these data, e.g., from such data as first differences in time series one would have to estimate first derivatives. Second, major, government policies, i.e., the control variables designed to influence the performance of the state variables (consumer spending and business investment) are generally made at discrete rather than continuous times. The governmental budget (fiscal policy) is formulated annually. Decisions regarding formulation and operations of components of monetary policy such as changes in reserve ration or changes in open market

operations are made at discrete time intervals. The deterministic optimal growth or stabilization models are inadequate in dealing with problems of actual economy since they are non-stochastic in design. The economy is a dynamic system in which the structural relationships tend to change with time. One cannot assume the structure existing before the utilization of the optimal control variable will remain the same after the control has been exerted. For instance, let us assume that in a given economy the government is to use the amount of investment expenditure it has available as a control variable to bring about a set of desired changes in a section of the economy. A given set of structural equations describing the existing technology, i.e., production functions in various industries in the area may change after the manipulation of the control variable. It is therefore necessary that equations describing the system be stochastic and that they be estimated based on a sample of the observed values of the variables involved.

The immediate challenge in the area of economic policy seems to be the design of sequential-stochastic optimal control models which would permit the choice of an optimal and stable growth path of national income.

II. **Other Areas of Application.** There are many problem areas in economics which have not yet been exposed to methods of optimal control. One such area is the general operations of organizations, industrial or administrative. Control theory has to a great extent been applied to physical operations of business firm in which the state and control variables have been fairly easily defined. Examples of these applications are the theory of growth of the firm and the theory of optimal inventory control [9, 10, 11, 12, 68].

Many complex organizations are set up in form of hierarchical structures composed of many subsystems. Recently, there has been increasing efforts to view the organization as a self-motivating and self-adapting system with the management as the control unit. In terms of system theory the usual subsystems within an organization are (a) a memory or information subsystem, (b) a decision making subsystem, (c) an information processing subsystem, (d) a sensor subsystem which is generally designed to measure changes within the organization, and finally, (e) a control unit which usually provides feedback control. In large organization where the decision making tasks are specialized and there are "sub-managers," the subsystems become a series of sequentially interated processes in which the output of one process becomes the input of the next. Considered as feedback control system the manager's function is that of measuring the deviation between the actual- versus the expected output and depending upon the magnitude of this deviation he either retains or changes the operation of the system. In this framework the expected output is pre-specified, thus requiring considerable planning as to resource allocation and operational capacity of the resources. It seems rather a natural extension of this framework that one would expect the optimal control theory to contribute significantly to this area of research. Many areas of managerial functions can potentially utilize the methods of optimal control. One such area, for example, is the decision on advertising policies of the firm in terms of media selection and the magnitude of adevertising outlays. The sales vector of a given firm can for example serve as the state vector and its advertising expenditure as the control variable. Areas such as accounting, budgeting, information, as well as organizational struc-

ture of the firm itself, appear to be potential fields open to utilization of methods of optimal control.

A further extension of the above ideas can be made to the field of agricultural economics policy, in which from both a macro point of view, e.g., agricultural price supports programs, or from a micro point of view, e.g., farm management, control theory can effectively be used. For instance, for many years the price has been used as a control variable to bring about an equilibrium between the supply of and the demand for many agricultural commodities [20].

In a different setting, for many years local governments have attempted to attract industries to their areas by providing favorable tax conditions for these industries. This problem can be considered from an optimal control standpoint in which the local property taxes are the control variables. Furthermore, many problems of urban centers, such as mass transit, industrial zoning, urban plight, pollution, water resource requirements and management can all be designed as problems capable of analysis and solution by optimal control methods. Many problems in developing countries such as mass unemployment or problems of economic planning, regional allocation of public expenditure, can be treated as optimal control problems. [56, 73]

