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A Macroeconomic Model of Forecasting the Long—Term Economic Growth

"The social and economic changes taking place at present in the world, require more and more the working out of some long-term developing strategies, the forecasting of the social-economic phenomena — which have become so complex — with a view to substantiating the decisions of economic policy."

Manea Mănescu

Forecasting of the social-economic development of the Socialist Romania, Viitorul social, no. 1, 1972.

In the Socialist Republic of Romania, the party and state leadership attaches special importance to the scientific tackling of problems on the strategy of economic-social development. As it is known, the economic and social transformations which are taking place in the world, today, make ever more necessary the elaboration of long-term prognoses and the substantiation of the decisions of economic policy based on a scientific forecast of the economic-social phenomena and processes.

In this connection, the macroeconomic models of prognosis of long-term economic growth are playing an important part. The increase in the role of the unique national plan necessarily implies intensified preoccupations for long-term prospective studies which aim at forecasting the probable dynamics for a period of 15—20 years and even more, of the national economy, of the economic and social phenomena and processes to take place within it *.

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The author assumes his responsibility for the possible errors within the model.

This work presents, in a general form, a unisectorial model of long-term prognosis of economic growth.

The proposed model can be used both for simulating various possible trajectories of development of the national economy and for establishing some optimum trajectories. In other words, the model can be considered either a simulation model or a dynamic optimization model.

In this work, we shall tackle only the problem of simulation, the problems linked with optimization (the optimum criterion, the establishment of the necessary and sufficient optimum conditions, the qualitative study of the model, etc.) going to be approached in another work.

As *elements characteristic* of the model, we shall mention the following:

a. the model is trying to model "*an open economy*".

As it is known, at present, foreign trade is playing an extremely important part in economic growth. Or, to elaborate a prognosis on the trajectory of development of the national economy without considering this important field of economic growth means to wander too much from the economic reality.

Though conscious of the difficulties arising in case one tries to model the evolution of an "open economy" within the model we are presenting, one tries to distinctly outline the foreign trade activity.

In connection with the foreign trade activity, the following elements are taken into consideration:

— *export* which is divided into two component parts: commerical export (cash) and export on the basis of credit;

— *import* which, on the one hand, is divided into its two components, namely commerical import (cash) and import on the basis of credits received, and on the other hand, it is divided, in keeping with the economic destination, into three components: import for investment in fixed assets, import of objects of labour and import of consumer goods.

The model is also trying to take into consideration the elements linked with the mechanism of the obtaining and granting of credits, the problem of the efficiency of foreign trade, etc.

b. As it is known, an essential element of any economic growth prognosis model is represented by the *production funds*. It is on their dynamics that greatly depends the very dynamics of economic growth. Taking into account the importance of this element, the model including the function of macroeconomic production, distinctly considers the following three components of the production funds:

— active fixed assets (heavy machines and energetic equipment, machines, devices and working installations; measurement, control and adjustment apparatus and installations; means of transport; animals; plantations; tools; production accessories, etc.).

— inactive fixed assets (buildings, special constructions);

— circulating funds (for raw and related materials, fuel and other objects of labour).

This separation of the fixed assets in the sphere of material production was suggested because the rhythm of the increase of labour producti-

vity and consequently the very rhythm of the economic growth greatly depends on the share held by the active fixed assets within the total fixed assets.

It should be mentioned that the share of the machines and installations in the total investments has increased in the Socialist Republic of Romania from 36.7% in 1965 to 38.9% in 1969, without however reaching the level of some advanced countries. For example, in 1968, the share of machiery and equipment in the total investments (excluding house building) was of 68% for the whole of West European countries, whereas in the Socialist Republic of Romania, it was of 41%.

Of course, in comparing the international data, one must take into account the fact that the Socialist Republic of Romania is witnessing a period of intensive industrialization, in which new branches and subbranches of the national economy must be created.

We consider that by the separation of the fixed assets into its two components: active and inactive and, the elaboration therefore of a *macroeconomic production function* which should take this separation into consideration, the economic reality can be better modelled. This also because of the fact that technical progress itself acts differently through the two types of fixed assets.

c. From the mathematical point of view, *the suggested model is a dynamic model with discrete variables* and contains lags both for the state variables and for the decision parameters. The model has *6 state variables and 8 decision parameters.*

The lags appear within the model because of the fact that the time lag existing between the moment of the start of the financing of investment and the moment at which these are materialized into fixed assets entering the economic circuit, is taken into consideration.

In the following, we shall present the economic content of the notions and categories with which we shall work as well as the working hypotheses we shall make.

