VENIAMIN AL. URSEANU
Centre of Mathematical Statistics
of the Academy of the Socialist Republic of Romania

Models of Stochastic Optimization and the Price Systems

"The price of goods is not only an economic category, but a category of stochastic type.
In this respect, a production linear stochastic model is considered to be the only able to reflect the essential characteristics of the real process of price formation".

1. GENERAL CONSIDERATIONS

The idea is emphasized that any attempt at building an economic-mathematical model, in general and, a mathematical model of the value of goods or prices, in particular, must be preceded by a thorough analysis of the conditions in which the modelled object is taking place and is developing and of the parameters characterizing it in space and time.

It is from this point of view that we have considered so far a series of joint models concerning production and prices, of a determinist type. We saw that what is characteristic of the models of this type is the fact that the parameters characterizing the modelled process are some constant values determined or known with the necessary precision for guaranteeing the exactness of the results. We have noted that such parameters can be, for example, the quantities of the necessary production material means for obtaining a certain product unit or the well established demands of the market, the available and necessary resources, etc. But, the same parameters can also be defined as mean values of random variables without the model changing its determinist type.

Thus, it results that in the case of the economic-mathematical models of a determinist type, we generally have a series of decision variables the variation field of which is established by the restrictions of the model. Thus,

an adopted decision-programme satisfies the restrictions of the problem and, at the same time, fulfils the chosen criterion of economic optimum, expressed by the objective-function.

Novojilov's model of the socially necessary labour time, differentially reflecting the conditions of production, the primal problem, on the one hand and, the conditions of consumption, the dual problem, on the other hand, is obviously a determinist model.

From our point of view, Novojilov's determinist model does not entirely reflect the essential features of the real process of the formation of prices, since the initial data and the main parameters of this process are, generally, random variables.

Thus, for example, in the primal model proposed by Novojilov, the coefficients c_j^l of the objective-function $\sum_{j,l} c_j^l q_j^l$ can be and, are in reality random variables with known or unknown allocations, a fact which confers the entire model the character of a stochastic programming problem [1].

The coefficients Q_i of the available resources and even the coefficients α_{ij} of matrix A might be random variables in the same model. There are situations when both the coefficients of matrix A and the coefficients of the available resources or of "the demand" are random variables. In all these cases, we are faced with problems of stochastic programming.

As it is known, the class of stochastic models differs from that of the determinist type models not so much through their form rather than through the *content* of information offered by them and through the *character* of the results obtained with their aid, a fact which necessarily implies interpretations with a *probabilistic, statistical character.*

Further on, we shall present a problem of linear stochastic programming to outlining the *optimum price system*, with a *statistical* character.

2. THE PROBLEM OF PRICES WITHIN A LINEAR STOCHASTIC MODEL

We shall assume that the optimization of the planned management of the socialist economy can be achieved by considering economy as a unitary system within which *the optimization of the formation of prices* is an inseparable, organic component *of the optimization of the production plans.* We should not forget that prices and economic management, worked out on the basis of the law of value, of the law of labour economy, in closely connected with the other laws of the socialist economy enable the harmonization of profitableness, at the level of the basic productive units, with the major requirements, tasks and objectives of the centralized plan of the national economy.

To emphasize the statistical character of the real price systems, we shall consider the following model in which, for simplification, the entire mechanism of the economy is presented monosectorally.

[1] According to some authors, if the allocations of all random parameters or of some of them are unknown, we have to do with programming problems with total or partial incertitudes. We think these distinctions are partly conventional and they do not pertain to the essence of the problems.

Thus, it is considered that the entire economy is specialized for producing three agricultural cultures A, B, and C taking into account certain restrictions and conditions.

For the whole of economy, we have at our disposal

1000 units [1] of arable land;

2 400 000 man-days (live and materialized work).