III. **The Long Range Perspective.** At the beginning of this paper we gave a definition of "standard" economics as the science of determining optimum allocation of scarce means. Thus any problem can be reduced, according to the standard economics, to allocating given resources among given ends by given rules. In this sense the economic system becomes a closed system which involves given means as conservation principle and optimal satisfaction as maximizing rule. Such a system is a "mechanical analoque." In this sense optimal control can further add to the arsenal of already existing maximizing rules in which case its main contribution will be in prescriptive economics. However, optimal control can make a far greater contribution. The standard economics view of economic system, many economists believe [42], is too narrow. The standard economics has treated the economic system for the most part as a closed system existing in a void. The reason for his has generally been the aspiration of economists to, for the last century and a half, promote economics to the status of physical sciences. This has led economists to abstract from non-observable, non-quantifyable factors no matter how importantly they have affected the economic system.

To apply optimal control to the problems of economics is to have a system-theoretic point of view. A system has been defined as a whole which functions as a whole by virtue of the interdependence of its parts. To be effective, therefore, system-theoretic view requires a fairly complete knowledge of the way in which the components of the system are interrelated. Economic system, if at all possible to analytically isolate it, cannot be assumed to exist in a void. It is an open system surrounded by psychological, social, institutional, political, biological, and physical environments. It is exposed to influences exerted from these systems and it influences them in turn. From this flows streams of events that make up the laws of behavior under which the components of the system, "economic men," must act, and to which they adapt. The economic system is adaptive in a sense that its components will develop mechanisms to cope with their environment and according to which

they regulate their own behavior. Furthermore, these components have the capacity to restructure and remodel their aims and goals according to the dynamical changes which continuously occur in the influences received from the other systems in their environment. The usual treatment in optimal growth theory has been to seek the optimal trajectory of the system without regards to dynamic changes occurring in its components. This system is only dynamic insofar as it moves through time.

To clarify what has been said above let's consider an example. No economist could have predicted the phenomenal economic recovery and growth of the Japanese society [42] since this prediction would have to rely on the usual economic aggregates used in growth theories which tend to abstract from the institutions, socio-political and cultural character of the Japanese society which shape and influence the Japanese economic system. This is not to suggest that optimal control cannot be applied to economic problems, on the contrary, since the economic existence is a social existence and a social existence is a controlled existence, economics will prove a fertile field for application of optimal control. What is needed is a fairly accurate description of the components, and their behavior, of the system to which the methods of optimal control are intended to be applied. Thus far very little is known about the actual behavior of the basic components of the economic system. The fundamental unit of an economic system is the consumer, yet economics knows little about his behavior. A whole edifice of highly sophisticated system of deductive thought has been built based on few premises that at best are "first approximations to a complicated reality". Yet there has been very little experimentation or research on the validity of the basic assumptions of micro economics theory regarding the consumer behavior. The advertising agencies perhaps have more practical knowledge of the consumer behavior than an average economist, [sitting in front of your television set viewing a commercial, you may be sure that you are a state variable in some advertising agency's crude control model].

At this juncture these questions may seem academic, but unfortunately they are not. One need only to consider governmental policy of the last three years in their attempt to combat inflationary spiral to note the seriousness of the shortcomings of standard economics. The government policy, being used as a control variable and a regulative mechanism, has been slow and extremely tardy in bringing about necessary changes. Economists blame this on political, institutional and social factors, not admitting that these factors are important to the behavior of economic system which is an open dynamic system in which processes are always changing. Individuals within the economic system do not subscribe to the rules derived from the textbook behavior of a static economic system moving through time. Their economic problem is not only how to allocate given means optimally to given ends but also how they can improve the quality of their means and/or how to increase their share of social wealth. It is through these behaviors that new economic relations, new economic means and new economic ends are continuously created.