It should be pointed out that on passing to the effective application of the model, because of the restrictions imposed by the available data stock, the concrete calculus possibilities, etc., another set of simplifying hypotheses will be necessary (for instance, certain functions which within the general model are considered nonlinear functions will have probably to be considered linear, etc.).

1. The added value *created* at stage n, which represents the new created value in the considered period.

In case the volume and structure of the objects of labour allow the complete use of the existing production capacities, the volume of the added value created in each period in depends on:

— the volume of labour in the sphere of material production noted by L (measured in efficiency units);

— the volume of active fixed assets in the sphere of material production, noted by K_n^1;

— the volume of inactive fixed assets in the sphere of material production (buildings, workshops, etc.) noted by K_n^2.

The two categories of fixed assets act differently on the formation of the added value in which the active fixed assets have a prevailing action. At the same time, technical progress acts differently on the two types of fixed assets.

We shall suppose that for each period n, the best volume of inactive fixed assets determined exogeneously to the model is given by the value of function:

$$\widetilde{k}_n^2 = h_n^2(k_n^1,\ L_n).$$

It is noticed that the optimum necessary of inactive fixed assets is put in correspondence with the volume of active fixed assets and with that of labour.

Functions:

$$h_n^2 : \mathbf{R}_+^2 \to \mathbf{R}_+$$

we suppose they are concave in both variables.

In the simplest case, it can be considered:

$$h_n^2\ (k_n^1,\ L_n) = \rho_n^1 \cdot k_n^1 + \rho_n^L \cdot L_n$$

where ρ_n^1 and ρ_n^L are constant data.

As regards the macroeconomic production functions $F_n : \mathbf{R}_+^3 \to \mathbf{R}_+$ which give the volume of the created added value, in the event of the complete use of the production capacities, we suppose they have not following properties:

a) $$\frac{\partial F_n(L,\ K^1,\ K^2)}{\partial L} > 0;$$

$$\frac{\partial F_n(L,\ K^1,\ K^2}{\partial K^1} > 0;$$

$$\frac{\partial F_n(L,\ K^1,\ K^2)}{\partial K^2} > 0; \qquad \text{for } K^2 \leqslant h_n^2(L,\ K^1);$$

$$\frac{\partial F_n(L,\ K^1,\ K^2}{\partial K^2} = 0; \qquad \text{for } K^2 > h_n^2(L,\ K^1);$$

b) all partial differentials of second order are non-positive, in the points in which they are defined.

c) $F_n(L,\ K^1,\ K^2) \geqslant F_{n-1}(L,\ K^1,\ K^2)$.

2. The added value is the total sum between the new created value and the part of the value of the fixed assets which was transmitted on the production.

Noting by:

A_n — the size of the obsolescence fund of the productive fixed assets, we have:

$$V_n^b = V_n + A_n$$

where by V_n^b we noted the added value.

As regards the calculus of the obsolescence fund, we shall suppose that:

$$A_n = h_n^a (K_n^1,\ K_n^2)$$

where functions

$$h_n^a : \mathbf{R}_+ \to \mathbf{R}_+$$

are given.

In the simplest case, we can consider:

$$A_n = a_n^1 \cdot k_n^1 + a_n^2 \cdot k_n^2$$

where a_n^1 and a_n^2 are the sinking coefficients for the active and respectively inactive fixed assets. For each period n, the best volume of *objects of labour (of circulating funds)* which ensure the normal unfolding of production is given by function:

$$K_n^3 = h_n^3(V_n^b).$$

It is noticed that the best volume of the circulating means depends on the volume of the added value.

As it is known, the necessary circulating means depend on a set of factors (the structure on branches of the national economy, the structure of production within each branch, the way of supply etc.) some of which have the tendency of leading to an increase of the need and others to its reduction.

Based on the analysis of these elements, we consider that the resultant of these contradictory actions has as an effect the reduction of the increase of necessary circulating means for an increase by a unit of the added value.

Thus, we shall suppose that functions:

$$h_n^3 : \mathbf{R}_+ \to \mathbf{R}_+$$

are concave.

We shall note:

$$F_n(L_n,\ K_n^1,\ K_n^2) + h_n^a(K_n^1,\ K_n^2) = \bar{f}_n(L_n,\ K_n^1,\ K_n^2).$$

About labour force in the sphere of material production, we suppose it has an exogeneous evolution in relation to the considered model.

The evolution law, in general, is:

$L_{n+1} = (1 + l_n) \cdot L_n$ where L_0 is given and l_n is the growth rate.

If $l_n = l =$ constant, we have an exponential law of evolution of the labour-force.