The three cultures can be sown under the following conditions:

Culture A produces, on an average, 1800 units of measure [2] per one unit of arable land [3]

Culture B produces, on an average, 2400 units of measure per one unit of arable land

Culture C produces, on an average, 2000 units of measure per one unit of arable land.

The labour costs for obtaining culture A are 2000 man-days 1 t.a.

The labour costs for obtaining culture B are 3000 man-days 1 t.a.

The labour costs for obtaining culture C are 2500 man-days 1 t.a.

A unit of measure of the production of culture A is sold at the "price" of 2 thousand man — days [4] ;

A unit of measure of the production of culture B is sold at the "price" of 2.58 thousand man-days;

A unit of measure of the production of culture C is sold at the "price" of 2.50 thousand man-days.

To raise profitableness at an ever higher level, the central planning bodies have proposed to establish the number of units of agricultural land on which cultures A, B, C should be produced, so that *a maximum possible national income* should be obtained, since there is no other economic consideration in establishing the areas for one of these cultures.

To formulate the problem mathematically, we shall introduce the variables x, y and z, representing the number of hectares cultivated with cultures A, B and C.

In the given conditions, we must have

$$\left.\begin{aligned} &x + y + z = 1000 \\ &x \geq 0 \quad y \geq 0, \; z \geq 0 \\ &2000x + 3000y + 2500z \leq 2\,400\,000. \end{aligned}\right\} \tag{1}$$

The national income will be given by the expression

$$2 \times 1\,800x + 2.58 \times 2\,400x + 2.5 \times 2\,000z - 2\,000x - 3000y - \\ - 2500z = 1600x + 3200y + 2500z. \tag{2}$$

---

[1] These units may be, for example, "a hundred" or "a thousand" hectares.

[2] A unit of measure (u.m.) may represent a ton, a hundred tons, a thousand tons, etc.

[3] Further on, the term "arable land" will be abbreviated by "t. a".

[4] If, for example, a man-day is equal to 25 lei, it means that the price of a unit of measure is 50 000 lei.

1. The determinist solution to the problem

The primal problem of the model consists in finding the maximum of function (2), conditions (1) being also fulfilled.

So, we must have

$$1600x + 3200^{1}y + 2500z = maxim. \tag{3}$$

We shall reduce the three unknown quantities to two, namely x and y by using relation

$$z = 1000 - x - y,$$

resulting from the first of relations (1)

Carrying out the substitution, in each of relations (1), the data of the problem simplify. We know we have

$$\left.\begin{aligned} x - y &\geq 200, \\ x + y &\leq 1000, \\ x &\geq 0, \\ y &\geq 0. \end{aligned}\right\} \tag{4}$$

And now, the problem is to determine the values of x and y so as to have

$$-9x + 7y = maxim. \tag{5}$$

The solution is obtained very simply either graphically (see Fig. 2), or through the simplex method; the result of the calculus is the following:

$$x = 200;\ y = 0;\ z = 800$$

Consequently, the maximum national income which we shall note by $\overline{V}$ corresponds to the use of 200 units of t.a. for culture A and of 800 units t.a. for culture C. It is equal to 2 320 000 man-days. Any other solution is leading to a smaller income.

The dual problem for problem (4), to which we add also a first condition $x + y + z = 1000$) can be formulated as follows:

$$\begin{gathered} u + v + \omega \geq 1\,600, \\ u + (-1)v + w \geq 3\,200, \\ u + 0 + 0 \geq 2\,500, \\ \text{any } -u- \\ v \leq 0, \\ w \geq 0, \\ 1000u = 200v + 1000w = min \end{gathered} \tag{6}$$

[1] The number was rounded by eight hundreds for facilitating the calculations.

As it can be seen, a dual variable (u, v, and w) is attached to each restriction in the primal problem, each of them acquiring the positive, negative or any sign just as the respective restriction is concordant or discordant in keeping with the sense of the objective-function, or is expressed by an equality. In our case: variable u may get any sign as the restriction in the primal problem to which it is attached bears the sign of equality; variable v has the minus sign, as the restriction in the primal problem to which ($\geq$) is attached is discordant with the maximum sense of the objective-function in this problem; variable w has the positive sign as the restriction in the primal problem to which ($\leq$) is attached is concordant with the sense of the objective-function.