At present these questions are not being explored through utilization of a system-theoretic point of view. But, one can expect that with the aid of the seductive potential of optimal control theory the economists can look at their optimal economic control model and see that these models are of

little use unless the components and their relationships are well explored, structured, and defined. The design of an optimal economic control model which would be truely dynamical and which would incorporate key factors in the society affecting the behavior of the economic system is an extremely formidable task and therein lies the greatest challenge to the experts in the field of optimal control, economics, political science, sociology, and psychology.

REFERENCES

The following is a list of references of work in application of control and system theory to the field of economics.

1. ARA KENJIRO: *Capital Theory and Economic Growth.* Economic Journal, No. 68, September 1958, pp. 511—527.
2. ARROW K.J.: *Applications of Control Theory to Economic Growth.* Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford, California, Technical Report No. 2, July 14, 1967.
3. ARROW K.J. and DEBREU G.: *Existence of an Equilibrium for a Competitive Economy.* Econometrica, Volume 22, pp. 265—290, 1964.
4. ARROW K.J.: *Optimal Capital Policy, the Cost of Capital, and Myopic Decision Rules.* Ann. Inst. Stat. Math., Volume 16, pp. 21—30, 1964.
5. ARROW K. J., KARLIN S. and SCARF H.: *Studies in the Mathematical Theory of Inventory and Production.* Stanford, Stanford University Press, 1958.
6. ARROW K. J., HARRIS T. and MARSCHAK J.: *Optimal Inventory Policy.* Econometrica, Volume 19, No. 3, 1951, pp. 250—272, Erratum: 20, 1, 133 (1952).
7. *ALLEN R.G.D.: Mathematical Analysis for Economists. London, MacMillan, 1949.*
8. ALLEN R.G.D.: *The Engineer's Approach to Economic Models.* Economica, pp. 158—168, 1955.
9. BECKMANN M.J.: *Dynamic Programming of Economic Decisions.* Springer-Verlag, Inc., New York, 1968.
10. BECKMANN M.J.: *Dynamic Programming and Inventory Control.* Operations Research Quarterly, 15, 4, pp. 389—400, 1964.
11. BELLMAN R.E.: *An Application of Dynamic Programming to Location-Allocation Problems.* SIAM Review 7, 1, pp. 126—128, 1965.
12. BELLMAN R.E.: *Dynamic Programming and Inventory Control.* Operations Research Quarterly 15, 4, pp. 221—236, 1967.
13. BERTALANFFY: *General System Theory: A New Approach to Unity of Science.* Human Biology. Volume *23, pp. 303—361, 1951.*
14. BOULDING K.E.: *General Systems Theory — The Skeleton of Science.* Management Science, Volume 2, No. 3, 1956.
15. BREDEHOEFT J.D. and YOUNG A.: *The Temporal Allocation of Ground Water — A Simulation Approach Water Resources Research,* Volume 6, No. 1, 1970, pp. 3—31.
16. BUCKLEY W.F., ed.: *Modern Systems Research for the Behavioral Scientist.* Chicago, Aldine Publishing Company, 1968.
17. BURAS N.: *A Three Dimensional Optimization Problem in Water Resources Engineering.* Operations Research Quarterly, Volume 16, 1965, pp. 419—438.
18. BURAS N.: *Dynamic Programming in Water Resources Development.* Advances in Hydroscience, Volume 3, Academic Press, New York, 1966.
19. BURMEISTER E. and DOBELL A.R.: *Mathematical Theories of Economic Growth.* London, The Macmillan Company/Collier-Macmillan Ltd., 1970
20. BURT O.R.: *Control Theory for Agricultural Policy: Methods and Problems in Operational Models.* American Journal of Agricultural Economics, Volume 51, No. 2, 1969, pp. 394.

