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About the Characteristics of the Triangulate Table of Inter—Branch Connections

„... great economists had to content themselves with verbal description and deductive reasoning, we can measuring and we can compute. Therein lise the real difference betweeu the post and the present state of economics"

W. Leontief

Shortly after the appearance of the input-output method, the problem of triangulation of the table of inter-branch connections was raised within the preocupations of its theoretical substantiation and application in practice. As W. Leontief pointed out in 1963, towards the end of the fifth decade, Marshall K. Wood, George D. Dantzig and collaborators carried out, as part of the "Scoop Project" of the United States Air Forces, the re-arrangement of the lines and columns in a table of the United States economy so as to reduce to the minimum the necessary calculations for obtaining some numerical solutions. [1, p. 72]. The results of these studies were presented for the first time in the paper of M.K. Wood and H. Burke Horton "An Experimental Dynamic Model of the U. S. Economy" and endorsed at the American Association of Statistics in December 1950 [2, p. 230]. M. K. Wood and M. A. Geisler in another work [3] solving a problem of dynamic programming have permutated the branches until they could notice that the delivery flows in the United States economy were considerably oriented in a single direction and that for obtaining a strictly triangulate table, only the unimportant flows had to be eliminated.

In the quoted works, the point in question is about a triangulation with a purely technical method of arranging and permutating the order of the branches in the table, by which advantages especially as regards calculation are obtained. However; triangulation has a major significance. The dispute has appeared in

the fourth decade whether the social production process was described more exactly by circular delivery flows or by linear flows. The abstract constructions, for example, of the sort of those of Böhm-Bawerk could not give a satisfactory answer. Empiric studies based on ample statistics were necessary. An excellent way for this was offered by the input-output method elaborated by W. Leontief *). It could be noticed from the analysis of some concrete tables that an important number of squares in which are written the flows of products between branches remain unoccupied. As the list of the branches is described in detail, the proportion of the occupied squares is rapidly decreasing. Thus, for example, in the input-output table of the United States of the size 17×17, 97% of the squares are occupied, in the table 85×85, the proportion is of 53% and in the table 188×188, only 29% of the squares are occupied. In the natural value balance of the U.S.S.R. for the 1970—1975 period, of the size 600×600 only about 2 thousand coefficients were counted, that is under one per cent of the $600^2 = 360{,}000$ possible coefficients. [5, p. 325]

It was also noticed that not only there is no symmetry between the mutual inter-branch deliveries, but, that in general the flows have a marked linear character, that is, even in cases that there are mutual deliveries between two branches, the quantity delivered in a direction exceeds several times the quantity delivered in the opposite direction. Naturally, the problem arose to arrange the branches by successive symmetrical permutation so that deliveries in a direction be grouped in a certain part of the table and the deliveries in the opposite direction in the opposite side.

As is known, [6 pp. 27—34], the flows of inter-branch products appear in quadrant I of the input-output table. Because of the fact that only the circulation in a single year of products of the sort of the objects of labour — and not also that of the investment goods — appears in this quadrant, the ampleness and extension of the connection flows are sensibly reduced.

To follow up the explanation, we reproduce the matrix of inter-branch flows (x_{ij}) and the corresponding matrix of the input coefficients $(a_{ij}) = A$:

$$(X_{ij}) = \begin{pmatrix} x_{11} \cdots x_{1j} \cdots x_{1n} \\ \vdots \\ x_{i1} \cdots x_{ij} \cdots x_{in} \\ \vdots \\ x_{n1} \cdots x_{nj} \cdots x_{nn} \end{pmatrix} \text{ and } A = \begin{pmatrix} a_{11} \cdots a_{1i} \cdots a_{1n} \\ \vdots \\ a_{i1} \cdots a_{ij} \cdots a_{in} \\ \vdots \\ a_{n1} \cdots a_{nj} \cdots a_{nn} \end{pmatrix}$$

If the delivery from the producing branch i to consumer branch j, x_{ij} is *direct*, the delivery from the producing branch j to the consumer branch i, x_{ji} is *indirect*. When, between branches i and j there exists both the flow x_{ij} and x_{ji}, their connection is *circular*. The circular flow is symmetrical only as regards position, but, not as regards size. One of the pair-terms surpasses by far the other one.