The sign $\geq$ of the first restrictions in (6) expresses the fact that they are concordant with the sense of the dual(minimum) objective-function, as the primal variables to which (x, y, z) correspond are positive.

The system (6) may be changed into an equivalent system which is more convenient for applying the simplex calculus algorithm.

If we introduce the following substitution variables:

$$\left.\begin{aligned} v' &= -v \\ u' &= u - 2\,500, \end{aligned}\right\} \quad (u \geq 2500 \Rightarrow u' \geq 0),$$

the dual problem can be formulated — after simplifying it by 100 and afterwards by 2 the coefficients of the objective-function, as follows

$$\begin{aligned} u' + 2\,500 - v' + w &\geq 1600, \\ u' + 2500 + v' + w &\geq 3\,200, \\ \hline 5u' - v' + 5w &= min \end{aligned}$$

or, in addition

$$\left.\begin{aligned} u' - v' + w &\geq -900, \\ u' + v' + w &\geq 700, \\ u',\ v',\ w &\geq 0, \\ \hline 5u' - v' + 5w &= min. \end{aligned}\right\} \quad (6')$$

To make possible the solving of the problem with the aid of the simplex algorithm, we shall introduce another two equalizing variables h_1 and h_2 and an artificial variable ξ. Finally, we obtain the following system of equations of the dual problem:

$$\left.\begin{aligned} -u' + v' - w + h_1 &= 900, \\ u' + v' + w - h_2 + \xi &= 700, \\ \hline 5u' - v' + 5w &= min \\ \xi &= min \end{aligned}\right\} \quad (7)$$

After three iterations, which we no longer reproduce here, the following optimum solution to the dual problem is obtained:

$$h_2 = 200,$$
$$v' = 900,$$

thus:

$$u' = w = 0;\ v' = 900.$$

Hence:

$$\left.\begin{aligned} u &= u' + 2\,500 = 2\,500 \\ v &= -v' = -900, \\ w &= 0. \end{aligned}\right\} \qquad (8)$$

The solution-vector of the dual problem being known, we immediately obtain the minimum value of the objective-function which we shall note by $\bar{C}$ (the minimum volume of the labour costs in thousands of man-days):

$$\bar{C} = 1000u + 200v + 1000w = 2\,500\,000 + 200(-900) + 1\,000 \times 0 =$$
$$= 2\,500\,000 - 180\,000 = 2\,320\,000.$$

As $\bar{V} = \bar{C}$, the fundamental theorem of duality, in our illustrative model is checked.

In the case of this example, let us specify the economic significance of the dual variables.

In fact, they are the same optimum prices we have dwelt on, at length, in the three previous chapters. For a more exact understanding, in particular, of these prices, we shall note the following:

"The price" u of 2500 man-days expresses the importance which is being attached, on the scale of the entire national economy, to a unit of measure of the arable land area (a thousand ha). The marginal increase of the surface of arable land, that is by *a single unit of measure*, will cause this means of production to bring about an increase of the *minimum* costs on the national economy as a whole, equal to *2500 thousand man-days* and at the same time, an increase of the maximum national income of the same value[1]

What does "the price" v equal to *900 thousand man-days* mean?

To understand more exactly the meaning of this "price" we shall first note that variable v was attached to the second restriction in the primal problem (4), $(x - y \geq 200)$. Or, this restriction is a changed form of the restriction of not surpassing the total fund on the economy of man-days $(200x + 300y + 2500z \leq 2\,4000\,000$ man-days), in (1.) which shows that for not surpassing this fund, at least 200 units more of arable land should be alloted to culture A than to culture B.

