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Mathematical Methods for Analyzing Foreign Trade Economic Efficiency

This study deals with the posibility of working out a classified list of export commodities ensuring the economic efficiency, by selecting the commodities on the basis of the calculation of a number of probability indices for each commodity. Taking into account the peculiarities of foreign trade the authors suggest the replacement of the determinist models worked out up to now by parametrized and, afterwards, stochastic models.

The development of international relations, intensified nowadays by a revolution in science and engineering and by increasingly various demands of material and cultural values, has caused trading exchanges — this main form of economic relations between peoples — to extend their sphere and deepen their contents.

Foreign trade enables any country to turn to good account the advantages resulting from a rational participation in the international division of labour, thus allowing some important savings of social labour to be performed in its production process. A most profitable structure — per commodities and per geographical orientation of exchange courses with foreign countries, may favourably affect the progress of national economy. In the last analysis this influence is materialized in foreign trade economic efficiency; a complex notion which, besides its financial and foreign currency aspects (contained in the notion of economic efficiency), embraces many other elements determined by a great number of factors.

The economic efficiency obtained in foreign trade activity is a component of a country's entire economic efficiency. The part played by foreign trade in national economy is bound to increase, provided export and import

structure — per goods and countries — be elaborated so as to ensure the greatest degree of labour saving, on the scale of the whole society. But society's labour cannot be measured in labour-hours, because of the difference between the various productive activities. It is therefore directly expressed in prices, as a concrete expression of the different products values. Natural indicators, not having a common character, can be only partially used. The money expression consequently becomes a most important instrument in estimating foreign trade economic efficiency. This accounts for the particular and growing interest granted, in socialist countries, to economic efficiency — as the expression of money results of foreign trade activity — as well as to the determination of certain indicators which would reflect it most amply and accurately. Although in socialism profit is not the aim of economic activity in different departments, nevertheless, economic efficiency is of great importance, because it reflects the saving degree of social labour.

Foreign trade relations are commodity-money relations in which, as in any other exchange the value law is in operation, with the peculiarity that, commodities being produced by different national economies and exchanged on the international market, they will have both a national and an international value, determined by socially necessary labour expenses, on national as well as world scale. The price oscillates around the value on world scale too. Certain variations may possibly appear between national and international values, owing to the differences between countries as regards natural conditions, technical development, labour productivity etc.

To establish the value, respectively the price of commodities, is a fundamental problem of foreign trade. According to what has been stated above, the prices of commodities, as object of foreign trade operations, are of two kinds. Inside the country, the "export complete price" is formed by adding to the prime wholesale price of the internal supplier the circulation expenses up to the frontier and the expenses and profit of the foreign trade enterprise[1]. But commodities are generally sold (bought) in foreign countries at the prices in use on the market, formed on the basis of commodities international value, under the influence of the "characteristic markets"[2].

By relating the export complete internal price (in national currency) to the external selling price (in contracted foreign currency), both considered "free on board" (F.O.B.), we obtain the indicator known in the practice of our

[1] Import complete price is formed by the equivalent value in lei (at the official rate) of the price in Romanian F.O.B. foreign currency (that is so say that outland circulation expenses are included) to which the inland circulation expenses and the profit of the foreign trade entreprise, are added. The import complete price does not usually correspond exactly to the internal selling price of the respective commodity, for this price is established at the level of, and according to the price policy of the country. Since import complete price cannot be compared to export complete price, it is not used in the economic efficiency calculation; the price of delivery to the beneficiary is used instead, being similar to the one applied to products obtained from internal production as well.

