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A Demo-Economic Model*
Preliminaries

"A model is a concentrated transformation into an adequate language, generally a mathematical one, of the information accompanying the material structure of the system and its interactions."

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0. We shall set out from the remark that a country's economy has a unitary and balanced character, being an indisputable reality, no matter how important its relations with and interdependence on the other countries are; this justifies the system approach we are proposing for the demo-economic modelling.

In the following, we shall present in turn: the elements of the model or the "actors" of the national economic life (in our treatment); *the model* of description of the economic action of these "actors", methods of mathematical description of these actions; the dynamics of the considered model and problems of describing model relations and structure; the sketch of a balanced, exploratory treatment of the model; the normative guidance, through established objectives, of economic life with the aid of the considered model.

1. The *elements* of the demo-economic model suggested by us-or the "actors" of our variant of representation of the national economic life- are:

— the persons P(possibly juridical persons) making up the people of the respective country π;

— the state S — as a distinct personality;

— the exterior of the state E (or the economic-social environment);

— natural resources N (physical-material and spiritual).

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* A work presented at the Seminar for the application of statistical-mathematical methods in economy, functioning under the Central Board of Statistics.

We shall describe the *condition* of these elements at a given moment, t, the *relations* (the structure of the system) and its *dynamics* with the aid of accounts rendered profitable in the currency of the respective country and of the *variations* of these accounts.

In a first approximation, these accounts will be considered as *certain* and their variations will be deterministic.

We shall thus associate with *each person* P of the population π, $(P \in \pi)$ at the moment t, the account 1, generally vectorial,

$$\overline{m}(P, t) = (m_0(P, t), ..., m_s(P, t), \mu(P, t)),^1$$

with

$$\mu(P, t) = \sum_{j=0}^{s} \alpha_j m_j(P, t).$$

s — the given natural number, α_j — parameteres $(j = 0, 1, ..., s)$.

The component accounts $\{m_j(P, t), j = 0, 1, ..., s\}$ have the significance of describing the "condition" of person P, at the moment t, in relation to certain general categories of incomes or expenses (wages, consumption, taxes, economies, etc.). The concrete specification of these accounts is to be made by specialists in statistics and in economic sciences, according to criteria of *relevance* of information and of the *possibility* for its practical collection. The account $\mu(P, t)$ has the significance of a "sum account" of the component accounts. Coefficients $\{\alpha_j; j = 0, 1, ..., s\}$ will be defined so that expression $\sum_j \alpha_j m_j$ should really represent the "sum account": by conveniently balancing the component accounts, or eliminating, with the aid of $\alpha_j = 0$, components which are occuring several times, or in several forms, in the sum account.

For the established $P \in \pi$ functions

$$\{m_0(P, \cdot), m_1(P, \cdot), ..., m_s(P, \cdot)\},$$

as time functions, express the social attitude of a person P towards the respective categories of incomes or expenses. Practice is expected to show to what extent *classes* of functions, with variable parameters depending on person P, which should describe this attitude for various social groups can be determined for the component accounts $\{m_j(P \cdot), j = 0, ..., s\}$. Being established by *retrospective* statistical investigations, these possible classes of functions may also play an important *prospective* part, in the variation of this social attitude. For the established (P, t), the quantities

$$\{m_0(P, t), m_1(P, t), ..., m_s(P, t)\}$$

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[1] Number s or the number of component accounts is chosen so as to be able to express the condition of *any* person $P \in \pi$; suffice for it to attach to each person P the number of component accounts $(s_p + 2)$ and, then to take for all persons P, $s = max\ s_p$. $P \in \pi$

represent the *stocks* existing in the respective accounts of person P, at the moment t.