But, as the solution to the model indicates $v = -900$, it results that if culture A would request to have not 200 but 201 units more arable land,

[1] Obviously, on the essential condition that the optimum basis should not be changed.

a number of *900 thousand man-days are lost* on the economy as a whole, obviously, representing an adequate loss from the maximum volume of the national income.

2. The stochastic solution to the problem and its economic interpretation

A. *The primal problem*

The determinist solution to the primal problem can be considered neither realistic nor *satisfactory* from the economic point of view. Indeed, upon drawing up the plan of culture, the planning bodies know the average production per unit of agricultural land, but, they *cannot be sure that, in reality, after the gathering in of the harvest this average will be obtained for sure.* The only estimation they can make is that the harvested production, *with a high probability*, will be comprised within a certain interval.

Thus, in connection with the three cultures, we shall suppose that the production per unit of surface of agricultural land follows a normal allocation law with the following typical values:

Culture	Average Production	Dispersion
A	1 600	180
B	3 200	360
C	2 500	250

In this case, the production of culture A is $1600 + 180k$, where k in most cases (in 997 per cent of the cases) a number ranging between -3 and 3. Equally, for cultures B and C, we have respectively the productions $3200 + 360k'$ and $2500 + 250k''$.

Conditions (1) and (2) remain the same and, function $F(x, y, z)$ which must be maximum, becomes in this case

$$F(x, y, z) = (1600 + 180k)x + (3200 + 360k')y + \\ + (2500 + 250k'')z. \tag{9}$$

The maximum of function $F(x, y, z)$ falls in this case also at one of the tops of the triangle $M\ N\ P$ in Figure 1.

Calculating the values of function $F(x, y, z)$, in each of the points M, P, N, we obtain the following values:

$$F = 2\,320\,000 + 36\,000k + 200\,000k'' \text{ in point } M,$$

$$F = 2\,240\,000 + 180\,000k + 144\,000k' \text{ in point } P,$$

$$F = 1\,600\,000 + 180\,000k \text{ in point } N,$$

resulting thereof that for the maximum to fall in point M, the following conditions must be fulfilled:

$$\left.\begin{aligned} 10 - 9k - 18k' + 25k'' > 0, \\ 90 - 18k + 25k'' > 0. \end{aligned}\right\} \tag{10}$$

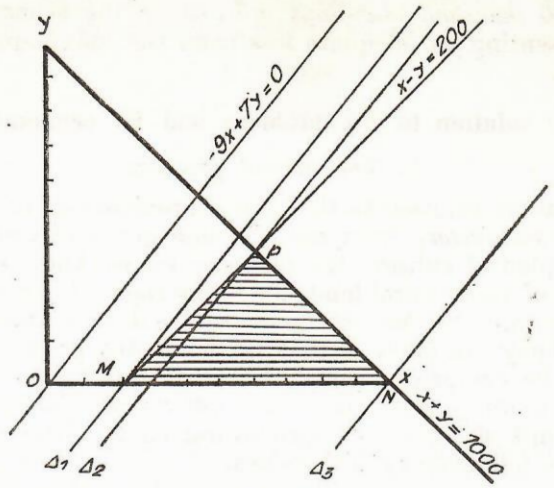


Fig. 1

The maximum falls in P if the following conditions are fulfilled

$$\left.\begin{aligned} 10 - 9k - 18k' + 25k'' < 0, \\ 80 - 9k + 18k' > 0. \end{aligned}\right\} \tag{11}$$

Finally, the maximum position corresponds to point N, if

$$\begin{aligned} 90 - 18k + 25k'' < 0, \\ 80 - 9k + 18k' < 0. \end{aligned} \tag{12}$$

Each of the numbers k, k', k'' follows a normal law with the mean zero and dispersion 1. If we suppose that these laws are independent, a simple calculus of probabilities shows that

$$(10 - 9k - 18k' + 25k'' > 0) = 0 \cdot 5120; \tag{13}$$

$$(90 - 18k + 25k'' > 0) = 0.9987; \tag{13'}$$

$$(80 - 9k + 18k' > 0) = 0.9999. \tag{13''}$$

Formulas (13) (13') and (13'') show that the inequalities (12) are being fulfilled with a probability less than 10^{-6}. In less than one case, for one million the maximum income might correspond to point N (1000, 0, 0). Practically, we reject this solution.