[2] "Characteristic markets" are usually considered those with the biggest transaction volume concerning various products, as for instance: for wheat — Winnipeg, New York, London, Chicago: for cotton-New York, Liverpool, Alexandria, Sao-Paulo, Karachi; for iron ore-New York, London

country as return-rate which can be either below the legal parity (official rate of the respective foreign currency) or above it. For instance:

	Example 1	*Example 2*
Export complete price	600,000 lei	296,300 lei
Price obtained on foreign market	$ 150,000	$ 30,000

$$\text{Return-rate} \quad \frac{600{,}000}{150{,}000} = 4 \text{ lei/\$ 1} \qquad \frac{296{,}300}{30{,}000} = 9.87\text{/\$ 1}$$

Official rate = 6 lei/ $ 1

The ratio can also be written: $\frac{\text{Export complete price}}{\text{Equivalent value in lei of foreign currency}}$

Thus expressed, it is known as "foreign exchange coefficient" which in the first example, is 0.66 lei/1 leu — foreign exchange (leu converted in foreign currency) and in the second 1.61 lei/1 leu foreign exchange.

The return-rate, also known as "indicator of financial and foreign currency economic efficiency" shows, in lei, the internal expenses for obtaining each foreign currency unit, by means of goods exportation.

Adopted as indicator of economic efficiency, with the possibility of being calculated per commodities and groups of commodities, per countries, per foreign trade entreprises and per export or import taken on the whole, the return-rate has not proved yet sufficiently expressive in the estimate of financial and foreign currency results in foreign trade. This is due, no doubt, to the defficiencies of the elements it is obtained from. The internal selling price of exports fails to reflect the social labour contained in each product (since deliveries are effected, as a rule, at prime wholesale prices, the clear profit created by the producing enterprises is only partially included in those prices). Exports delivered by foreign trade enterprises indirectly contain some accumulation quotas, differentiated according to the earlier process stages, the raw material used, the degree of co-operation, etc., all of them bearing a negative or positive influence upon the return- rate.

One must generally bear in mind that the level of export internal prices deviates from the value, as a result of the use of prime wholesale prices and as a consequence of the defficiencies in price formation. This, and other domestic and external factors which are to be taken into consideration (as for instance the production capacities and the available labour power, the necessary quantity of internal consumption, the processing capacity of raw material and the increase of the reserves, the availabilities of orders and supplies on foreign market, the customs policy of partner countries, the position in the balance of payments) entail a number of fairly difficult theoretical, but mostly practical, problems.

In the process of working out methods of research and building up new models with a view to optimize foreign trade operations, three categories of proceedings can be marked out, which constitute at the same time three stages in the analysis of foreign trade economic efficiency:

1) Determination of indicators which we shall call "primary" obtained by "correcting" the relation which the return-rate is derived from;

2) Use of the index method;

3) Application of mathematical methods.

Socialist countries are producing nowadyas a very rich economic literature dealing with the theoretical research and the results scored by applying economic efficiency coefficients (obtained by improving the contents of the return-rate calculation formula)[1]. Among the numerous variants elaborated, we shall point out the "net return-rate", obtained by eliminating the value of imported raw materials, fuel etc., from the price; a similar indicator is in certain countries called "foreign currency equivalent of labour" or "coefficient of foreign economic efficiency of labour". For agricultural produces one uses the term "foreign currency income per hectare". One also tries to take into consideration a certain efficiency coefficient of the investments. Nevertheless, all those coefficients fail to indicate the economic efficiency of producing certain export commodities, and they sometimes happen to bring forth conflicting results.

The index method in analyzing foreign trade economic efficiency is mostly materialized in some countries by the use of the "terms of trade" index, resulting from the relation between export and import price indices, calculated on the same basis. A group of indices has been built on this set-up. They appear in three forms:

1) The "terms of trade" general index, calculated by comparing two price categories: a) actually obtained prices (export), respectively paid (import), and b) obtainable prices (world market prices); the calculation formula is:

$$I_{Rng} = \frac{I_p\ (e)}{I_p\ (i)} = \frac{\dfrac{\Sigma\, p_n q_n}{\Sigma\, p_{int} q_n}\ \text{(export)}}{\dfrac{\Sigma\, p_n q_n}{\Sigma\, p_{int} q_n}\ \text{(import)}}$$

in which:

q_n = delivered quantities (export), respectively bought (import);
p_n = obtained prices (export), respectively paid (import);
p_{int} = world market prices.