We can thus describe the population π, at the moment t, through the general file of the accounts

$$\overline{m}_o(\pi, t) = \{\overline{m}(P, t), P \in \pi\}.$$

The knowing of this file and of its variations seems to be sufficient for developing some *sociological analyses* allowing the classification of the population, the identification of its various typologies, the characterization — and, thus the diagnosis and prognosis — of the active population, etc. For example, the active population $L(t)$ at the moment t may be defined through relations of the type

$$L(t) = \{P \mid \overline{m}(P, t) \in \Delta_a(t)\}$$

by $\Delta_a(t)$ a conveniently defined region of space R^{s+2}. The social groups, sectors, etc. can be defined by conveniently partitioning the space R^{s+2}. Thus, the possibility appears for the introduction and systematic examination of social *topologies*, with the aid of *topologies induced* in Euclidean spaces. A vast research field opens up here, at the intersection of econometry and sociometry.

We shall further associate with the *state* S, at the moment t, the vectorial account

$$\overline{\mathfrak{M}}(t) = \{M_o(t), ..., M_r(t), M(t)\}$$

with $M(t) = \sum_{j=0}^{r} \beta_j M_j(t)$, r — the given natural number, β_i — parameters $(j = 0, 1, ..., r)$.

The component accounts $\{M_j(t), j = 0, 1, ..., r\}$ may have different interpretations. Thus, for some values of j, $0 \leqslant j \leqslant r$, $M_j(t)$ may be the correspondents of the personal accounts $m_j(P, t)$, at a state level, thus sizes of the form $f_j \sum_{P \in \pi} m_j(P, t)$; also included among the components of $\overline{\mathfrak{M}}(t)$ can be various components of the national production funds at the moment t (active fixed assets, passive fixed assets, circulating funds), of the final social product at moment t (investments concerning the development, modernization and renewal of the productive fixed assets, the ensurance of circulating means, the ensurance of the resources necessary for "generalized consumption" [2] etc,), state reserves, etc. The size $M(t)$ has the significance of a *global account* of the state, a synthesis of these components. In the expression of the synthetic account $M(t)$, coefficients $\{\beta_j, j = 0, 1, ..., r\}$ are playing a role similar to that played by coefficients $\{\alpha_j, j = 0, 1, ..., s\}$ in ensuring the *synthetic* and *representative* character of the account μ towards m_j, $j = 0, 1, ..., s$. The concrete structure of account

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[2] A detailed economic analysis of this concept is to be found in the work of Prof. Dr. Em. Dobrescu, Corelaţia dintre acumulare şi consum (A Correlation between Accumulation and Consumption), Editura politică, Bucharest, 1971.

$\overline{\pi}(t)$ obviously remains an open matter which is to be solved through the contribution of specialists in economic sciences and planning. It is obvious that, generally, $M(t) > \sum_{P \in \pi} \mu(P, t)$. We also consider that the quantity

$$S(t) = M(t) - \sum_{P \in \pi} \mu(P, t)$$

is not without an economic significance, expressing, in a way, the share of the intervention of the state as a distinct personality in the national economic life; within the socialist economy, for example, this indicator is expected to play a very important part.

We shall attach to the *economic-social environment* E ("the foreign countries as a whole"), at moment t, the vectorial account

$$\overline{\Omega}(t) = \{\Omega_0(t), \Omega_1(t), ..., \Omega_l(t), \Omega(t)\}$$

with $\Omega(t) = \sum_{j=0}^{l} \gamma_j \Omega_j(t)$, l-the given natural number, γ — parameters, $j = 0, 1, ..., l$. The component accounts $\{\Omega_j(t), j = 0, 1, ..., l\}$ can be introduced in various ways. Thus, for certain values of j, $0 \leqslant j \leqslant l$, $\Omega_j(t)$ can be the "external" correspondent of some accounts of the type $M_j(t)$, $0 \leqslant j \leqslant r$ or, possibly $m_j(P, t)$, $0 \leqslant j \leqslant s$, thus sizes of the form $f_j(M_j(t)$ or $g_j(m_j(P, t), P \in \pi)$ or combinations of these; also included among the components of $\Omega(t)$ can be elements corresponding to foreign trade (import, export, credits). The size $\Omega(t)$ has the significance of a *synthetic global account* of the foreign countries as a whole. Coefficients $\{\gamma_j, j = 0, 1, ..., l\}$ are playing for $\Omega(t)$ a role similar to that played by $\{\alpha_j, j = 0, 1, ..., s\}$ and $\{\beta_j, j = 0, 1, ..., r\}$ for $\mu(P, t)$ and, respectively, $M(t)$.