Thus, we have to examine only the inequalities (10) and (11) which correspond to points M and P.

Taking into account that the last two probabilities in (13) are big, we can enunciate the following result: *with a probability equal to 0.51 the optimum solution is $x = 200$, $y = 0$, $z = 800$ and with a probability equal to 0.49 the optimum solution is $x = 600$, $y = 400$, $z = 0$.*

The result we have obtained shows the statistical character of the method of linear programming. We have shown above (the graphical method) that the maximum income is obtained if culture A is applied to 200 units of arable land and culture C to 800 units of t.a. The completion of a statistical

character of this result shows that only in 51 per cent of the cases, the maximum income corresponds to this solution and that in 49 per cent of the cases, the maximum income is obtained if 600 units of t.a. are reserved to culture A and 400 units of t.a. to culture B. The result is explained by the fact that production per one unit of agricultural land is not given by a before established number but by a number ranging, in most cases, within a known interval.

The choice with a certain probability of the optimum solution is thus due to the fact that we cannot envisage from the beginning the exact value of production per one unit of arable land.

It should be noticed that the problem was solved in the hypothesis that productions A, B, C, are *independent* between them. In the case that the exact probability laws of these productions are given, the solution is obtained in the same way, so that the hypothesis made is leading only to eventual calculus simplifications.

We note by α the production for culture A, by β the production for culture B and by γ the production for culture C. Consider $f(\alpha, \beta, \gamma)$ the probability density for variables α, β, γ.

Obviously, we have

$$f(\alpha, \beta, \gamma) \geqslant 0.$$

$$\iiint_T f(\alpha, \beta, \gamma)\, d\alpha d\beta d\gamma = 1.$$

the integral being extended to space T: $\alpha \geqslant 0$, $\beta \geqslant 0$, $\gamma \geqslant 0$.

Function $F(x, y, z)$ the maximum of which we are seeking is in this case

$$F(x, y, z) = (2\alpha - 2000)\, x + (2.58\beta - 3000)\, y + (2.5\gamma - 2500)\, z.$$

$F(x, y, z)$ becomes maximum also in one of the points;

$$M(200{,}0,\ 800),\ P(600,\ 400,\ 0),\ N(1\,000,\ 0,\ 0).$$

Within the values α, β, $F(x, y, z)$ reaches the maximum value in point M, when (α, β, γ) is placed in region T_1 established through the relations

$$2\alpha + 2 \cdot 58\beta \leqslant 5\gamma,$$

$$4\alpha + 1000 \leqslant 5\gamma.$$

The maximum corresponds to point P, in region T_2, determined by the equalities

$$2\alpha + 2.58 \geq 5\gamma$$

$$2\alpha + 1000 \leq 2.58\beta$$

Finally, the maximum is given by point N in region T_3 denoted by

$$2.58\beta \leq 2\alpha + 1000,$$

$$5\gamma \leq 4\alpha + 1000.$$

The probability for the optimum solution to correspond to point M is given by the integral:

$$\iiint_{T_1} f(\alpha, \beta, \gamma) d\alpha d\beta d\gamma.$$

The probabilities concerning the solutions given by points P and N are determined similarly.

The solution to the problem is made by taking into consideration the mean value of function $F(x, y, z)$ the maximum of which we are seeking.