21. BURT O.R.: *Temporal Allocation of Ground Water.* Water Resources Research, Volume 3, No. 1, pp. 45—56, 1967.
22. CASS D.: *Optimum Growth in an Aggregative Model of Capital Accumulation.* Review of Economic Studies, No. 32, 1965, pp. 233.
23. CHAKRAVARTY S.: *Capital and Development Planning.* The MIT Press, Cambridge, Mass., 1969.
24. CHAKRAVARTY S.: *The Logic of Investment Planning.* Amsterdam, North-Holland Publishing Company, 1959.
25. COLE J.A. and SCHWIEG Z.: *Optimal Control of Linked Reservoirs.* Water Resources Research, Volume 4, No. 3, 1968.
26. DAVIS R.K.: *The Range of Choice in Water Management.* Baltimore, Johns Hopkins Press, 1968.
27. DEIGINGER R.A.: *Water Quality Management — The Planning of Economically Optimal Pollution Control Systems.* Proceedings, First Annual Meeting of American Water Resources Association, Chicago, Illinois, 1965, pp. 254—282.
28. DOBELL A.R.: *Some Characteristic Features of Optimal Control Problems in Economic Theory.* Expository Papers, IEEE Trans. on Automatic Control, Volume AC-14, No. 1, 1969.
29. DOBELL A.R. and HO Y.C.: *Optimal Investment Policy: An Example of a Control Problem in Economic Theory.* IEEE Trans. Automatic Control, Volume AC-12, pp. 4-14, 1967.
30. DORFMAN R.: *Regional Allocation of Investment.* Comment. Quarterly Journal of Economics, No. 77, pp. 162—165, 1963.
31. DORFMAN R.: *An Economic Interpretation of Optimal Control Theory.* American Economic Review. December 1969.
32. DORFMAN R. and STEINER P.O.: *Optimal Advertising and Optimal Quality.* American Economic Review, Volume 45, No. 5, 1954, pp. 826—836.
33. DORFMAN R., SAMUELSON P.A. and SOLOW R.N.: *Linear Programming and Economic Analysis.* New York, McGraw-Hill, 1958.
34. DREYFUS S.E.: *A Generalized Equipment Replacement Study.* SIAM 8, No. 3, pp. 425—435, 1960.
35. DUESENBERRY J.S., ECKSTEIN O. and FROMM G.: *Simulation of the U. S. Economy in Recession.* Econometrica, Volume 28, 1960, pp. 749—809.
36. EMERY J.C.: *Organizational Planning and Control Systems: Theory and Technology.* The MacMillan Company/Collier MacMillan Ltd., London, 1969.
37. ENKE S.: *Equilibrium Among Spatially Separated Markets, Solution by Electric Analogue.* Econometrica, Volume 19, 1951, pp. 40—47.
38. FEDORENKO N.: *Questions Pertaining to Optimization of the Growth and Location of Production.* Problems of Economics, 1969, Volume 11, No. 9.
39. FRISCH R.: *The Mathematical Structure of a Decision Model: The Oslo Submodel.* Metroeconomica, Volume 7, December 1955, pp. 111—136.
40. FRISCH R.: *Propagation Problems and Impulse Problems in Dynamic Economics.* Econonomic Essays in Honor of Gustav Cassell, London, Allen, pp. 171—205, 1933.
41. FOX K.A., SENGUPTA J.K. and Thorbecke E.: *The Theory of Quantitative Economic Policy.* Volume 5 of Studies in Mathematical and Managerial Economics, H. Theil, ed., Rand McNally and Company, Chicago, 1966.
42. GEORGESCU-ROEGEN N.: *Analytical Economics.* Cambridge, Massachusetts, Harvard University Press, 1966.
43. HABERSTROH C.J.: *Controls as an Organization Process.* Management Science, January, 1960.
44. HAGUE W.: *A Pseudo-Classical Dynamic Programming Model of Capital Accumulation.* International Economic Review, Volume 6, pp. 32—46, January, 1965.
45. HAWKINS D. and SIMON H.A.: *Note: Some Conditions of Macro-Economic Stability.* Econometrica, No. 17, 1949, p. 245.
46. HOLMES W.L.: *Derivation of Application of a Discrete Maximum Principal.* Western Economic Journal, December, 1968.
47. HOLT C.C.: *Linear Decision Rules for Economic Stabilization and Growth.* Quarterly Journal of Economics, February, 1962.
48. HWANG C.L. and FAN L.T.: *The Application of the Maximum Principle to Industrial Management Systems.* Journal of Industrial Engineering, November, 1966, pp. 589—592.