Finally, we shall attach to resources N (nature) at the moment t, the vectorial account

$$\overline{N}(t) = \{N_0(t), N_1(t), ..., N_m(t), N(t)\}$$

with $N(t) = \sum_{j=0}^{m} \varepsilon_j N_j(t)$, m — the given natural number, ε_j — parameters, $j = 0, 1, ..., m$. The component accounts $\{N_j(t), j = 0, 1, ..., m\}$ may be introduced in various ways, taking into consideration the structures of accounts $\overline{m}$, $\overline{M}$, $\overline{\Omega}$ and other components which were not taken into consideration in these (for exemple, technological or social innovation). In our opinion, also natural would be the distinction between the account of natural physical resources $N_f(t)$ and that of intellectual resources $N_i(t)$, accounts which may be introduced or added to $\overline{N}(t)$ and whose management may measure the development of science and culture. The size $N(t)$ has the significance of a *global account* of the resources at the moment t. We specify that through restrictions on the condition (value) of $\overline{N}(t)$, (or of some of its components), or on the speed of variation of $\overline{N}(t)$, the model may include ecological aspects (pollution, the preservation of environment, etc.) which are ever more important in the modern epoch. Just as in the

previous developments, the specification of these problems (the specification of parameters, quantification, the formalization of restrictions) remains a field open to interdisciplinary studies in which the contribution of statisticians, economists and ecologists is decisive.

We have shown how the economic "actions" of the "actors" of the economic model suggested by us can be described. But, how can the condition of this model be studied in a global perspective? For this, we suggest the utilization of *informational entropy and energy*, allowing an integrated analysis of these aspects. Let us suppose, for instance, that we are interested in the informational "condition" of the demo-economic model in relation to a set of properties $\mathfrak{L}$ attached to accounts $\overline{m}$, $\overline{\mathfrak{M}}$, $\overline{\Omega}$, $\overline{N}$ (or of some of their components), these properties may refer to the values of these accounts (respectively, the property of "appartenance" to certain fields of variation in the spaces of the possible values of these vectorial accounts), or to the speed of variation of the accounts or to some of their combinations. In this connection, one can speak about the probability, for certain elements of the model, to have properties $\mathfrak{L}$, thus, informational entropy and energy can be introduced. Let us consider, for instance, the property for the account $\overline{m}(P, t)$ to pertain to a certain region Δ in the space R^{s+2}; it is reasonable to introduce the probability for a certain person $P \in \pi$ to verify $\overline{m}(P, t) \in \Delta$, thus, we can calculate the informational states in relation to various regions $\Delta \subset R^{s+2}$, with other properties, etc. We shall not dwell on these aspects.

2. How the *structure and functioning* of the demo-economic model we have presented can be described?

We shall set out from the remark that the relations between the elements of the model are relations between their economic „actions" and, thus they can be followed through the *variations* of various accounts, resulting only from the *operations* (the flows) taking place between them. We shall note that any variations are taking place within a time interval $(t, t + \delta t)$.

Various types of elements, of connections between elements, of types of functioning of the model can be defined with the aid of these variations. A group of persons $\mathfrak{M} \subset \pi$ is isolated on the interval $(t, t + \delta t)$ from the point of view of the model, if

$$\delta\left[\sum_{P \in N} \mu(P, t)\right] = 0;$$

partial isolations, linked with certain partial accounts, may also be introduced. A person P is isolated from the rest of the persons in the interval $(t, t + \delta t)$ — from the point of view of the model-, if

$$\delta[\mu(P, t) + M(t)] = 0$$

or

$$\delta[\mu(P, t) + N(t)] = 0$$

or

$$\delta[\mu(P, t) + \Omega(t)] = 0$$

Similarly, economic "isolations" of the state from the "exterior" may also be introduced, in the interval $(t, t + \delta t)$, defined by

$$\delta[M(t) + N(t)] = 0$$

or complete „integrations" in the environment, if

$$\delta[M(t) + \Omega(t)] = 0$$

By introducing, besides the variations, also their "speed", we can more exactly describe the functioning of the system. We shall not develop these considerations here.