It results thereof that the endogeneous variables on which depends the size of the created added value are only:

$$K^1,\ K^2 \text{ and } K^3$$

We shall define the *macroeconomic production function:*

$$V_n^b = min\ \{\bar{f}_n(k_n, k_n^1, k_n^2)\ (h_n^3)^{-1}\ (K_n^3)\}$$
$$= \hat{f}_n(K_n^1, K_n^2, K_n^3).$$

3. For the needs of the model we introduce the notion of the *utilizable added value*, this being the sum between the created total added value and the balance between imports and exports in the respective stage.

Thus, the structure of the utilizable total added value according to its way of formation, is:

$$V_n^u = V_n^b + M_n - E_n$$

where:

V_n^u — utilizable total added value
M_n — imports in the period n
E_n — exports in the period n.

We shall assume that as regards the necessary imports for replacing the objects of labour used in the production process (raw and related materials, etc.) they do not intervene in the estimation of E and thus E represents a "net export", respectively the part of the total added value which is exported and N "the net import", that is the total imports less those necessary for renewing the consumed stock of circulating means.

Of course, with this convention, the $M-E$ balance does not change nor does the size of the utilizable total added value.

As it is known, the utilizable total added value has the following destinations:

a) investments for the development, modernization and renewal of the productive fixed assets in industry, agriculture, constructions, transport and in other branches of material production;

b) the supply of the economic units with the additional quantities of raw and related materials, fuel, energy and other circulating means necessary for the unfolding of their economic activity;

c) the ensurance of total resources for the satisfaction of the social needs of consumption of the population. This index which we shall also call *generalized consumption*, according to the definition given by prof. dr. E. Dobrescu, [6] represents the sum of the consumption fund in the added value and of the unproductive investments;

d) the formation of reserves;

e) other destinations.

Thus, the structure of the utilizable total added value, according to its economic destination, is:

$$V_n^u = I_n^f + I_n^c + C_n + \Delta_n + S_n$$

where:

I_n^f — investments for productive fixed assets;
I_n^c — the increase of circulating funds in the sphere of material production;
C_n — total resources meant for the satisfaction of the social needs of consumption of the population;
Δ_n — the variation of the reserve fund in relation to the previous period;
S_n — other expenses.

As regards the generalized consumption C_n, we shall suppose that a certain *minimum level* which has ensured to be compulsorily was established for each stage n of the prognosis period.

Consider:

C_n^1 — the minimum level of the generalized consumption which has to be ensured in the period n

Hence

$$C_n \geqslant C_n^1$$

We shall also suppose that the sizes of indices Δ_n and S_n were exogeneously established for each period of the considered prognosis horizon, based on adequate prognosis models.

We shall note:

$$Q_n = C_n^1 + \Delta_n + S_n$$

Of course, for each stage, the sizes of the index Q_n are given exogeneously in keeping with the necessities of ensuring a minimum consumption level, with the policy of the state on state reserves, expenditure for the defence of the country, etc.

We shall also suppose that a certain part of Q_n must be ensured from import.

Thus:

$$Q_n = Q_n^i + Q_n^m$$

where Q_n^m is the share which has to be ensured from imports and Q_n^i from home production. We suppose that both components are given exogeneously.

As regards relation (*) i can also be written:

$$V_n^u = I_n^f + I_n^c + (C_n - C_n^1) + Q_n.$$

As part of the model proposed, each of the components I_n^f, I_n^c, $C_n - C_n^1$ and O_n will be divided, according to the criterion of source, into two components, respectively the share deriving from the home production and the share deriving from import.

We shall also introduce the following index:

$$V_n^r = V_n^b - O_n^i = \bar{f}(K_n^1,\ K_n^2,\ K_n^3) - Q_n^i = f_n(K_n^1,\ K_n^2,\ K_n^3)$$

We shall call this index (V_n^r) total added value *to be allotted* and it is defined as being the difference between the total added value and the size which is exogeneously allotted from home production.

Finally, *this is, in fact, the index in connection with which economic decisions can be taken on its way of allocation into the share for accumulation, the share for consumption and the share for export.*

The aim of the model suggested is to allow the selection of *the best variant* from the various possible variants of allocation of V_n^r.

4. Based on previous calculations, we suppose that the following shares are known for each period n:

$$\lambda^1_{n-\theta_1,\,n},\ \lambda^1_{n-\theta_1+1,\,n},\ \ldots,\ \lambda^1_{n,\,n}$$

$$\lambda^2_{n-\theta_2,\,n},\ \lambda^2_{n-\theta_2+1},\ \ldots,\ \lambda^2_{n,\,n}$$

where $\lambda^1_{n-\tau,\,n}$ represents the share of the investments made in the period $n-\tau$ which is materialized in active fixed assets becoming effective in the period $n(\tau = 0, 1, \ldots \theta_1)$; $\lambda^2_{n-\tau,n}$ have the same significance, but, they refer to the inactive fixed assets.