This indicator, appearing under the form of a territorial index, reflects, on the whole, the favourable or unfavourable financial results of foreign trade, within a certain period.

2) The "terms of trade" special (territorial) index, characterizing similar results but referring to the trade with one country, or a group of countries, or a particular area.

3) The commodities price index, separately calculated for export and import, and enabling us to estimate the advantages offered by different ranges of goods; it is calculated on the basis of the following formula:

$$I_{Rsm}^{e\,(i)} = \frac{\Sigma\, p_n q_n}{\Sigma\, p_{int} q_n}$$

in which:

p_n = the obtained price (export) or the paid price (import);
p_{int} = the respective commodity price on the international market.

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[1] In our country, the most important contributions to this scope were brought forth by T. Pavel in "Probleme economice" (Economic Problems) No. 5/1957 and in his report at the Vth Scientific Session of the Institute of Economic Sciences, and by I. Craiu: "Probleme ale rentabilității și eficienței economice a comerțului exterior" (Problems of Foreign Trade Profitability and Economic Efficiency), in "Finanțe și Credit" (Finance and Credit) No. 3/1965

The three indices shown above can be of use in evaluating the price evolution in a country's foreign trade, in comparison with the international average prices, and for indicating the prices influence both upon the balance of trade and the direct economic efficiency. But they can't be employed for determining the financial and foreign currency economic efficiency.

We mention the fact that combinations were tried between those indices and the return-rate; thereby, it has become possible to know the advantage of export distribution per different countries (rank indicator), to estimate direct economic efficiency per commodity and draw conclusions with regard to a favourable structure change per commodities; some supplementary possibilities of analysis have been provided as well.

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We try to analyse in this paper the application possibilities of certain mathematical methods in the study of foreign trade economic efficiency; our object lies in the task of reducing social labour consumption by using the advantages of international division of labour.

Among the means used with that end in view, particular attention has to be paid to mathematical programming. Working out models meant to optimize foreign trade activity, and finding the methods for the solution of problems connected to this purpose constitute a steady concern — especially in the Polish People's Republic, the German Democratic Republic, the Hungarian People's Republic and Sweden — remaining, however, a moot question [2], [3] [4], [5].

The problem of determining the efficiency is linked to the possibilities of making a choice. Foreign trade is able to provide these possibilities as far as structure per commodities and geographical orientation are concerned.

Building a foreign trade optimization model thus involves the working out of a classified list of the goods to be imported and of those available for exportation. If the first task is easier to achieve — since it takes into account the national economy development plan and the requirements of our home market — the determination of a classified list of export commodities, meant to ensure a highest economic efficiency of foreign trade, is far more difficult. Difficulties are due to the absence of a unique indicator, unanimously acknowledged and capable of estimating the economic efficiency of a product.

An attempt to remove this difficulty can be made as follows:

For a commodity i a number of indicators can be calculated. They will be marked

$$\alpha_{i1},\ \alpha_{i2},\ \ldots,\ \alpha_{in}$$

Commodities available for exportation (either as finished articles or as semi-products or raw materials) are marked: $p_i (i = 1, \ldots, m)$. The indicators calculated for all these commodities constitute a matrix $m \times n$ dimensioned

$$A_{m,n} \begin{bmatrix} \alpha_{11} & \alpha_{12} & \ldots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \ldots & \alpha_{2n} \\ \alpha_{m1} & \alpha_{m2} & \ldots & \alpha_{mn} \end{bmatrix}$$

For two commodities whatever, p_r and p_s, we can have:
$\alpha_{rj} > \alpha_{sj}$ for an subset of indicators j belonging to the set $N = \{1, 2, \ldots, n\}$, $j \in J_1$, CN being the subset for which this inequality is satisfied; the following relations are also liable to appear:

$$\alpha_{rj} < \alpha_{sj} \qquad \text{for } j \in J_2 \subset N$$
$$\alpha_{rj} = \alpha_{sj} \qquad \text{for } j \in J_3 \subset N$$