3. Before passing to exploratory and normative approaches of the dynamics of the described model, we should mention that it is capable of allowing the construction, through its elements, of the concepts currently used in macroeconomic modelling. Space does not allow us to develop this aspect. We shall give only two examples. We have shown before that the active population (and thus, *man power*) at the moment t, can be described with the aid of the general file of accounts $\overline{m}(\pi, t)$. The national revenue $V(t)$ can also be obtained as the combination of some elements of $\overline{M}(t)$, $\overline{N}(t)$, sometimes $\overline{\Omega}(t)$, for example, of the form

$$V(t) = m(t)M(t) + n_f(t)\, N_f(t) + n_i(t)\, N_i(t)$$

(sometimes $+ \omega(t)\, \Omega(t)$, where $m(t)$, $n_f(t)$, $n_f(t)$, respectively $\omega(t)$ have the significance of some fractions (or ponderation) of $M(t)$, $N_f(t)$, $N_f(t)$ respectively $\Omega(t)$ which contribute to the creation of the national revenue; the dynamics of these elements, retrospective and prospective, may offer numerous data and eventually suggestions about the structure and variation of the quantity $V(t)$. Finally, it is obvious that investments (or the capital) $K(t)$ can be described with the aid of elements of $\overline{M}(t)$, $\overline{N}(t)$, sometimes $\overline{\Omega}(t)$. It results that the sizes appearing in the expression of the common production functions (for example, of the Cobb-Douglas type) can be described with the aid of the proposed demo-economic model and they may be retrieved by putting convenient conditions to the elements of this model.

The second example refers to the possibility of describing, with the aid of the demo-economic model, the classical analyses of the input-output balance type. Obviously, an economic sector can be modelled through certain combinations of the accounts $\overline{m}(\pi, t)$ $\overline{M}(t)$, $\overline{N}(t)$, $\overline{\Omega}(t)$: indeed, it is characterized by certain *regions* of the "spaces" R^{s+2}, R^{r+2}, R^{l+2}, R^{m+2}, thus, by certain groups of the population, by the structure of the accounts $\overline{M}$, $\overline{N}$, $\overline{\Omega}$, by the relations between them, etc. An inter-branch model can thus be described through functions of the elements of the demo-economic model. The reciprocal of this affirmation is not true: an inter-branch model does not determine, or at least, does not generate *a single* demo-economic model. It results that for some problems of a detailed analysis, the demo-economic study presents advantages, remaining a possible extension of a classical type model which can be retrieved by convenient syntheses.

4. The proper functioning of the demo-economic model (in an explorative approach) can be described with the aid of an equilibration or balance

equation of the accounts expressing the fact that the system is closed (if all the variations of accounts are taken into consideration). This equation of "conservation of the flows" has the form

$$\Delta N(t) = \sum_{P \in \pi} \Delta \mu(P,\ t) + \Delta M(t) + \Delta \Omega(t) \tag{1}$$

or, under obvious conditions referring to the class of functions $N(t)$, $\mu(P,\ t)$, $M(t)$, $\Omega(t)$, of the form

$$\frac{dN}{dt} = \sum_{P \in \pi} \frac{d\mu(P,\ t)}{dt} + \frac{dM}{dt} + \frac{d\Omega}{dt} \tag{2}$$