We supposed that the maximum lag in the case of investments in active fixed assets is of θ_1 and, in case of investments in inactive fixed assets is of θ_2.

5. For each of the two categories of fixed assets and for each period n a function is given:

$$\bar{g}^i_n\colon \mathbf{R}_+ \to \mathbf{R}_+ \qquad i = 1,\ 2,$$

where $\bar{g}^i_n(K^i_n)$ represents the volume of fixed assets turned ineffective in the period n following the physical and moral wear.

As the length of use, especially for the active fixed assets is not a purely technological parameter, but is to a great extent linked with economic decisions, we shall suppose that the bigger the volume of funds, the better one can take into consideration the moral wear (obsolescence).

Therefore, we shall suppose that function $\bar{g}^1_n(K^1_n)$ has the following properties:

$$\bar{g}^1_n(K^1_n) < K^1_n;\ \frac{d\bar{g}^1_n(K^1)}{dK^1} > 0;\ \frac{d\bar{g}^1_n(K^1)}{d(K^1)^2} \geqslant 0.$$

In other words, we suppose that $\bar{g}^1_n(K^1)$ is a convex function. About $\bar{g}^2_n(K^2)$ we suppose it to be a linear function of the form:

$$\bar{g}^2_n(K^2_n) = \mu^2_n \cdot K^2_n$$

We shall note:

$$g^i_n(K^i_n) = K^i_n - \bar{g}^i_n(K^i_n), \qquad i = 1,\ 2.$$

6. *Foreign trade activity.* We shall note:

M_n — the total volume of imports at stage n;
E_n — the total volume of exports at stage n;
M^1_n, M^2_n — the volume of commercial import (cash), respectively that made on credit (in/lei);
$E^1_n,\ E^2_n$ — the volume of commercial export (cash), respectively that made on credit (in/lei);
x_n — the balance of the credits received and not-repayed until period n (lei — currency)
y_n — the balance of credits granted and not-cashed until period n (lei-currency);

For each period n of the considered prognosis horizon, the following functions are known, on the basis of adequate models;

$$\pi_n^e : \mathbf{R}_+ \to \mathbf{R}_+$$

$$\pi_n^m : \mathbf{R}_+ \to \mathbf{R}_+$$

functions which give the recurrence rate for exports and imports respectively.

If E is the volume of exports, expressed in lei, $\pi_n^e(E)$ shows the volume of value expressed in lei-currency obtained from exports; and if M is the volume of imports expressed in lei, at the invoice price, the foreign purchasing price, expressed in lei-currency is $\pi_n^m(M)$.

Generally, the more the volume of exports increases the bigger the difficulties in selling the products on the foreign market and, hence the necessity of approaching less advantageous markets and of including less profitable products on the export list.

These considerations make us admit the following properties for functions $\pi_n^e(E)$:

$$\frac{d\pi_n^e(E)}{dE} > 0; \quad \frac{d^2\pi_n^e(E)}{dE^2} \leqslant 0.$$

In the simplest case, we can consider:

$$\pi_n^e(E) = e_n \cdot E$$

that is a linear function.

Similar considerations make for functions $\pi_n^m(M)$ admit the following properties:

$$\frac{d\pi_n(M)}{dM} > 0; \quad \frac{d^2\pi_n^m(M)}{dM^2} \geqslant 0.$$

Obviously, here too, the simplest case is that of the linear function:

$$\pi_n^m(M) = m_n \cdot M$$

For simplifying the writing, we supposed that the recurrence rate is the same for both categories of imports and exports respectively (the cash and the credit ones). Of course, the consideration of some different recurrence rates does not change the structure of the model.

We shall also introduce the following notations:

r_n^m — the share of the volume of credits received and not-repayed, which has to be paid at stage n;

r_n^e — the share of the volume of credits granted which is to be paid at stage n;

d_u^m, d_n^e — the interest applied on the credits received, respectively granted;

R_n — currency reserves (in lei-currency) at the beginning of period n;

b_n — the surplus stock of the balance of non-commercial payments. This size is exogeneous to the model.

In period n, we have the following currency availabilities:

$$R_n + b_n + \pi_n^e(E_n^1) + (r_n^e + d_n^e)\, y_n$$

and the currency expenses are:

$$\pi_n^m(M_n^1) + (\gamma_n^m + d_n^m)\, x_n$$

We shall introduce the following *decision parameter:*

w_n^1 — the share of the currency expenses in the total currency availabilities existing in the stage n.