Since all indicators are not equally important in estimating the economic efficiency, a system of numbers $\lambda_1, \ldots, \lambda_n$, can be determined and used as weights: $\sum_{j \in N} \lambda_j = 1$. The product of matrix A by column vector $\lambda(\lambda_1 \lambda_2, \ldots, \lambda_n$. is a column vector

$$A\lambda = \left[\sum_{j \in N} \alpha_{ij} \quad \lambda_j \right]_{i \in M}$$

The components of this vector can be regarded as complex indicators attached to commodities $p_1, p_2, \ldots, pm$. Disposing the indicators thus determined into a increasing order, a preference order is established concerning the export of the commodities taken into consideration. The success of the proposed method depends to a large extent on the selection of initial indicators and on the weight system employed.

The elaboration of a classified list of necessary and exportable goods is the first stage in building a foreign trade optimization model. For such a model — valid for as short a period as to allow us to disregard the modifications occuring in the demand and supply, as a consequence of national economy development — the following notations are used:

Q_i; $i = 1, \ldots, m$ — quantity of commodities i available for exportation;

Q_i^{Er}; $i = 1, \ldots, m$; $r = 1, \ldots, t$ — quantities of commodities i requested by foreign market;

q_j; $j = 1, \ldots, n$ — quantity of commodities j necessary on home market;

q_j^{Ir}; $j = 1, \ldots, n$; $r = 1, \ldots, t$ — quantity of commodities j offered on different markets;

d_i^{Er} and d_j^{Ir} — obtainable price, respectively that should be paid for a commodity unit;

l_r, $= 1, \ldots, t$ — balance of payments result with market r.

Variables of the model are:

y_j^{Ir}; $j = 1, \ldots, n$; $r = 1, \ldots, t$ — quantities of imported commodities;
y_i^{Er}; $i = 1, \ldots, m$; $r = 1, \ldots, t$ — quantities of exported commodities.

Restrictions of the model are:

1. Conditions of balance:

$$\sum_{i=1}^{m} d_i^{Er} \, y_i^{Er} - \sum_{j=1}^{n} d_j^{Ir} \, y_j^{Ir} \geqslant l_r; \; r = 1, \ldots, t \qquad (1)$$

Certain 1_r can also take negative values, if it is agreed that some particular balance of payments should be adverse in order to satisfy other major objects of national economy.

2. Conditions ensuring satisfaction of the final demand on home market:

$$\sum_{r=1}^{t} y_j^{I_r} \geqslant q_j ; j = 1, \ldots, n \tag{2}$$

3. Conditions of export limitation:

$$\sum_{r=1}^{t} y_j^{E_r} \leqslant Q_i \quad i = 1, \ldots, m \tag{3}$$

4. Conditions ensuring observance of the demand and supply for all markets and commodities:

$$\begin{aligned} &0 \leqslant y_j^{I_r} \leqslant q_j^{I_r} \; ; \; j = 1, \ldots, n ; \; r = 1, \ldots, t \\ &0 \leqslant y_i^{E_r} \leqslant Q_i^{E_r} \; ; \; i = 1, \ldots, m ; \; r = 1, \ldots, t \end{aligned} \tag{4}$$

A foreign trade optimization model aims at achieving several objects. Some of those are:

a) Maximization of the balance of payments with a particular country or a group of counrties. In this case the object function is:

$$\max. \left(\sum_{i=1}^{m} d_i^{E_r} \, y_i^{E_r} - \sum_{=1}^{n} d_j^{I_r} \, y_j^{I_r} \right)$$

or

$$\max \sum_{r} \left(\sum_{i=1}^{m} d_i^{E_r} \, y_i^{E_r} - \sum_{=1}^{n} d_j^{I_r} \, y_j^{I_r} \right)$$