In region T_1, the value of function $F(x, y, z)$ is:

$$400\alpha + 2\,000\beta - 2\,400\,000.$$

In regions T_2 and T_3 the values of $F(x, y, z)$ are respectively

$$1\,200\alpha + 1\,032\beta - 2\,400\,000,$$

$$2\,000\alpha - 2\,000\,000.$$

Consequently, we have to calculate the integrals:

$$\iiint_{T_1} (400\alpha + 2\,000\gamma - 2\,400\,000) f(\alpha, \beta, \gamma)\ d\alpha d\beta d\gamma,$$

$$\iiint_{T_2} (1\,200\alpha + 1\,032\beta - 2\,400\,000) f(\alpha, \beta, \gamma)\ d\alpha d\beta d\gamma,$$

$$\iiint_{T_3} (2\,000\alpha + 2000\,000)\ f(\alpha, \beta, \gamma)\ d\alpha\ d\beta\ d\gamma$$

The optimum solution is given by the highest of the values of the above integrals. Thus, if the first is the highest, the optimum solution is given by M (200, 0, 800).

Generally, the above integrals cannot be calculated directly. In this case, the Monte Carlo method, excellently a statistical method, is used.

B. *The dual problem*

The restrictions of the dual model, for the formulation (1) of the primal problem, under the hypothesis that the productions per unit of surface of arable land are normal random variables can be explained as follows:

$$\begin{aligned} &u + 20v \geq 1\,600 + 180k, \\ &u + 30v \geq 3\,200 + 360k', \\ &u + 25v \geq 2\,500 + 250k'', \\ &\text{any } u - \\ &\qquad v \geq 0, \\ &\hline \\ &1\,000u + 24\,000v = min \end{aligned} \tag{14}$$

If, instead of the first and the third inequality in (14) we consider the case of equalities, we can infer the value of v and u under this hypothesis.

$$\begin{aligned} u + 20v &= 1600 + 180k, \\ u + 25\omega &= 2500 + 250k'' \\ \hline 5v &= 900 + 250k'' - 180k, \\ v &= 180 + 50k'' - 36k. \end{aligned}$$

The value of u also results immediately:

$$u = 1600 + 180k - 20v = 1600 + 180k - 3600 - 1000k'' - 720k =$$
$$= -2000 - 540k - 1000k''.$$

Consequently, the two normal variables sought are

$$\boxed{\begin{aligned} u &= -300 - 540k = 1000\,k'', \\ v &= 180 + 50k'' - 36k. \end{aligned}} \tag{15}$$

If point P is optimum ($x = 600$, $y = 400$, $z = 0$), the values of v and u will be different

$$\begin{aligned} u + 20v &= 1600 + 180k \\ u + 30v &= 3200 + 360k' \\ \hline 10v &= 1600 - 180k + 360k' = 160 - \\ & 18k + 36\ k', \\ u &= 1600 + 180k - 3200 + 360k - \\ & 720k' = -1600 + 540k - 720k'. \end{aligned}$$

Thus, under this hypothesis the two normal random variables are

$$\boxed{\begin{aligned} v &= 160 - 18k + 36k', \\ u &= -1600 + 540 - 720k'. \end{aligned}} \tag{16}$$

We should establish also the interval in which these values will oscillate as well as the probabilistic degree of its exactness.

It is known that if the random variables are independent, the values k, k' and k'' are comprised with a probability of 0,999 in the following interval

$$-3 < k,\ k',\ k'' < 3$$

and, thus the interval for v in (15) will be

$$160 - 18 \times 3 - 36 \times 3 < v < 160 + 18 \times 3 + 36 \times 3,$$

that is:

$$\boxed{-2 < v < 322} \tag{17}$$

with a probability equal to $0.999 \times 0.49 \approx 0.48951$.

With the same probability of 0.48951 we shall have for u in (15) the following interval:

$$-1600 - 540 \times 3 - 720 \times 3 < u < -1600 + 540 \times 3 + 720 \times 3$$

or

$$\boxed{-5380 < u < 1\,180} \tag{18}$$

The interval for v in (14) will be:

$$180 - 50 \times 3 - 36 \times 3 < v < 180 + 50 \times 3 + 36 \times 3$$

or

$$\boxed{98 < v < 418} \tag{19}$$

with a probability of $0.999 \times 0.51 = 0.50849$.
And, the interval for u in (14) is:

$$\boxed{-6620 < u < 2620} \tag{20}$$

How can these results be interpreted from the economic point of view?