The inter-branch connections can be systematized according to the sense and intensity of the material flows [7]. In the simplest case, branch B linearly

*) It is interesting to mention, in this connection, that the young W. Leontief in his dissertation "*Die Wirtschaft als Kreislauf*" published in Archiv für Sozialwissenschaft und Sozialpolitik, Tübingen, Bd. 60,3,1928) approaches circularity in the national economy (4, p. 236).

depends on branch A, because B takes supply from A (or A delivers production to B), but, there is no feed-back:

$$A \rightarrow B$$

Thus, we have a flow from A to B, but, not an inverse flow from B to A. In this case, the relation is of linear *dependence.*

If between the two branches exists a relation of circular *interdependence* between A and B there exist mutual, direct deliveries (supplies), flow and inverse flow:

$$A \rightleftarrows B \text{ or } A \overset{\cdot}{\longleftrightarrow} B$$

The interdependence of this type is *direct.* In keeping with the intensity of the flows, we speak of *downstream* or *upstream* direct interdependence.

We agree that branch A (having the number i in the balance) occupies an inferior hierarchial position and branch B (with the number k) a superior position, that is $i < k$.

If the flow from A to B is higher than that from B to A, that is $x_{ik} > x_{ki}$ or

$$A \rightleftarrows B$$

The direct interdependence is *downstream.*

If, inversely, the flow from B to A is higher than that from A to B, that is $x_{ik} < x_{ki}$ or $A \rightleftarrows B$

The direct interdependence is *upstream.*

The interdependence may be also *indirect.* In this case, the flow A to B exists, $x_{ik} > 0$, but, not the inverse flow from B to A, $x_{ki} = 0$. Yet, the circularity of flows is achieved through the third branch C:

$$\begin{array}{ccc} A & \rightarrow & B \\ & \nwarrow \quad \swarrow & \\ & C & \end{array}$$

It is noticed that the indirect interdependence is a particular case of the upstream direct interdependence.

Triangulation consists in the symmetrical re-arrangement (simultaneously after lines and columns), based on a criterion and algorithm, of the branches so that in one part of the main diagonal there be as many of the greater terms of the pairs (x_{ik}, x_{ki}) or (a_{ik}, a_{ki}) as possible, and on the other side of the diagonal, as many of the smaller terms of these pairs as possible.

It can be easily seen that perfect triangulation, that is the bringing of all flows in one part of the main diagonal, is possible only in the case of the exclusive linearity of the interbranch connections. If there are inverse connections (the circularity of flows), triangulation is formulated only as an optimization problem, in the sense of those specified.

A graph with n nodes and with arces oriented in the direction of the inter-branch production flows can be associated with the technological matrix A.

According to the degree of triangularity, the technological matrices A may be classified as follows:

— the matrix of a *general shape* which cannot be brought to a triangular shape, because after any symmetrical re-arrangement of the branches, approximately the same number of positive coefficients is left on the two parts of the diagonal. Corresponding to this type of matrix is the graph of the type of that in Figure 1.;

— the matrix of *an approximately triangular shape* which cannot be brought to a strictly triangular shape either, but, after permutations, the

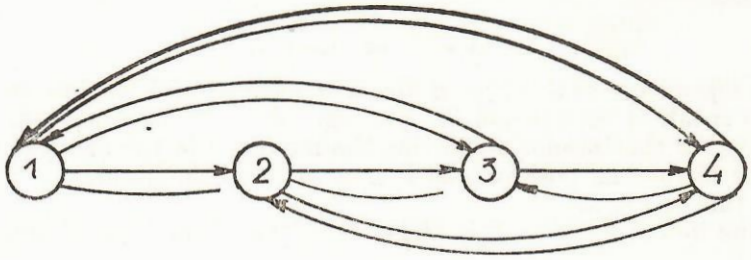


Fig. 1

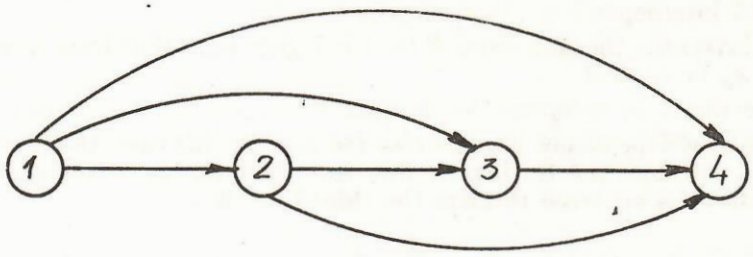


Fig. 2

bigger mass of coefficients is placed on one part of the diagonal and the smaller mass on the other side. In principle, a graph of the type of that in Figure 1 also corresponds to this type of matrix;

— the matrix of a *triangular shape* to which corresponds the graph of the type of that in Figure 2.