By totalizing r to the indicators characterizing the group of countries selected, or by using a weight system ensuring a preferential regime to some particular markets, the object function becomes:

$$\max. \sum_{r} \alpha_r \left(\sum_{i=1}^{m} d_i^{E_r} \, y_i^{E_r} - \sum_{j=1}^{n} d_j^{I_r} \, y_j^{I_r} \right)$$

with the supplementary condition $\sum_{r} \alpha_r = 1$

b) maximization of export collections:

$$\max. \sum_{r} \sum_{i} d_i \, y_I^{E_r}$$

c) minimization of import expenses:

$$\min. \sum_{r} \sum_{j} d_j \, y_j^{I_r}$$

The difficulties arising when trying to determine the models concerning foreign trade activity are connected to their dimensions. Indeed, for a classified list of export commodities with 1,000 positions, one of import commodities with 600 positions and a foreign market covering 20 countries, the mathematical model comprises 32,000 unknowns and 33,620 restrictions. It has therefore prohibitive dimensions for many electronic computers.

The assumed dimensions of the above model are quite modest as compared to the situations met with in real practice. In fact, many of the unknown quantities fail to appear in the model, since a particular commodity required on home market is not offered by *all* countries with which trade exchanges are carried out. Also, certain commodities available for exportation are not wanted on *all* foreign markets. On the other hand, the form restriction (4) are superior limitations of the variables, the model being solved by means of special algorithms. Solving mathematical models of great dimensions is a serious concern [8].

Foreign trade activity is carried out under conditions conveying an uncertain or stochastic character to mathematical models, since most of the coefficients appearing both in the restrictions and in the object function are, at the best, stochastic variables.

We find though, in the literature regarding optimizations in foreign trade, that the models used are linear programmings, because of the difficulties arising in the construction of the distribution function of the different stochastic variables included in the model.

In order to get a more accurate reflection of the real situation, we think that the determinist models should be first replaced by parametrized models and afterwards by stochastic ones. We know that parametric programming allows the determination of the parameters critical values and the optimum solutions for each variation interval comprised between two critical values.

For a stochastic model where the demands on foreign market are considered aleatory, the problem may be thus formulated:

To determine

$$\min_{x} \; E \min_{y} \; [\varphi(x) + \psi(y)]$$

with the conditions

$$g(x) + h(y) \geqslant b$$

where x and y are the export and import vectors and b the aleatory vector of the demand with a known distribution. It is demonstrated that in the case of both linear and non-linear programmings, where the functions $\varphi(x)$ and $\psi(y)$ are convex, while $g(x)$ and $h(y)$ are concave, the inequalities are satisfied [9].

$$E\gamma\,[b,\ \bar{x}\,(Eb)] \geqslant \min_{x} E\,\gamma\,(b,x) \geqslant E \min_{x} \gamma\,(b,x) \geqslant \min_{x} \gamma(Eb,x)$$

where we noted:

$$\gamma\,(b,x) = \min\ \{\varphi(x) + \psi(y) / g\,(x) + h\,(y) \geqslant b\}$$

$$\bar{x}\ (Eb) \text{ the solution for } \min_{x}\ (Eb,\ x)$$

These inequalities allow the calculation of the superior and inferior margin of the object-function; those limits give a satisfactory answer.

The use of mathematical methods with a view to optimize foreign trade structure per commodities and countries meets with numerous difficulties as far as long term programmings are concerned. Beside calculation difficulties and the fact that of internal and international prices commodities deviate from their value, there is a series of other essential problems. As for instance

the difficulty of forecasting the foreign trade market demand for even a relatively short period and the modifications liable to appear in forecastings made for several years in the field of home and foreign economy policy. Experimentation of the mathematical models theoretically worked out in different countries has also revealed the serious troubles caused, during the application stage, by inaccurate accounts. In order to determine expressive indicators of the economic efficiency it is imperative that a unique system of accounts should be used in all branches engaged in foreign trade.