The optimum prices offered by the solution to the dual of the stochastic problem are not *certain* values; they are oscillating, with the above shown probabilities, within the limit of variation intervals determined by expressions (16)—(20).

For example, interval (20) shows that the optimum "estimation" of a supplementary unit of surface of arable land, introduced into the economic circuit will not be exactly at the level of an economic effect of 2 500 thousand man-days, just as in case of the determinist solution (8), but, it can be still higher, going up to 2620 thousand man-days, yet it can be lower, going up to maximum losses of 6 620 thousand man-days and, this with a probability of approximatively 0.50 849.

The consideration of such possibilities in the best, planned management of the national economy reflects *a realistic* way of approaching, in the future, the problems of the production process as well as the problems of the real process, with *a statistical* character of the formation of the price value and systems.

REFERENCES

1. DRAGOMIRESCU M., ZIDĂROIU C.: *Modele stohastice pentru determinarea structurii optime a producției agricole.* (Stochastic Models for Establishing the Optimum Structure of Agricultural Production). Revista de statistică 6, 31—41 (1968).
2. DRAGOMIRESCU M.: *An Algorithm for the Minimum Risk Solution of Stochastic Programming.*
3. FEDORENKO N.: *Țena i optimalnoe planirovanie.* In "Kommunist", 8 (1966).
4. GUITTON N.: *Les modes nouveaux de formation des prix. Prix avec et prix sans marché.* Revue d'Economie Politique, 3 (1967).
5. KANTOROVICI L. V.: *Calculul economic al folosirii optime a resurselor.* (Economic Calculation for the Best Use of Resources). Editura științifică, Bucharest, 1966.
6. KANTOROVICI L. V., MAKAROV, V. L.: *Optimalnie modeli perspektivnogo planirovania.* Primenenie matematiki v economiceskih issledovaniiah, 3, Izd-vo, Misl, 1965.
7. LURIE A. L.: *Abstraktnaia model optimalnogo hozeaistvennogo protessa i obiektivno obuslovlennie oțenki.* Economika i matematiceskie metodi, II, fascicula 1, (1966).
8. MIHOC GH. and URSEANU V.: *Matematici aplicate în statistică* (Mathematics Applied in Statistics). Editura Academiei, Bucharest, 1962.
9. MIHOC GH.: *Calcule economice de natură probabilistă* (Economic Calculations of a Probabilistic Character). Probleme economice 10, (1963).
10. I d e m: *Metode statistice-matematice în economie* (Statistical-Mathematical Methods in Economics). În știința-forță de producție. Editura politică, Bucharest, 1964.
11. MIHOC GH., IOSIFESCU M., URSEANU V.: *Elemente de teorie a probabilităților și aplicațiile ei* (Elements of the Theory of Probabilities and Its Applications). Editura științifică, Bucharest, 1966.
12. MIHOC GH., NADEJDE I.: *Programarea matematică vol. I — Programarea parametrică și neliniară* (Mathematical Programming Vol. I — Parametrical and Nonlinear Programming). Editura științifică, Bucharest, 1966.
13. I d e m: *Programarea matematică vol. II — Programarea stochastică* (Mathematical Programming Vol. II — Stochastic Programming). Editura științifică, Bucharest, 1967.
14. NOVOJILOV V. V.: *Izmerenie zatrat i ih rezultatov v sotialisticeskom hozeaistve.* Primenenie matematiki v economiceskih issledovaniiah, Sotekghiz, Moscow, 1959.
15. NOVOJILOV V. V. : *Problemî planovogo țenoobrazovaniia i reforma upravleniia promîșlennostiu.* Ekonomika i matematiceskie Metodi, 3 (1966).
16. ONICESCU O.: *Strategia jocurilor* (The Strategy of Games). Editura Academiei, Bucharest, 1961.
17. URSEANU V.: (under the editorship of) *Elemente de statistică matematică și aplicațiile ei* (Elements of Mathematical Statistics and Its Applications). Editura științifică, Bucharest, 1966.
18. I d e m: *Asupra unor proprietăți ale prețurilor-umbră* (On Some of the Properties of Shade-Prices). Probleme de prețuri, 6 and 7 (1968).
19. I d e m: *Un model al dezvoltării optime a economiei socialiste și dinamica prețurilor* (A model of the Optimum Development of the Socialist Economy and the Dynamics of Prices). Studii de statistică, D.C.S., 1968.