It can be easily seen that perfect triangulation, namely the bringing of all flows on one side of the main diagonal is possible only in the case of the exclusive linearity of inter-branch connections.* If inverse connections exist (the circularity of flows) triangulation is formulated only as an optimization problem in the sense of those specified.

A first complex analysis based on the triangulate tables was made by H. B. Chenery and T. Watanabe in the paper "International Comparisons of

*) Generally, if in all the pairs of symmetrical coefficients (a_{ij}, a_{ji}) for $i, j = 1, 2, ..., n$, one of the coefficients is null, this does not mean that the matrix is of a triangular shape.

If the graph associated with the matrix is a contour graph, even if the enunciated condition is fulfilled, the matrix is undecomposable.

the Structure of Production" (Econometrics, vol. 26 (1958), p. 496). The results of the study are taken over and commented in the book of H. B. Chenery and P. G. Clark [2, chapter 8].

They calculate indices $u_j = (\sum_{i \neq j} x_{ij})/X_j$ and $v_i = \left(\sum_{j \neq 1} x_{ij}\right)/X_i$ where X_i and X_j represent the raw product of branches i and j) which shows respectively the share of intermediate consumption and the share of deliveries for the intermediate consumption in the value of the raw product of the branch. According to the use ob production, on the basis of the first index, branches are divided into a) "*primary production*" (for which u_j is small, that is it consumes relatively little from the production of the other branches) and b) *processing branches* (with u_j big, that is consuming relatively much from the other branches) and on the basis of the second criterion, we distinguish c) *final branches* (with v_i small, that is those producing especially for satisfying the final demand) and d) *intermediate branches* (with v_i big and which produce especially for satisfying the intermediate demand). Four groups of branches result from the combined classification according to the two criteria: I-branches which from exogeneous resources produce (especially for the intermediate demand, II — processing branches which satisfy with priority the intermediate demand, III — processing branches satisfying with priority the final demand and IV — branches which from exogenous resources produce especially for the final demand. Statistical estimations from the balances of four countries (Japan, Italy, Norway and the USA) have confirmed, with some exceptions, that the order of branches in the case of optimum scheduling is almost the same. It was uniformity which led the authors to the formulation of the hypothesis on the "natural order of branches". The groups of branches I and III follow this diagram. The situation of group II is different: its branches deliver, to a great extent, the intermediate production and are also processing big quantities of products from the other branches. Most of the circular connections appear in this group.

In an analysis made in 1970 on the structure of the United States economy, Em. Dobrescu [8], to establish the place of the branches in the general economic circuit resorts to a similar classification, the groups of branches are delimited on the basis of the critical value of 0.50 of indices u_j and v_i, the delimitation criteria being:

— for group I, $u_j < 0.50$ and $v_i > 0.50$

— for group II, $u_j > 0.50$ and $v_i > 0.50$

— for group III, $u_j > 0.50$ and $v_i < 0.50$

— for group IV, $u_j < 0.50$ and $v_i < 0.50$

Examining the problems of planning in France, H. Aujac [9], in 1960, came up against similar problems.

F. Helmstädter's works bring sensible progress in the problem of triangulation. In his dissertation [10] notes that a barrier in time appears in the proportionally balanced growth if the production of raw materials and semi-fabs does not outrun, to some extent, the final stages of production. The concrete solution depends on the intensity of mutual (circular) deliveries between

branches. The hierarchy of the production branches is established with the triangulation method. In his subsequent works, F. Helmstädter elaborates the method, the calculus algorithms and makes an important number of international comparisons [4], [11] and [12]. E. F. Baranov elaborates a program for the manual operation of Helmstädter's algorithm [13].

W. Leontief [1] in the structural analysis of the economy distinguishes two pairs of fundamental notions: *dependence — independence* and *hierarchy-circularity*. He visually demonstrates the definition and practical importance of these ideas by a number of input-output tables. Among them is a complete table, in which all coefficients are significant and consequently, a universal interdependence results, in which any change is influencing the entire system. In a table with some squares empty, if an inverse circular connection does not correspond to a direct connection, the two branches are considered independent on each other. "If intersectorial dependence is indicated even by a single square in this table, such a square can give birth to a whole chain of indirect demands, finally implying both members of a pair of sectors apparently independent" [1, p. 73]. In the triangulate system, the hierarchical relation between various branches is clear: the circularity of the flows is eliminated and the indirect dependences are propagated in a direction from the top downstream to the final branches of the triangulate system. Conferring theoretical proportions on this property of the scheduled balance, I. V. Suhotin [14] presents a theory on the organization on stages of the process of material production with all consequences deriving from this theory as regards the propagation of local impulses on the entire economy, the planning of investments and of the proportions between the economic branches.