Equations (1) or (2) underline the *relation* between the variations of quantities $N(t)$, $\mu(P,\ t)$, $M(t)$ and between their variation speed, a relation which seems to be neglected in certain global models. In plausible hypotheses concerning only the *form* of functions $N(t)$, $\mu(P,\ t)$, $M(t)$, $\Omega(t)$, the afore-mentioned equations are leading to interesting results as regards the relation between these "increases". Thus, it results from (2)

$$\sum_{P \in \pi} \frac{d\mu(P,\ t)}{dt} = \frac{dN}{dt} - \frac{dM}{dt} - \frac{d\Omega}{dt} \tag{3}$$

It results that, for

$$\sum_{P \in \pi} \frac{d\mu(P,\ t)}{dt} \geqslant 0,$$

meaning that the sum of the personal accounts does not decrease, it is necessary that

$$\frac{dN}{dt} - \frac{dM}{dt} - \frac{d\Omega}{dt} \geqslant 0,$$

or

$$\frac{dN}{dt} \geqslant \frac{dM}{dt} + \frac{d\Omega}{dt},$$

which shows that the increase of the exploitation of resources should surpass the speed of increase of the account of the state and of the account of interaction with the environment.

Further, let us suppose, in a first approximation, that

$$\mu(P,\ t) = \varphi(P)\ \nu(t),$$

thus

$$\sum_{P \in \pi} \mu(P,\ t) = \left[\sum_{P \in \pi} \varphi(P)\right] \nu(t) = \Phi(\pi)\ \nu(t), \qquad \text{with} \qquad \Phi(\pi) = \sum_{P \in \pi} \varphi(P)$$

The element $\Phi(\pi)$ measures the weight of the population π in the quantity $\sum_{P \in \pi} \mu(P,\ t)$. Unlike in the previous approximations, let us suppose that the

population π varies in time; it is obvious that its weight $\Phi(\pi)$ in the quantity $\sum_{P \in \pi} \mu(P, t)$ should also depend on time. Let us consider, for simplification, the case of the exponential growth of the population; this implies to replace the constant:

$$\Phi(\pi)$$

by:

$$\psi(\pi, t) = \pi_0 e^{at}$$

where π_0, are positive constants. From (3), we find:

$$[\pi_0 e^{at}] \frac{d\nu}{dt} = \frac{dN}{dt} - \frac{dM}{dt} - \frac{d\Omega}{dt},$$

or:

$$\frac{d\nu}{dt} = \left[\frac{dN}{dt} - \frac{dM}{dt} - \frac{d\Omega}{dt}\right] \pi_0^{-1} e^{-at}. \tag{4}$$

Thus, it is necessary that the difference between the speed of growth of the exploitation of resources and the speed of increase of the state accounts and of the inter-action with the foreign countries should exponentially increase, by a module at least equal to the module a of the growth of the population in order to obtain a variation of "the personal account" $\nu(t), = \frac{d\nu}{dt}$ — which ought not decrease exponentially. Thus, generally, for $\frac{d\nu}{dt}$ not to be of the order e^{-at}, it is necessary that:

$$\frac{dN}{dt} - \frac{dM}{dt} - \frac{d\Omega}{dt} \sim ke^{bt}, \qquad b \geqslant a.$$

This occurs, for example, if:

— function $N(t)$ increases exponentially by a module at least equal to a, and $M(t)$, $\Omega(t)$ are linear or at most exponential, by a module lower or equal to a.

— function $N(t)$ increases lineary, while at least one of the functions $M(t)$, $\Omega(t)$ varies exponentially from the module at least equal to a. The results may be used in analysing the much discussed problem of the "exponential growth". We shall not develop these considerations, bearing in mind only the possibility of a complete study of equations (1), (2), with the functions $N(t)$, $M(t)$, $\Omega(t)$, $\mu(t)$ in various established classes.

