V. DUMITRU
S. MOGA
Department of Economic Cybernetics
Academy of Economic Studies

Testing the Efficiency of the Sequential Optimizing Technique

"One of the recent optimizing methods of the non-linear programming problems is the sequential optimizing technique. In this paper there are presented some experiments performed by the authors, on testing problems, in order to estimate the efficiency of the computation taking into account the number of function and gradient evaluations and the accuracy. The method proves to be efficient for the tested examples."

One of the techniques for solving non linear programming problems is that of sequential optimization, which consists in the following [1]:

Consider the problem

$$\min f(x), \qquad x \in E^n$$

with the conditions

$$g_i(x) \geqslant 0, \qquad i = 1, \ldots, m. \tag{I}$$

Let us define

$$D^\circ = \{x \mid g_i(x) > 0, \qquad i = 1, \ldots, m\}$$

and let $P(x)$ be a real valued function of x with the following properties:

$\mathbf{P}_1$: $P(x)$ is continuous in D°;

$\mathbf{P}_2$: For any infinite sequence $\{x^n\}$, $x^n \in D^\circ$ convergent to a border point x^f, $g(x^f) = 0$, for at least a value of i, we have

$$\lim_{k \to \infty} P(x^k) = +\infty$$

and, consider $Z(r)$ a function of real variable r with real values with the properties:

$\mathbf{Z_1}$: If $r_1 > r_2 > 0$, then $Z(r_1) > Z(r_2) > 0$

$\mathbf{Z_2}$: If $\{r_k\}$ is an infinite sequence with the property

$$\lim_{k\to\infty} r_k = 0, \qquad \text{then} \qquad \lim_{k\to\infty} Z(r_k) = 0.$$

We define the problem

$$min\ U(x, r) = f(x) + Z(r_1)\ P(x) \tag{II}$$

where r_1 is a positive number.

An unconstrained minimization technique is applied to Problem II sequentially, starting from a $x^\circ \in D^\circ$ (the case of the interior point algorithm). An $x^1(r_1)$ is obtained.

Setting out from x^{k-1} the method is repeated with an $r_k < r_{k-1}$ obtaining x^k and it is pointed out [1 page 47] that in certain conditions, the series x^k of optimal values for the series of unconstrained Problems II has a limit point which is a local solution to Problem I.

As part of the Laboratory of Economic Informatics of the Economic Cybernetics Department — the Academy of Economic Studies — the authors have programmed a series of algorithms for optimizing an unconstrained function of a variables with the aid of which it experimented the sequential optimizing technique for certain test problems. We mean by the efficiency of a programming technique the capacity of the method of solving the problem with a certain given accuracy, under relatively acceptable conditions of time for man and the machine.

In this study, we present some of the results in which the algorithms of Fletcher-Reeves [2], [4] and Davidon-Fletcher Powell [3], [4] are used for optimizing problem II.

For all examples given below, problem II has the form

$$min\ U(x, r) = f(x) - r\sum_{i=1}^{m} \log\ g_i(x)\cdot$$

and the stopping criterion $\| \Delta f(x^k) \| < \varepsilon, \qquad \varepsilon = 0.001$

The results were the following:

1) Beale's function with constraints:

$$min\ f = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3;$$

$$x_1 + x_2 + 2x_3 \leqslant 3, \qquad x_1, x_2, x_3 \geqslant 0.$$

Theoretical optimum

$$x^* = \left(\frac{4}{3}, \frac{7}{9}, \frac{4}{9}\right) = (1\cdot33;\ 0\cdot77;\ 0\cdot44);$$

$$f(x^*) = \frac{1}{9}\cdot$$

a) *Fletcher- Reeves's method*
Initial point $x^\circ = (0.5;\ 0.2;\ 0.8)$

	r	$x_1(r)$	$x_2(r)$	$x_3(t)$	Number iterations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	1.2724	0.7235	0.3023	16	0.00072	146	17
r_2	0.25	1.3411	0.7242	0.3040	11	0.00094	115	14
r_3	0.1	1.3720	0.7366	0.3333	10	0.0009	131	11
r_4	0.01	1.3444	0.7696	0.4225	38	0.00067	570	39
r_5	0.001	1.3341	0.7767	0.4423	100	0.0008	1695	102

b) *The Davidon-Fletcher-Powell method*
Initial point $x^\circ = (0.5;\ 0.2;\ 0.8)$

	r	$x_1(r)$	$x_2(r)$	$x_3(r)$	Number iterations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	1.2722	0.7236	0.3024	7	0.0002	56	8
r_2	0.25	1.341	0.7242	0.304	7	0.0004	73	8
r_3	0.1	1.3720	0.7366	0.3333	9	0.0009	88	10
r_4	0.01	1.3445	0.7697	0.4223	6	0.0001	95	7
r_5	0.001	1.3351	0.7763	0.4419	6	0.0007	106	8