The studies made in the last few years are focused in two directions. One of the pre-eminently methodological orientation is concerned with establishing the criteria of triangulation and the elaboration of the algorithms for its fulfilment (with the aid of the computer). Worth mentioning in the first direction are the studies of B. Korte and W. Oberhofer [15] and [16]. Part of the second direction are the concrete analyses of comparing the balances in various countries and of examining the levels and economic structures, analyses made by W. Leontief himself [1] or by his collaborators, such as D. Simpson and J. Tsukui [17], or within the United Nations organisms, such as the research owed to J. Lamel, J. Richter and W. Teufelsbauer [7].

DEFINING THE PROPERTY OF TRIANGULARITY OF THE BALANCE OF INTER-BRANCH CONNECTIONS

The property of triangularity of material production was correctly defined from the very beginning, but, not also the concrete criteria according to which an adequate arrangement of the branches can be achieved.

There is no single solution as regards the way of defining the triangulate matrix of the balance. W. Leontief, H. B. Chenery, P. G. Clark, I. V. Suhotin, D. Simpson, J. Tsukui and others consider that the inferior triangulate matrix should be achieved (under the main diagonal); F. Helmstädter, V. V. Kossov, B. Korte, W. Oberhofer, E. F. Baranov, J. Lamel, J. Richter, W. Teufelsbauer and others consider that the triangulate matrix should be superior (above the main diagonal). There is no essential difference between the two solutions:

any of the matrices is the transposition of the other one. Preference may be of a practical character, which of them is more convenient to be used. A triangulate table is solved in several iterations with the Gauss-Seidel method. The method leads to a system of the shape of the superior triangulate matrix. According to sense, the presentation in the form of the inferior triangulate matrix is also justified. The solution in this case begins in a natural order: downwards.

Superior triangulation leads to the hierarchization of the branches according to the number and volume of the delivery transactions and the inferior one, according to the number and volume of supply (consumption) transactions. In the first case, all the dependence connections and most of the direct interdependence ones are concentrated above the main diagonal and the hierarchical succession means the passing from brances producing for intermediate consumption to final branches. The branches with a smaller contribution to the obtaining of the final production but with a big share are placed at the top of economic hierarchy and the branches with a bigger contribution to the creation of the final product occupy the lowest levels. If one accepts the idea of the organization on stages of the process of social production, the inferior triangulate matrix is preferred, the successive processing stages may superpose on line groups of the matrix.

Triangulation may be achieved on the basis of various criteria, such as:

1. the existence or inexistence of the flow between branches,
2. the flows between branches x_{ij},
3. the input coefficients a_{ij}.

There are two types of solutions in approaching these criteria: 1. one operates with all the elements of the matrix, chosen as a basis for triangulation, or 2. the "insignificant" elements are removed and one operates only with the "significant" elements. Starting with the first ample research made by H. B. Chenery and T. Watanabe [2] the most important concrete analyses [1], [4], [7], [13] were made on the basis o the real flows matrix (x_{ij}). The maximization of the sum of flows situated above the main diagonal $\sum_{i}\sum_{j>i} x_{ij}$ (and, at the same time, the minimization of the sum of flows situated under the diagonal) was used as a criterion of triangulation, or the maximization of the sum under the diagonal $\sum_{j}\sum_{i>j} x_{ij}$ (and, at the same time, the minimization of the complementary sum). This approach takes into consideration the totality of flows and avoids arbitrariness in choosing the "significance threshold". Of course perfect triangulation is not obtained; the "trifling" flows between branches exist for various technological or non-technological reasons, especially in not very detailed balances.

D. Simpson and J. Tsukui [17] have triangulated the input matrix (a_{ij}), from which they had previously eliminated, on the basis of criterion $a_{ij} \leqslant (100/n)\%$ the smaller coefficients (n being the number of branches in the balance).

A. Marciniak [18] uses an intersting method of triangulation in analysing the similitude of inter-branch connections in Poland and the European capitalist countries. He builds a matrix in which each coefficient a_{ij} not null is replaced by 1. He thus obtains a matrix A, made up of elements 1 and O (of the Boole type) which means the existence or inexistence of direct interbranch connections. As is known [6, p. 75], through the successive powers of

matrix A, the matrix of indirect connections of various orders is designed, as they propagate through ever larger "concentrates" on the entire system of circular connections.