49. INGLEHART D.: *Capital Accumulation and Production for the Frim: Optimal Policies.* Management Science, Volume 12, November, 1965, pp. 193—205.
50. JIND K.: *Control Systems with Jumps in the State Variables.* Econometrica, Volume 35, April, 1967.
51. KOOPMANS T.C.: *Economic Growth of the Maximal Rate Quarterly Journal of Economics.* No. 78, 1964, p. 355.
52. KURZ M.: *Optimal Paths of Capital Accumuation under the Minimum Time Objective.* Econometrica, No. 33, January, 1965, pp. 42—46.
53. LOFTSGARD L.D. and HEADY E.O.: *Application of Dynamic Programming Models for Optimum Farm and Home Plans.* Journal of Farm Economics. Volume 41, No. 1, 1959, pp. 51—62.
54. MORISHIMA M.: *Equilibrium, Stability, and Growth.* London, Oxford University Press, 1964.
55. MURPHY R.E., JR.: *Adaptive Processes in Economic Systems.* Academic Press, New York, 1965.
56. NARASIMHAM G.V.L. and MIRAKHOR ABBAS: *Multistage Allocation of Resources in National Planning.* The Engineering Economist, Volume 16, No. 2, 1971.
57. NARASIMHAM G.V.L. and MIRAKHOR ABBAS: *Utilitarian Basis for Optimum Savings.* The Indian Journal of Economics, Volume 50, No. 196, July 1969.
58. NEWMAN P.: *Approaches to Stability Analysis,* Economica Volume 28, No. 109, February 1961, pp. 12—29.
59. PHILLIPS A.W.: *Mechanical Models in Economic Dynamics.* Economica, Volume 17, 1950, pp. 382—395.
60. PHILLIPS A.W.: *Stabilization Policy in a Closed Economy.* Economic Journal, No. 64, pp. 290—323, 1954.
61. PHILLIPS A.W.,: *Stabilization Policy and the Time Form of Lagged Responses.* Economic Journal, No. 67, 1957, p. 265.
62. PHELPS E.S.: *Golden Rules of Economic Growth.* Studies of Efficient and Optimal Investment. New York, Norton, 1966.
63. RADNER R.: *Paths of Economic Growth that are Optimal with Regard Only to Final States.* Review of Economic Studies, Volume 28, February, 1961, pp. 98—104.
64. RAMSEY R.P.: *A Mathematical Theory of Savings.* Economic Journal, Volume 38, December, 1928.
65. SAMUELSON P.A.: *Classical Orbital Stability Deduce for Discrete-Time Maximum Systems.* Western Economic Journal, Volume 8, No. 2, June 1970, p. 110.
66. SCHWEIG Z. and COLE J.A.: *Optimum Control of Linked Reservoirs.* Water Resources Research, Volume 4, No. 3, 1968, pp. 479—497.
67. SENGUPTA J.K.: *On the Relative Stability and Optimality of Consumption in Aggregative Growth Models.* Economica, February, 1964, p. 33.
68. SIMON H.A.: *On the Application of Servomechanism Theory in the Study of Production Control.* Econometrica, Volume 20, 1952, pp. 247—268.
69. SHELL K., ed., *Essays on the Theory of Optimal Economic Growth.* Cambridge, Massachusetts, MIT Press, 1967.
70. STROTZ R.H., McNULTY J.C. and NAINES J.B. JR,: *Goodwin's Nonlinear Theory of the Business Cycle: An Electo-Analog Solution.* Econometrica, No. 21, July, 1953, p. 390.
71. STROTZ R.H., CALVERT J.F. and MOOREHOUSE N.F.: *Analog Computing Techniques Applied to Economics.* American Institute of Electrical Engineers, Transactions, Volume 70, Part I, 1951, pp. 557—563.
72. STROTZ, R.H., MOOREHOUSE N.F. and HORWITZ J.: *An Electro-Analog Method for Investigating Problems in Economic Dynamics: Inventory Oscillations,* Econometrica, Volume 18, pp. 313—328, 1950.
73. TAKAYAMA A.: *Regional Allocation of Investment: A Further Analysis.* Quarterly Journal of Economics, Numer 81, 1967, p. 330.
74. TINTNER G.: *What Does Control Theory Have to Offer?* American Journal of Agricultural Economics, Volume 51, No. 2, May, 1969, p. 383.
75. THEIL H.: *Optimal Decision Rules for Government and Industry.* Amsterdam, North-Holland Publishing Company, 1964.
76. TOBIN H.: *Economic Growth as an Objective of Government Policy,* American Economic Review, Papers and Proceedings, Volume 54, May, 1964, pp. 1—20.