Obviously:

$$W_n^1 \in [0,\ 1]$$

With this, we have the following relation of balance:

$$\pi_n^m(M_n^1) + (\gamma_n^m + d_n^m)\, x_n = W_n^1[R_n + b_n + \pi_n^e(E_n^1) + (\gamma_n^e + d_n^e)\, y_n]$$

from where it results:

$$M_n^1 = (\pi_n^m)^{-1}\{W_n^1[R_n + b_n + \pi_n^e(E_n^1) + (\gamma_n^e + d_n^e) - y_n - (\gamma_n^m + d_n^m)\, x_n]\}$$

For simplification, we shall note the function on the right side by:

$$M_n^1 = \overline{\varphi}_n^{m,1}(E_n^1,\ x_n,\ y_n,\ R_n,\ W_n^1)$$

As regards the volume of imports which can be made on credit, we shall suppose that it cannot surpass a certain extreme limit, which depends, on the one hand, on the volume of commercial import and on the volume of credits received and not-repayed yet, and, on the other hand, on a certain "political credit" on the world market.

Thus:

$$M_n^2 \leqslant \hat{\varphi}_n^{m,2}(M_n^1,\ x_n) + p_n$$

where p_n quantifies the "political credit", which is considered an exogenous size to the model.

For simplifying the writing, we shall note:

$$M_n^2 \leqslant \overline{\varphi}_n^{m,2}(M_n^1,\ x_n)$$

where we noted:

$$\overline{\varphi}_n^{m,2}(M_n^1,\ x_n) = \hat{\varphi}_n^{m,2}(M_n^1,\ x_n) + p_n$$

We also note:

$$M_n^2 = W_n^2\, \overline{\varphi}_n^{m,2}(M_n^1,\ x_n)$$

where:

$$W_n^2 \in [0,\ 1]$$

We noted by W_n^2 the following *decision parameter* — the share of imports, on credit, in the total volume of credits which can be obtained in the stage n.

As regards the volume of exports we make on credit, we shall suppose it is an increasing function in relation to the volume of commercial export.

Consider this function:

$$\overline{\varphi}_n^{e,2} : \mathbf{R}_+ \to \mathbf{R}_+$$

We consider that:

$$\frac{d\overline{\varphi}_n^{e,2}(E_n^1)}{dE_n^1} \geqslant 0.$$

We can write:

$$E_n^2 \geqslant \overline{\varphi}_n^{e,2}(E_n^1).$$

As, in general, we suppose we are not interested in exporting on credit more than the minimum volume imposed by the foreign partners, we shall consider instead of the above relation, a similar relation, however, with a strict equality:

$$E_n^2 = \overline{\varphi}_n^{e,2}(E_n^1).$$

The total import is:

$$M_n = M_n^1 + M_n^2 = M_n^1 + W_n^2\,\overline{\varphi}_n^{m,2}(M_n^1,\ x_n)$$

where

$$M_n^1 = \overline{\varphi}_n^{m,1}(E_n^1,\ x_n,\ y_n,\ R_n,\ W_n^1)$$

and the total export is:

$$E_n = E_n^1 + E_n^2 = E_n^1 + \overline{\varphi}_n^{e,2}(E_n^1).$$

We shall also introduce the following *decision parameter*:

U_n^0 — the share of total exports in the total added value to be allotted in stage n.

Thus:

$$E_n = U_n^0 f(K_n^1,\ K_n^2,\ K_n^3)$$

We have the following equality:

$$E_n^1 + \overline{\varphi}_n^{e,2}(E_n^1) = U_n^0 f_n(K_n^1,\ K_n^2,\ K_n^3)$$

Consider:

$$E_n^1 = \varphi_n^{e,1}(K_n^1,\ K_n^2,\ K_n^3,\ U_n^0)$$

The solution to the above equation. We shall suppose that this solution exists and is also unique:

It results:

$$E_n^2 = \overline{\varphi}_n^{e,2}(E_n^1) = \overline{\varphi}_n^{e,1}(\varphi_n^{e,1}(K_n^1,\ K_n^2,\ K_n^3,\ U_n^0)) = \varphi_n^{e,2}(K_n^1,\ K_n^2,\ K_n^3,\ U_n^0)$$

With this, the commercial import is written:

$$M_n^1 = \varphi_n^{m,1}(E_n^1,\ x_n,\ y_n,\ R_n,\ W_n^1) =$$

$$= \varphi_n^{m,1}(\varphi_n^{e,1}(K_n^1,\ K_n^2,\ K_n^3,\ U_n^0),\ x_n,\ y_n,\ R_n,\ W_n^1)$$

$$= \varphi_u^{m,1}(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^1)$$

and the import on credit is:

$$M_n^2 = W_n^2\overline{\varphi}_n^{m,2}(M_n^1,\ x_n) =$$

$$= W_n^2\overline{\varphi}_n^{m,2}(\varphi_n^{m,1}(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^1),\ x_n)$$

$$= W_n^2\varphi_n^{m,2}(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^1)$$

The total import is:

$$M_n = M_n^1 + M_n^2 = \varphi_n^{m,1}(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^1) +$$

$$+ W_n^2\varphi_n^{m,2}(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^2)$$

We shall note:

$$M_n = \varphi_n^m(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^1,\ W_n^2)$$

and will bear in mind that function $\hat{\varphi}_n^m$ (1) is linear in the decision parameter W_n^2.