With the raising to power of matrix A some (or several) of its null elements become not null, namely with the appearance of indirect connections of various levels in those squares where there are no direct connections. Matrix D of the same size as A, whose elements will be the exponents of power for which the null elements in A become for the first time not null, is introduced in the analysis. From the elements of this matrix, by generalizing the notion of density the index:

$$0 \leqslant \frac{1}{n^2} \sum_i \sum_j \frac{1}{d_{ij}} \leqslant 1$$

is built, whose value increases with the intensification of the interbranch connections. Besides the property of being the mean of the partial indices built on branches, similarly with the above relation, the index gives the synthetical extent of the development of the inter-branch connections and consequently, it can be used for international comparisons.

As a somehow unilateral way of approach and consequently with a limited cognitive value, the triangulation made on the basis of the Boole type matrix of the existence or inexistence of inter-branch connection may however be preferred in certain circumstances. The triangulation of such a matrix is achieved relatively easily with the method used by J. N. Warfield [19].

The establishment of the connections—without taking into account their importance — is made directly. The identification of the propagation of indirect connections is facilitated through the relative easiness of raising to power the basic matrix.

The elimination of the "insignificant" connections can also be justified. In a developed economy, with a high level of complexity and diversification, with a network of multiple and intense connections, the elimination of the flows or of the insignificant coefficients may facilitate triangulation and the grasping of the essential connections. D. Simpson and J. Tsukui have used for this the afore mentioned criterion $a_{ij} \leqslant (100/n)\%$. The following objections may be expressed in connection with this solution: 1) the input coefficients characterize only the connections on the line of supply and consumption (that is the connections which usually are called inverse connections), but, they say nothing about the delivery connections (direction connections); 2) the comparison of the coefficients to $100/n$ is not conclusive because the sum total $(\sum_i a_{ij})\%$ for each branch being different, they cannot represent a common basis for comparison.

However, a more consistent "significance threshold" may be offered for eliminating the insignificant connections. In case of perfect, "entropic" diffusion of the production connections, the total volume of the intermediate consumption $\sum_i \sum_j x_{ij}$ is uniformly distributed on all celles of the table. As selfconsumption x_{ij} (for $i = j$) behaves indifferently in the process of triangulation (in any order of the branches, selfconsumption is situated on the main diagonal), usually, the diagonal elements of matri (X_{ij}) or A, are considered null. Thus,

the possible number of cells that can be filled by the interbranch flows is $n\,(n-1)$, and the equally distributed quantity is $\left(\sum_i \sum_j x_{ij}/[n(n-1)]\right)$. Thus, the adequate threshold for eliminating the insignificant flows will be:

$$x_{ij} \geq \left(\sum_i \sum_j x_{ij}\right)/n(n-1).$$

The experience of triangulation shows that in an economy, no matter how developed, there are interdependences and circular flows of products between branches. Thus, a perfect triangulation, the turning of all connections into linear dependences, cannot be achieved in statistical balances. This means that through the very intrinsic character of the economic system, the matrix of flows (X_{ij}) and the matrix of input coefficients A are undecomposable, that is they cannot be brought to the triangulate or quasi-triangulate shape. Consequently, the criterion of triangulation must be formulated so as to take into consideration this property of the matrix. Besides, the fact should also be taken into account that a certain hierarchical order of the branches may appear in the empiric tables, outlining a polarization of connections, a concentration of the inter-branch flows.

Choosing the triangulation of the matrix of flows (X_{ij}) in the form of superior triangulation, the criterion of triangulation is formulated as an optimization problem, namely a symmetrical re-arrangement of the branches so that a maximum sum of flows above the main diagonal is obtained:

$$max \sum_i \sum_{j>i} x_{ij}$$

and a minimum sum of those placed under the diagonal

$$min \sum_i \sum_{j<i}$$

Among the multiple problems of principle or of methology of triangulation, we should dwell on the way of approaching imports. At first sight, it seems that being a question of structure of the national economy, imports should not be included in the basic matrix of inter-branch connections (x_{ij}). But, for "quenching the appetite" of consumption, conditioned by the applied technology and the fabrication processes, no distinction is made as regards the internal or external source of the necessary materials. The technological structure is outlined on the basis of complete flows (including imports). The incidences of the analyses or plans on the necessary import-export materials may be determined with various methods, for example, with the method used by W. Leontief [1].