5. In conclusion, we shall outline some elements linked with the *normative* study, through goals of the suggested demo-economic model. This aspect appears in the *prospective* control and in the demoeconomic *prognosis*. From the mathematical point of view, a future goal is described by a condition referring to the future value of a certain *functional* of

the elements of the model; a set of goals will thus lead to conditions of the form

$$J_k(\overline{m}(\pi, t+\theta);\ \overline{M}(t+\theta);\ \overline{N}(t+\theta);\ \overline{\Omega}(t+\theta)) = \\ = J_k(\theta),\ k = 1, 2, ..., p \tag{5}$$

where θ is the prognosis horizon and $I_k(\theta)$, $k = 1, 2, ..., p$ are the proposed future goals. In the most general case, the problem of the *management* of the system described by the proposed demo-economic model towards the goals $J_k(\theta)$, $k = 1, 2, ..., p$ recurs to the finding of a system of solutions to the equations of the (1) or (2) type, verifying (5). Among the goal $J_k(\theta)$, $k = 1, 2, ..., p$ we can include, for example, the raw national product or other quantities of the same kind; in equations (1) or (2), some of the functions $\overline{m}$, $\overline{M}$, $\overline{N}$, $\overline{\Omega}$, or some of their components, may be considered as *given*, and others — control variables. We specify that the problem of the *existence* of the solutions is not banal and it essentially depends on the goals $J_k(\theta)$, $k = 1, 2, ..., p$; other forms of objectives, such as time interval functionals $[\theta_1, \theta_2]$, goals of the type $J_k([\theta_1, \theta_2])$ or other forms of control can also be imagined. We specify that the same type of developments allows to solve the problem of *goal design* compatible with certain restrictions, of various forms, on the quantities $\overline{m}$, $\overline{M}$, $\overline{N}$, $\overline{\Omega}$, on some of their components, on some combinations of these components, etc; we can thus examine the existence and building of those strategies — defined by the controlled variations of some established control variables or by choosing some convenient control variables, — which with certain restrictions concerning functions $\overline{m}$, M, N (for example: non-decreasing functions, or the increase of maximum M), should lead to optimum

$$J_k[\overline{m}(\pi, t+\theta);\ \overline{M}(t+\theta);\ \overline{N}(t+\theta);\ \overline{\Omega}(t+\theta)],\ k = 1, 2, ..., p, \tag{6}$$

the optimum (maximum, minimum, etc.) being taken the set of controls compatible with equations of the type (1) or (2) and, relations (6) defining possible goals (coherences) for the previsional management of the system. The space of decisions for the management of the economic system is thus appearing considerably extended, and, the problems of analysis and future development find a natural framework for a rigorous investigation.

6. The considerations in the present study have only a preliminary character, of a draught — project which is to be discussed critically, explained, developed and moreover materialized by groups of specialists with a multidisciplinary structure (statisticians, economists, sociologists and mathematicians).

UN MODELE DEMO-ECONOMIQUE. PRELIMINAIRES

Le modèle démo-économique proposé décrit le système économique national par l'introduction de certains comptes valorisés dans la monnaie du pays respectif appartenant: aux personnes (population), à l'Etat — comme personnalité distincte, à l'étranger (globalement)

et aux ressources naturelles (matérielles et spirituelles). Le mécanisme du système économique peut être poursuivi par les relations et les variations réciproques de ces comptes et dirigé sur cette même base. En dehors de l'esquisse du cadre général d'étude de cette problématique, les auteurs présentent un premier résultat concernant la liaison entre la balance des comptes et la croissance économique.

ДЕМОЭКОНОМИЧЕСКАЯ МОДЕЛЬ. ПРЕДВАРИТЕЛЬНЫЕ АСПЕКТЫ

Предлагаемая демоэкономическая модель описывает национальную экономическую систему при помощи счетов, проводимых в валюте соответствующей страны и принадлежащих отдельным людям (населению), государству как отдельному лицу, заграница (в совокупности) и природным ресурсам (материальным и духовным), а также при помощи возможных промежуточных счетов. Наблюдение за функциональностью экономической системы, а также управление ею можно вести при помощи отношений и взаимных изменений указанных счетов на одной и той же основе. Помимо обрисовывания общих рамок рассмотрения этой проблематики, настоящая статья содержит первый результат, касающийся связи между балансом счетов и экономическим ростом.

5.IX.1973