We shall now suppose that, on the basis of analyses or adequate prognosis models, the following elements are given exogeneously for each period:

η_n^1 — the minimum share of the investments of period n in active fixed assets which have to be ensured from import;

η_n^3 — the minimum share of the increase of circulating means which have to be ensured from import;

Q_n^m — compulsory import for Q_n;

η_n^c — the minimum share of the additional generalized consumption (which is given above the level C_n^1) which has to be ensured from import.

We shall also introduce the following *decision parameters* :

$U_n^1,\ U_n^2,\ U_n^3$ — the share of accumulation for active and inactive fixed assets, respectively for increasing the circulating funds in the total added value to be allotted in period n.

Of course, $1 - (U_n^0 + U_n^1 + U_n^2 + U_n^3)$ represents the share of the additional consumption (above the compulsory minimum level C_n^1) in the total total added value to be allotted in period n.

For each stage n, the following minimum volume of import noted by M_n^0 has to be ensured compulsorily:

$$M_n^0 = [\eta_n^1 U_n^1 + \eta_n^3 U_n^3 + \eta_n^c(1 - U_n^0 - U_n^1 - U_n^2 - U_n^3)]\ f_n(K_n^1,\ K_n^2,\ K_n^3) + Q_n^m$$

We note:

$$M_n^0 = \varphi_n^{m,0}(K_n^1,\ K_n^2,\ K_n^3,\ U_n^0,\ U_n^1,\ U_n^2,\ U_n^3)$$

We shall call *supplementary import* and will note by M_n^s the size:

$$M_n^s = M_n - M_n^0$$

Thus:

$$M_n^s = M_n - M_n^0 = \hat{\varphi}_n^m(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ W_n^1,\ W_n^2) -$$
$$- \varphi_n^{m,\,0}(K_n^1,\ K_n^2,\ K_n^3,\ U_n^0,\ U_n^1,\ U_n^2,\ U_n^3)$$

We note

$$M_n^s = \varphi_n^m(K_n^1,\ K_n^2,\ K_n^3,\ x_n,\ y_n,\ R_n,\ U_n^0,\ U_n^1,\ U_n^2,\ U_n^3,\ W_n^1,\ W_n^2)$$

We shall introduce the last three *decision* (control) *parameters.*

$V_n^1,\ V_n^3,\ V_n^c$ — the share of import for active fixed assets, for increasing the circulating funds, respectively for the additional consumption in the total supplementary import in period n.

Obviously:

$$V_n^1 + V_n^3 + V_n^c = 1$$

THE VARIABLES OF THE MATHEMATICAL MODEL

For systematization, in the following, we shall again enumerate the variables of the model. As has already been said, the mathematical model we present has *6 state variables* with the aid of which the evolution of the considered system can be described, namely:

$K_n^1,\ K_n^2,\ K_n^3$ — the volume of active, inactive fixed assets, respectively of the circulating funds in the sphere of material production, in period n;

x_n — the balance of credits received and not-repayed until period n (in lei-currency);

y_n — the balance of credits granted and not-cashed until period n (in lei-currency);

R_n — currency reserves at the beginning of period n (in lei-currency);

The *control (decision) parameters* with the aid of which the system can be guided *are eight in number*, namely:

U_n^0 — the share of total exports in the total added value to be allotted in period n;

$U_n^1,\ U_n^2,\ U_u^3$ — the share of accumulation for active, inactive fixed assets, respectively for increasing the circulating funds in the total added value to be allotted in period n;

W_n^1 — the share of currency expenses *for commercial import (cash)* in the total currency availabilities existing in period n;

W_n^2 — the share of imports on credit, in the total volume of credits which can be obtained in period n;

$V_n^1,\ V_n^2$ — the share of imports for active fixed assets, respectively for increasing the circulating funds in the total additional imports in period n;