77. TUSTIN A.: *The Mechanism of Economic Systems.* New York, James H. Heinemann, Inc., 1953.
78. YAARI M.: *On the Existence of an Optimal Plan in a Continuous Time Allocation Process.* Econometrica, Volume 32, October, 1964, pp. 576—590.

APPLICATION DE LA THEORIE DU CONTROLE OPTIME DANS L'ANALYSE ECONOMIQUE

L'auteur présente toute une série d'idées touchant le mécanisme économique, considéré à ses différents niveaux comme un système et il aborde tout spécialement la théorie du contrôle optime en ce qui concerne ses applications actuelles et celles possibles.

Ayant pour point de départ, le fait que les problèmes économiques sont divisés en deux domaines, l'un comprenant la théorie de la croissance économique optime et l'autre la théorie de la politique de stabilisation, l'auteur rattache la théorie du contrôle optime à la politique économique et révèle l'apport de celle-ci tant à la résolution du problème de la croissance économique optime qu'à celle du problème de la stabilité économique. Par la suite, on met en évidence les possibilités d'application de cette théorie au niveau macroéconomique, à celui de l'entreprise, ainsi que ses fonctions. Finalement il est relevé que la théorie du contrôle optime aura à l'avenir une large application suite aux développements théoriques ayant trait au concept de système dynamique. C'est pourquoi le problème principal auquel devront faire face les experts du contrôle optime, ceux de l'économie, de la science politique, de la sociologie et de la psychologie est celui d'établir un modèle de contrôle économique optime à caractère dynamique, qui devra incorporer les facteurs-clef d'une société agissant sur le système économique.

ПРИМЕНЕНИЕ ТЕОРИИ ОПТИМАЛЬНОГО КОНТРОЛЯ В ЭКОНОМИЧЕСКОМ АНАЛИЗЕ

Автор представляет ряд соображений об экономическом механизме на различных уровнях в качестве системы, касаясь, в особенности, теории оптимального контроля с точки зрения существующих и возможных его применений.

Исходя из разделения экономической проблематики на две области — теорию оптимального экономического знания и теорию политики стабилизации, автор указывает, что теория оптимального контроля входит, прежде всего, в теорию экономической политики, соответственно возможный вклад в разрешение оптимального экономического роста и вопроса экономической стабильности. Затем обсуждаются возможности применения этой теории на макроэкономическом уровне, на предприятиях и ее функции. В заключение указывается, что в перспективе теория оптимального контроля получит широкое применение путем теоретического развития концепции динамической системы. Поэтому важнейшим вопросом, который будет стоять перед экспертами оптимального контроля экономики, политической науки, социологии и психологии, является проектирование модели оптимального экономического контроля, которая была бы динамической и охватывала бы основные факторы общества, воздействующие на экономическую систему.