Real progress has been scored as regards the techniques of achieving the triangulate form of the balance of inter-branch connections. In the first work quoted by us, in which triangulation is achieved [3] no specifications are made in connection with the method used. H. B. Chenery and T. Watanabe use, however, an empiric method of scheduling according to the value of indices. The methods used by H. Aujac and D. Masson [24] also have an approximate character. For the first time in E. Helmstädter's works, the problem of triangulation is formulated as an optimization problem in the sense already shown. The optimization techniques owed

to E. Helmstädter, E. F. Baranov, K. Hellmich [23] have given correct solutions, even if not always the most efficient and rapid ones. B. Korte and W. Oberhofer [16] present a number of exact algorithms of triangulation, such as: linear programming in whole numbers elaborated by W. Krelle, the Branch-and-Bound method, dynamic programming, the retrieval lexicographical algorithm, the modified Jacobi method, the triangulation with the square allocation method. The obtaining of the algorithms is made in electronic computers. The algorithms elaborated within the Computing Centre of the State Planning Committee, the results of M. Capătă and I. Niculescu and those of Al. Taşnadi and V. Dobrescu of the Faculty of Economic Cybernetics and Statistics are known in our country.

UTILIZATIONS OF THE TRIANGULATE INPUT-OUTPUT BALANCE

At the beginning when the calculus of the variants, on the basis of the square matrix of the input coefficients was difficult, because of the insufficient development of the computing technique, the calculating effects of triangulation came out in the forefront. Indeed, on the basis of a triangulate table in a few iterations (or, in any case, after a limited number of iterations), variants can be calculated or the effects of the change in various structural elements can be established.

The calculus of the production levels in a triangulate system is simplified to the maximum:

$$X_i = \sum_{j=i+1}^{n} a_{ij} X_j + Y_i \ (i = 1, 2, ..., n)$$

The triangulate matrix corresponds to a strictly recurrent system, is decomposable and also nilpotent. In this case, the serial development of Leontief's inverse

$$(I - A)^{-1} = I + A + A^2 + ... + A^k$$

reveals some interesting properties. All matrices $(I - A)^{-1}$, A, A^2, ..., A^k are triangulate. Besides, the elements of the indirect costs matrices $A^{(1)}$, $A^{(2)}$, ..., $A^{(k-1)}$ become even smaller, turning — with the propagation of indirect effects — into null elements. The number of the order of direct costs is finite.

The property of undecomposibility is of a qualitative character, more exactly topological and, consequently, it depends not on the size but on the way of allocation of elements x_{ij} in the table. As it was seen before, the allocation of the flows in the table depends, to some extent, on the method of composition of the balance, especially on the classification of branches. Thus, undecomposability, is not a "pure absolute" property of the economic system, but, rather a relative, empiric and methodological one.

If matrix (x_{ij}) is undecomposable, each branch is, directly or indirectly, interdependent on all the other branches. In other words, any pair of branches is a reciprocal interdependence. Hence:

1. all the coefficients of the integral costs are positive and bigger than the directly corresponding coefficients $c_{ij} > a_{ij} \geqslant 0$

$$(\text{for } i, j = 1, 2, ..., n);$$

2. any coefficient of the integral costs depends on all direct coefficients and the change even in a direct coefficient a_{kl} changes all coefficients $c_{ij}(i, j, k, l = 1, 2, ..., n)$;

3. the coefficients of the indirect costs of a certain cycle are by all means positive (there is a number $s \leqslant n$ for which $A^{(S)} = A^{(S+1)} > 0$).

On the contrary, if the matrix is decomposable, certain pairs of groups of branches are only in a multilateral direct or indirect connection. There are always pairs of coefficients (a_{ij}, a_{ji}) and (c_{ij}, c_{ji}), where only one of the elementes is positive.

It can be easily demonstrated that in a triangulate system, there exist the relations:

$$b_{ij} = 0 \text{ for } i \geqslant j,$$

$$b_{ij} = 1 \text{ for } i = j$$

$$b_{ij} = \sum_{k=i+1}^{n} a_{ik} b_{kj} \text{ for } i < j.$$

Thus, the coefficients of the integral costs depend only on those direct coefficients which are found on their right and on the next lines of the table.