2) $min\ f = x_1 + x_2$

$$-x_1^2 + x_2 \geqslant 0, \qquad x_1,\ x_2 \geqslant 0.$$

Theoretical optimum $x^* = (0,\ 0)$, $f(x^*) = 0$.

a) *The Fletcher-Reeves method*
Initial point $x^0 = (1,\ 2)$.

	r	$x_1(r)$	$x_2(r)$	Number interations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for function	No. of calls for gradient
r_1	1	0.6031	2.1977	3	0.0003	34	4
r_2	0.5	0.3610	1.069	6	0.00038	60	7
r_3	0.25	0.2058	0.522	5	0.00029	41	6
r_4	0.1	0.0915	0.2042	6	0.0008	92	7
r_5	0.01	0.099	0.0200	7	0.0006	92	8
r_6	0.001	0.001	0.002	22	0.0008	242	23

b) *The Davidon-Fletcher-Powell method*
Initial point $x^0 = (1, 2)$

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for function	No. of calls for gradient
r_1	1	0.6031	2.198	3	0.0003	22	4
r_2	0.5	0.3611	1.069	4	0.0003	38	5
r_3	0.25	0.2058	0.5221	4	0.0001	38	5
r_4	0.01	0.0914	0.2049	4	0.0004	56	5
r_5	0.1	0.0099	0.0200	4	0.0005	41	5
r_6	0.001	0.001	0.002	3	0.0001	37	4

3) $max\ f = 6x_1 + 4x_2 + 2x_3 - 3x_1^2 - 2x_2^2 - \frac{1}{3}x_3^2,$

$$x_1 + 2x_2 + x_3 \leqslant 4.$$

Theoretical optimum $x^* = (0.875;\ 0.625;\ 1.875)$, $f(x^*) = 7.25$.

a) *The Fletcher-Reeves method*
Initial point $x^0 = (0, 0.1)$

	r	$x_1(r)$	$x_2(r)$	$x_3(r)$	Number iterations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for function	No. of calls for gradient
r_1	1	0.8177	0.4532	1.3608	5	0.0008	49	6
r_2	0.5	0.8419	0.5259	1.5787	6	0.0007	45	7
r_3	0.25	0.8567	0.5703	1.7115	6	0.0005	47	7
r_4	0.1	0.8671	0.6014	1.8045	22	0.0002	177	23
r_5	0.01	0.8742	0.6227	1.8670	52	0.0008	561	53
r_6	0.001	0.8751	0.6249	1.8736	100	0.0009	1413	102

b) *The Davidon-Fletcher-Powell method*
Initial point $x^0 = (0, 0.1)$

	r	$x_1(r)$	$x_2(r)$	$x_3(r)$	Number iterations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for function	No. of calls for gradient
r_1	1	0.8178	0.4534	1.3603	5	0.0001	43	4
r_2	0.5	0.8421	0.5261	1.5776	6	0.001	45	5
r_3	0.25	0.8568	0.5704	1.7113	5	0.00007	47	5
r_4	0.1	0.8671	0.6015	1.8042	8	0.001	50	5
r_5	0.01	0.8741	0.6225	0.8675	5	0.0001	60	6
r_6	0.001	0.8748	0.6247	1.8743	7	0.006	55	5

4) $min\, f = x_1^2 + x_2^2 - x_1 - 8x_2;$

$$-2x_1 + 3x_2 \leqslant 3; \qquad x_1, x_2 \geqslant 0;$$

$$x_1 + x_2 \leqslant 3;$$

$$2x_1 - x_2 \leqslant 4.$$

Theoretical optimum $x^* = (1.2; 1.8)$.

a) *The Fletcher-Reeves method*
Initial point $x^0 = (0.1; 0.8)$

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\nabla f(x^k)\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	1.0819	1.5885	17	0.0007	185	18
r_2	0.25	1.1211	1.6783	16	0.0004	179	17
r_3	0.1	1.1604	1.7453	100	0.003	1405	102
r_4	0.01	1.1953	1.7939	100	0.002	1353	102
r_5	0.001	1.1995	1.7993	100	0.001	1486	102

b) *The Davidon-Fletcher-Powell method*
Initial point $x^0 = (0.1; 0.8)$

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\nabla f(x^k)\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	1.0819	1.5885	6	0.00001	48	7
r_2	0.25	1.12127	1.67839	7	0.001	49	8
r_3	0.1	1.1605	1.7454	8	0.003	50	9
r_4	0.01	1.1952	1.7939	5	0.002	40	6
r_5	0.001	1.1995	1.7993	5	0.003	40	6

5) $min\, f = \frac{1}{2}(x_1^2 + x_2^2) - x_1 - 2x_2;$

$$2x_1 + 3x_2 \leqslant 6;$$

$$x_1 + 4x_2 \leqslant 5;$$

$$x_1, x_2 \geqslant 0.$$

Theoretical optimum $x^* = (13/17, 18/17) = (0.764; 1.05)$.