The achievement, at a steady rate, of socialist industrialization, the development of all economic branches, the increase of internal consumption have created the favourable and necessary conditions to an ever larger participation of Romania to international relations. In 1964 the foreign trade exchange volume exceeded by 4.8 times the one of 1960. The changes occurring during this period in export and import structure were particularly important too. In 1964 industrial finished articles had a proportion of approximately 50% of the total export of our country, as against 2% in 1938, and over 90% of the total import was formed by complex equipments, machines and tools, raw materials and materials intended to industrial production. During the first five years of the six-year plan our national revenue increased by 54%, the industrial production volume by 99%, and the foreign trade volume by 112%. This fact eloquently shows that in the whole economic activity of our country the weight of international exchanges of commodities has considerably increased.

For the last years, state and party documents have emphasized the necessity of increasing our foreign trade economic efficiency. This task requires an immediate rising of the level of fundamental research work as well as of the applications in this field. It is our opinion that, carried out under the concrete conditions offered by our country, this work must profit by the experience aquired both here and abroad, and have in view a combination of economic efficiency primar coèfficiente with the index methods and with up-to-date mathematical methods — according to the differents aspects of the problem which consists of incressing, in a concrete way, the economic efficiency and profitability of the Socialist Republic of Romania's trade with other countries.

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MÉTHODES MATHÉMATIQUES D'ANALYSE DE LA RENTABILITÉ DU COMMERCE EXTÉRIEUR

Dans le processus d'élaboration des méthodes de recherche et de construction de modèles d'optimisation des opérations de commerce extérieur, on peut distinguer trois catégories de procédés, marquant trois etapes dans l'analyse de l'éfficacité économique dans ce domaine: 1) l'établissement d'indices „primaires" en partant du soi-disant „cours de récurrence" ; 2) l'utilisation de la méthode des indices ; 3) l'applications des méthodes mathématiques.

Après une caractérisation succinte des trois étapes, respectivement des catégories des procédés, l'étude traite la possibilité de déterminer une nomenclature des marchandises d'exportation qui puisse assurer l'efficacité économique, en choisissant les marchandises au moyen du calcul, dans un système matriciel, d'un nombre d'indices de rentabilité pour chaque marchandise. L'établissement sur cette base de la nomenclature constituerait la première étape dans la construction du modèle d'optimisation et les résultats de l'opération dépendent considérablement des indices et du système pondéré utilisé.

Ensuite, en partant de la considération selon laquelle dans les conditions du commerce extérieur, les modèles mathématiques ont un caractère d'incertitude ou stochastique, les auteurs proposent le remplacement des modèles déterministes élaborés jusqu'à présent, par des modèles paramétrisés et ensuite stochastiques.

МАТЕМАТИЧЕСКИЕ МЕТОДЫ АНАЛИЗА РЕНТАБЕЛЬНОСТИ ВНЕШНЕЙ ТОРГОВЛИ

В процессе разработки методов исследования и построения моделей оптимизации внешнеторговых организаций можно различать три категории методов, отмечающие и три этапа работы по анализу экономической эффективности в этой области: 1) определение «первичных» показателей, исходя из так-называемого «курса пересчета»; 2) применение метода индексов; 3) применение математических методов.

После очень краткой характеристики трех этапов, соответственно категорий методов в статье рассматривается возможность разработки номенклатуры экспортных товаров, обеспечивающих экономическую эффективность с отбором товаров на основе калькуляции — в матричной системе — отдельных показателей рентабельности по каждому виду товаров. Установление номенклатуры на этой основе явилось бы первым этапом в построении модели оптимизации и результаты операций зависят в значительной мере от применяемых показателей и системы весов.

Затем, исходя из соображения, что в условиях развертывания внешней торговли предлагаемые математические модели имеют характер неопределенности или стохастический характер, авторы предлагают заменить разработанные до сих детерминистские модел параметризированными и затем стохастическими моделями.