Even if the matrix is not perfectly triangulate, approximate numerical solutions can be obtained [17]. If the technological matrix is A(with null diagonal elements), two perfectly triangular matrices can be defined: A_1 made of the elementes placed above the the main diagonal and A_2 made up of the elements under the main diagonal, so that:

$$A = A_1 + A_2$$

The matrical equation known of the allocation of production

becomes, $$X = AX + Y$$

hence $$X = A_1X + A_2X + Y$$

therefore: $$Y = (I - A_1)\, X - A_2X \qquad (1$$

The approximate solution of the vector of the final product can be obtained only on the basis of the superior triangulate matrix A_1 neglecting matrix A_2:

$$\tilde{Y} = (I - A_1)\, X \qquad (2)$$

The approximate calculus error is $\tilde{Y} - Y = A_2X$. Thus, for this problem the minimization of the error is equivalent to the minimization of product A_2X.

The approximate calculus of the vector of the raw product $\widetilde{X}$, by taking into consideration that from (1) results

$$(I - A_1)\, X = A_2 X + Y$$

$$X = (I - A_1)^{-1}\,(A_2 X + Y)$$

can be made only on the basis of matrix A_1:

$$\widetilde{X} = (I - A_1)^{-1}\, Y \tag{3}$$

The calculus error:

$$\Delta X = \widetilde{X} - X = (I + A_1 + A_1^2 + \ldots)\,(A_2 X + Y) - (I + A + A^2 + \ldots)\, Y =$$

$$(I + A_1 + A_1^2 + \ldots)\, A_2 X = (I - A_1)^{-1} A_2 X = A_2 Y$$

is non-negative, because A_1, A_2, Y, are all non-negative. If $A_2 = 0$, the calculus is not affected by errors. This, however, implies the possibility of perfect triangulation. Since, this cannot be achieved, the calculus made on the basis of the triangulate matrix is approximate. The degree of approximation depends on the sum of the elements of vector $A_2 X$, and in the second case also on the size of the elements of the inverse matrix $(I - A_1)^{-1}$. Thus, the two submatrices A_1 and A_2, but, also the vector of the approximate final product $\widetilde{Y}$ appear as sources of error. The approximate calculus made on the basis of the triangulate matrix are better proved scientifically if the "insignificant" flows are considered conventionally constant costs in the extended vector of the free terms of the system.

The major values of the triangulate table do not consist, however, in advantages as regards calculus. They represent real posibilities of analysing the economic structure, the hierarchy of the branches in the succession of the stages of social production, of the volume and intensity of interpendences in the economy.

The concentration in the triangulate table of flows is often achieved in the form of compact groups of branches. Indeed, the most general form of technological matrices is the quasitriangulate matrix:

$$A = \begin{pmatrix} A_{11} & ** & \ldots & * \\ 0 & A_{22} & \ldots & * \\ \vdots & \vdots & & \\ 0 & 0 & \cdots & A_{kk} \end{pmatrix}$$

where:

A_{11}, A_{22}, ... A_{kk} are undecomposable diagonal square blocks of matrix, blocs of the matrix in which the elements may have any value (null or non-zero).

A number of experiments show that the real matrices A may be arranged so as to assume approximately a quasitriangular shape. The blocks, especially the diagonal ones, cumulate a big share of the flows. The possibility of "approaching" on blocks allows the detailed description of the list of branches until

the separate outlining of the main products, especially of those of the kind of the objects of labour.

The analysis of the scheduled tables in which blocks of homogenous branches were formed, such as those of W. Leontief [1] and Em. Dobrescu [7], has allowed the formulation of remarks and conclusions both interesting and important.

In a recent study made as part of the "Dolj Experiment", P. Andre has succeded in identifiyng in the agriculture of the county several blocks of products kindred as regards production technology. The blocks may be considered "modules" of the system from the blending of which the whole system can be built.

Another aspect related to the utilization of the triangulate balance in the optimization calculus should also be underlined. Due to its synthetical character, the balance of interbranch connections, at least in the present stage of development of the macro-economic optimization calculus, allows the correlation and equilibration of various investigations of partial optimization (at a branch level or on groups of branches). For many reasons, it is hardly probable that the input-output method or another method of multisectorial macroeconomic optimization can give satisfactory solutions for the substantiation of the plans of economic-social development. It seems more reasonable to us to approach, at the beginning, the optimization of the development of sectors, branches or top branch groups. The method of partial optimization is dwelt on, at length, in the literature of speciality, for example B. D. Belkin [20]. The connection to the whole is ensured through the equilibration rendered by the internal logic of the balance. The techniques which may be developed on the basis of the solutions proposed by A. G. Aganbeghian, A. G. Granberg and K. A. Bagrinovsky [21], [22], may give full satisfaction.

The triangulate table offers new possibilities for characterizing the hierarchical construction of the inter-branch structure of the national economy.