For simplifying the writing of the mathematical model, we shall use the following notations:

$$K_n = (K_n^1,\ K_n^2,\ K_n^3),$$
$$Z_n = (x_n,\ y_n,\ R_n),$$
$$U_n = (U_n^0,\ U_n^1,\ U_n^2,\ U_n^3),$$
$$W_n = (W_n^1,\ W_n^2).$$

A. Equations of dynamics

A.1. The dynamics of active fixed assets

$$K_{n+1}^1 = g_n^1(K_n^1) + \sum_{\tau=0}^{\theta_1} \lambda_{n-\tau,\,n}^1 [(1 + \eta_{n-\tau}^1)\, U_{n-\tau}^1 f_{n-\tau}(K_{n-\tau}) +$$
$$+ V_{n-\tau}^1 \varphi_{n-\tau}^m(K_{n-\tau},\ Z_{n-\tau},\ U_{n-\tau},\ W_{n-\tau})]$$

A.2. The dynamics of inactive fixed assets

$$K_{n+2}^2 = g_n^2(K_n^2) + \sum_{\tau=0}^{\theta_2} \lambda_{n-\tau,\,n}^2 U_{c-\tau}^2 f_{n-\tau}(K_{n-\tau})$$

A.3. The dynamics of circulating funds

$$K_{n+1}^3 = K_n^3 + (1 + \eta_n^3)\, U_n^3 f_n(K_n) + V_n^3\, \varphi_n^m(K_n,\ Z_n,\ U_n,\ W_n)$$

A.4. The dynamics of the balance of credits received

$$x_{n+1} = (1 - r_n^m)\, x_n + \pi_n^m [W_n^2 \varphi_n^{m,\,2}(K_n,\ Z_n,\ U_n^0,\ W_n^1)]$$

A.5. The dynamics of the balance of credits granted

$$y_{n+1} = (1 - r_n^e)\, y_n + \pi_n^e [\varphi_n^{e,\,2}(K_n,\ U_n^0)]$$

A.6. The dynamics of currency reserves

$$R_{n+1} = (1 - W_n^1)\, \{R_n + b_n + \pi_n^e [\varphi_n^{e,\,1}(K_n,\ U_n^0)] + (r_n^e + d_n^e)\, y_n\}$$

B. Restrictions

For a solution to be admissible, it must compulsorily satisfy the following restrictions:

B.1. *Restriction on the non-negativity of the total added value to be allotted*

$$f_n(K_n) \geqslant 0$$

B.2. *Restriction on the non-negativity of the volume of supplementary imports*

$$\varphi_n^m(K_n,\ Z_n,\ U_n,\ W_n) \geqslant 0$$

The *initial conditions* are known:

$$K^1_{-\theta_1},\ K^1_{-\theta_1+1},\ K^1_{-\theta_1+2},\ \ldots,\ K^1_0;$$

$$K^2_{-\theta_2},\ K^2_{-\theta_2+1},\ K^2_{-\theta_2+2},\ \ldots,\ K^2_0;$$

$$K^3_0$$

$$U^1_{-\theta_1},\ U^1_{-\theta_1+1},\ U^1_{-\theta_1+2},\ \ldots,\ U^1_{-1};$$

$$U^2_{-\theta_2},\ U^2_{-\theta_2+1},\ U^2_{-\theta_2+2},\ \ldots,\ U^2_{-1};$$

$$x_0,\ y_0,\ R_0.$$

UTILIZATION OF THE MODEL FOR SIMULATION

The above-described economic growth model can be used for simulating (generating) various possible trajectories of development of the national economy.

To achieve simulation, various hypotheses can be formulated on the policy of investment, on the policy in the field of foreign trade, in the field of consumption, etc. For each trajectory, various indices (out-put sizes) are to be calculated, on the basis of which various admissible trajectories should be analysed comparatively.

Of course, an important role in the simulation of various trajectories of development of the national economy has to be played by the formulation of various hypotheses on the efficiency of foreign trade. In other words, we consider it proper to adopt as many possible plausible hypotheses on the dynamics of the recurrence trend of imports and exports, respectively, the possibilities for obtaining credits, etc. and to analyses the sensitivity of the trajectories of development of the national economy in keeping with these elements.

Obviously, simulation with the aid of the model does not exclude its transformation into an optimization model.

But, we consider that this has to be made in a subsequent period, after an ever bigger volume of information on the behaviour of various admissible trajectories was obtained by simulation.

We say this because the transformation of the model into an optimization model raises a series of additional difficulties.

Firstly, it is known that although special attention was given in the literature of speciality to the problem of elaborating a criterion of *macroeconomic optimum*, no unanimously accepted result was reached so far. In other words, we can say that we can formulate various criteria of macroeconomic optimum, but, each of them (we refer to concrete quantifications) is disputable from various points of view.