MODÈLES DE L'OPTIMISATION STOCHASTIQUE ET LES SYSTEMES DES PRIX

L'auteur prend comme point de départ l'idée fondamentale selon laquelle dans une économie planifiée le système réel des prix peut et doit être mis en concordance avec le système des prix-ombre résultant de la résolution d'un macro-problème de programmation linéaire, adapté au niveau d'ensemble de l'économie nationale.

Dans ce sens, l'économie est considéré en tant que *système* unitaire dans le cadre duquel l'optimisation de la formation des prix est une composante organique, inséparable de l'optimisations des plans de production. D'autre part, l'auteur développe l'idée que l'obtention d'un système optimum de prix doit être le résultat non d'un modèle de type déterministe, mais d'un modèle stochastique qui, tant par le contenu de l'information qu'il offre, que par la nature des résultats qu'il engendre implique nécessairement des interprétations à caractère probable, statistique.

Ainsi, par la résolution de la dualité d'un problème de programmation linéaire stochastique on obtient un vecteur des prix dont les composantes ne sont pas des valeurs certaines, mais des valeurs qui oscillent, ayant certaines probabilités, dans les limites de quelques intervalles de variation résultant du modèle.

L'auteur estime que c'est justement la prise en considération de pareils systèmes de prix dans la gestion planifiée, optimale, de l'économie nationale qui assure un mode *réaliste* d'aborder, en perspective, les problèmes du processus de production, ansi que les problèmes du processus réel — à caractère statistique — de la formation des valeurs des marchandises et des systèmes correspondants des prix.

La théorie est illustrée par un exemple concret où — afin de simplifier — l'économie en son ensemble est présentée de façon monosectorielle.

МОДЕЛИ СТОХАСТИЧЕСКОЙ ОПТИМИЗАЦИИ И СИСТЕМЫ ЦЕН

Автор исходит из основной мысли о том, что в плановом хозяйстве реальная система цен может и должна быть приведена в соответствие с системой теневых цен, получаемых при разрешении макрозадачи линейного программирования, приспособленного к общему уровню народного хозяйства.

В этом отношении народное хозяйство рассматривается как единая *система*, в рамках которой оптимизация ценообразования является неотъемлемой составной частью оптимизации производственных планов. С другой стороны, автор развивает мысль о том, что получение оптимальной системы цен должно вытекать не из модели детерминистского типа, а из стохастической модели, которая по содержанию предоставляемой информации, а также по природе получаемых результатов обязательно требует толкования вероятностного, статистического характера.

Так, путем разрешения двойственности задачи стохастического линейного программирования получается вектор цен, компоненты которого не являются точно определенными величинами, а величинами, отмечающими колебания — при определенных вероятностях — в пределах интервалов вариации, вытекающих из модели.

Автор считает, что именно рассмотрение подобных систем цен при плановом, оптимальном руководстве народным хозяйством обеспечивает *реалистический* подход в перспективе к вопросам процесса производства, а также к вопросам реального процесса *статистического* характера — образования стоимости товаров и соответствующих систем цен.

Теория автора иллюстрируется конкретным примером, в котором — для упрощения — все народное хозяйство представлено односекториально.

6.VII.1973