a) *The Fletcher-Reeves method*
Initial point $x^0 = (0.1; 0.1)$

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\nabla f(x^k)\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	0.8236	0.6849	6	0.0001	37	7
r_2	0.25	0.7871	0.8248	4	0.0008	42	5
r_3	0.1	0.7659	0.9546	5	0.0009	49	6
r_4	0.01	0.7621	1.0483	59	0.0002	650	60
r_5	0.001	0.7653	1.057	73	0.008	956	74

b) *The Davidon-Fletcher-Powell method*
Initial point $x^0 = (0.1; 0.1)$

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\nabla f(x^k)\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	0.8238	0.6848	4	0.0002	28	5
r_2	0.25	0.7876	0.8246	5	0.00001	35	6
r_3	0.1	0.7652	0.9549	3	0.0007	30	4
r_4	0.01	0.7636	1.0483	4	0.0041	30	5
r_5	0.001	0.7645	1.0578	5	0.0004	32	6

6) $$min\ f = \frac{1}{3}(2x_1^3 + 3x_1^2x_2 + 6x_1x_2^2 + 5x_2^3) + 2x_1^2 - 6x_1x_2 +$$
$$+ 5x_2^2 + 10x_1 - 15x_2;$$
$$x_1 + 2x_2 \geqslant 8;$$
$$2x_1 - x_2 \leqslant 6;$$
$$x_1 \geqslant 0,\ x_2 \geqslant 0.$$

Theoretical optimum $x^* = (2, 3)$.

a) *The Fletcher-Reeves method*
Initial point $x^0 = (1, 4)$.

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\nabla f(x^k)\|$	No. of calls for function	No. of calls for gradient
r_1	0.5	1.9937	3.0098	100	0.1	1033	102
r_2	0.25	1.9982	3.0043	200	0.1	1940	202
r_3	0.1	2.1356	2.9336	100	0.01	1116	102
r_4	0.01	2.0927	2.9537	100	0.01	1412	102
r_5	0.001	2.0095	3.012	100	0.02	1315	102

b) *The Davidon-Fletcher-Powell method*
Initial point $x^0 = (1,4)$

	r	$x_1(r)$	$x_2(r)$	Number iterations	Precision $\|\|\nabla f(x^k)\|\|$	No. of calls for	
						function	gradient
r_1	0.5	2.0003	3.006	17	0,01	205	18
r_2	0.25	2.0002	3.003	6	0.01	57	7
r_3	0.1	2.00008	3.001	11	0.1	100	12
r_4	0.01	1.9996	3.0003	10	0.002	97	11
r_5	0.001	1.9991	3.0004	6	0.001	57	7

From the analysis of the above results, two conclusions are drawn for the chosen test functions:

a) The method of sequential optimization solves with a good accuracy the constrained non-linear programming problems tested;

b) The efficiency of the method concerning the number of function and gradient evaluations, depends on the optimization algorithm used for the unconstrained Problem II.

We refind in these tests the result claimed in other experiments that the Davidon-Fletcher-Powell method is more efficient than the Fletcher-Reeves method.

REFERENCES

1. FIACCO A. V., MC CORMICK G. P.: *Nonlinear Programming, Sequential Unconstrained Minimization Techniques:* John Wiley, New York, 1968.
2. FLETCHER R., REEVES C. M.: *Function Minimization by Conjugate Gradients.* Computer J., no. 7, 1964.
3. FLETCHER R., POWELL M. I. D.: *A Rapidly Convergent Descent Method for Minimization.* Computer J., no. 6, 1963.
4. AMUNATEGUI J., DUMITRU V.: *Computer Experiments with an Unconstrained Nonlinear Programme.* Economic Computation and Economic Cybernetics Studies and Research, No. 3, 1970.

TESTS SUR DE L'EFFICIENCE LA TECHNIQUE D'OPTIMISATION SÉQUENTIELLE

La technique d'optimisation séquentielle est l'une des méthodes relativement récentes d'optimisation des problèmes de programmation non linéaire. Dans cette étude ont présente quelques expérimentations effectuées par les auteurs sur des problèmes test afin d'évaluer l'efficacité de la méthode en ce qui concerne le temps et l'exactitude du calcul. Pour les exemples soumis aux tests la méthode s'avère être efficace.

ПРОВЕРКА ЭФФЕКТИВНОСТИ ТЕХНИКИ СЕКВЕНЦИОННОЙ ОПТИМИЗАЦИИ

Одним из сравнительно новых методов оптимизации задач нелинейного программирования является техника секвенционной оптимизации. В настоящей статье представлены некоторые эксперименты, проведенные авторами по опытным задачам при попытке дать оценку эффективности метода в отношении времени расчета и аккуратности. Для опытных примеров метод оказывается эффективным.

5.X.1973