Secondly, we know that the optimum trajectories which, in our case, will be calculated by applying the Principle of Pontreaghin's maximum for discrete systems, are "*vertex solutions*" which often have a "capricious" dynamics from the point of view of economic significance.

Thus, it is known that in case we formulate a criterion of linear optimum for instance, the maximization of the consumption cumulated for the whole

prognosis period, the optimal solution we shall obtain by applying an adequate mathematical apparatus is a "bang-bang" type solution. This means that, for a certain period of time, the optimum solution will show that the entire total added value to be alloted should be assigned to accumulation, after which, in the next period, it should be entirely assigned for consumption.

Such a behaviour, which is unreasonable economically, does not arise from the nature of the economic problem, but, from a series of qualitative properties of the mathematical apparatus used.

Naturally, these arguments should not make us give up the optimization models, but, they only justify the combined utilization of the two categories of techniques.

As regards simulation proper, we consider that one must start with the calculus of "balanced", trajectories, by this meaning those trajectories in which in any of the periods (eventually, except for the former) circulating funds and inactive fixed assets are not accumulated in "excess", or "in deficit".

In other words, for each period, accumulation for circulating funds and for inactive fixed assets is established so that the following equalities should be fulfilled:

$$K_n^2 = h_n^1(K_n^1,\ L_n)$$

$$K_n^3 = h_n^3(\bar{f}_n(L_n,\ K_n^1,\ K_n^2))$$

Of course, besides the family of "balanced" trajectories, it is interesting to calculate also other types of trajectories.

For each trajectory, we consider that the following *output sizes* should be calculated:

— annual total added value

$$Q_n^i + f_n(K_n);$$

— the volume of the total added value accumulated for the entire prognosis period:

$$\sum_{n=0}^{T} [Q_n^i + f_n(K_n)];$$

— the rate of increase of the added value

— annual consumption:

$$\bar{C}_n = C_n^1 + (1 + \eta_n^c)\ U_n^c f_n(K_n) + V_n^c \varphi_n^m(K_n,\ Z_n,\ U_n,\ W_n);$$

where:

$$U_n^c = 1 - (U_n^0 + U_n^1 + U_n^2 + U_n^3);$$

— the volume of consumption accumulated for the entire prognosis period:

$$\sum_{n=0}^{T} \bar{C}_n;$$

— the rate of increase of consumption;

— production accumulation:

$$[(1 + \eta_n^1)\ U_n^1 + U_n^2 + (1 + \eta_n^3)\ U_n^3]\ f_n(K_n) + (V_n^1 + V_n^3)\ \varphi_n^m(K_n, Z_n, U_n, W_n);$$

— rate of accumulation:

$$(1 + \eta_n^1)\ U_n^1 + U_n^2 + (1 + \eta_n^3)\ U_n^3 + (V_n^1 + V_n^3)\ \frac{\varphi_n^m(K_n, Z_n, U_n, W_n)}{f_n(K_n)};$$

— the final situation of the balance of payments:

$$R_T + Y_T - x_T.$$

Of course, the list can be completed from case to case.

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UN MODELE MACROECONOMIQUE POUR LA PRÉVISION DE LA CROISSANCE ÉCONOMIQUE A LONG TERME

Dans cet artide on présente, sous une forme générale, un modèle unisectoriel de prévision de la croissance économique.

Ce modèle a en vue une économie ouverte. Reliés aux rapports extérieurs, on y prend en considérations les importations, les exportations et toute une série d'éléments relevant du mécanisme de créances extérieures.

En ce qui concerne les fonds de production, le modèle prend en considération d'une façon distincte les fonds fixes actifs (machines, outils, etc.), les fonds fixes inactifs (bâtiments, etc.) et les fonds circulants. Au point de vue mathématiques, le modèle est un modèle dynamique à variables discrètes, ayant aussi des retards. Le modèle comporte 6 variables de stock et 8 paramètres de décision.

МАКРОЭКОНОМИЧЕСКАЯ МОДЕЛЬ ПРОГНОЗА ДОЛГОСРОЧНОГО ЭКОНОМИЧЕСКОГО РОСТА

В настоящей статье представлена в общей форме односекторная модель прогноза экономического роста.

Модель имеет в виду незамкнутую экономику. В области внешних связей принимаются во внимание импорт, экспорт и ряд элементов, связанных с внешнекредитным механизмом.

В отношении производственных фондов в модели учитываются раздельно активные основные фонды (машины, оборудование и т.д.), неактивные основные фонды (здания и т.д.) и оборотные фонды. В математическом отношении модель является динамической моделью с дискретными переменными, имея и запоздания. Модель имеет 6 переменных запаса и 8 параметров принятия решений.

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