As it was pointed out, the stastistical studies have revealed a well defined similarity in the order of the branches in the triangulate balances of various countries at a similar level of economic development, especially for the highly developed economies, offering a hierarchical model of the inter-branch transactions. This similarity is put on account of the complete "cookery book" on the modern technology of the identity of fabrication recipes [1, pages 71 and 76]. To measure the degree of correspondence between hierarchies * the methods of order statistics (of Spearman's coefficient, etc.) are used. Though these indices bring valuable information, they cannot be considered as perfect. The indices are constructed only on the basis of the order number in the hierarchy of branches, disconsidering however the volume of the branch production or the volume of inter-branch transactions. The similarity of the hierarchical or structural order is characterized with the classical methods of the analysis of correlation (the correlation coefficient between the vectors of the final product, between the vectors of the raw product, between the elements of the technological matrix, etc.).

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*) Hierarchy has a different meaning here. The order of branches is not uni-or-bi-dimensional but results from optimization on the basis of a general criterion.

To characterize "the degree of organization" of the economy (the concentration or dispersion of flows between branches), the enthropy or Onicescu's information energy is used. A smaller enthropy and a bigger information energy correspond to an increased possibility of triangulation. Thus, "the degree of triangulation" may be characterized through the indices of information statistics.

The measuring of the complexity of the economic structure remains a problem still insufficiently elaborated.

However, a number of indices characterizing the dependence-inter dependence, respectively linearity-circularity in the economy can be defined.

Some auxiliary sizes are calculated at the beginning:

$$T = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}$$

$$V = \sum_{i=1}^{n} \sum_{j>i=1}^{n} x_{ij}$$

$$R = \sum_{j=1}^{n} \sum_{i>j=1}^{n} x_{ij}$$

$$\overline{V} = \sum_{i} \sum_{j} [max(x_{ij}, x_{ji})]$$

$$\overline{R} = \sum_{i} \sum_{j} [min(x_{ij}, x_{ji})]$$

$$D = \sum_{i=1}^{n} \sum_{j>i=1}^{n} | x_{ij} - x_{ji} |$$

between which the following relations exist:

$$T = V + R = \overline{V} + \overline{R}$$

$$\overline{V} = \frac{\overline{I} + D}{2}$$

The degree of linearity is calculated for measuring the dependence or interdependence in an economic system. It results from those discussed above that the sum of the flows above the main diagonal in the triangulate table V has to be taken into consideration and compare it to the sum of the flows on the entire table T or to the sum of flows under the diagonal R.

For several reasons, *the degree of absolute linearity* is calculated first:

$$\lambda^* = \frac{\overline{V}}{T} 100$$

The sum V of each pair of symmetrical flows (x_{ij}, x_{ji}) cumulates the bigger term. Thus, for an undecomposable matrix, the sum $\overline{V}$ is the biggest that can be achieved above the main diagonal following triangulation. The degree of absolute linearity λ^* as well as the sum $\overline{V}$ can be calculated without

triangulation. Both sizes are some limits which can be approached by the empiric sizes but which they can probably never attain.

The result of the comparison λ^* is a specific weight and shows the ideal degree of linearization of interdependences in an input—output table. In a perfect triangulating table $\lambda^* = 1$. Thus, before starting triangulation the value of λ^* already offers an image about the possibilities and limits of triangulation.

The degree of linearity can be calculated on the basis of the triangulate table:

$$\lambda = \frac{V}{T} 100$$

achieved as a result of the solution, namely the optimization of the triangulation problem. Index λ has as a superior limit the degree of absolute linearity λ^* that is $\lambda \leqslant \lambda^*$. A high value of λ shows a high degree of linearity, connections of preponderant dependence and a weak interdependent structure and a low value — an inverse situation: close interdependences, a marked circularity of inter-branch connections. Thus, the degree of the approach of λ to λ^* shows the degree of success of triangulation, it is a sort of measure of the success of the operation.

The degree of partial linearity can be calculated for each branch separately:

$$\lambda_i = \left(\sum_{j>i} x_{ij}\right) / \left(\sum_{i>j} x_{ij}\right)$$

namely, as a relation between the sum of deliveries to next branches in the triangulate hierarchy $\sum_{j>i} x_{ij}$ and the sum of flows received (supplies) from these branches.

The degree of general linearity can be calculated, on their basis, as a ponderal aggregated mean

$$\bar{\lambda} = \sum_i \lambda_i g_{ji}$$

If the weights g_i of the branches are chosen correspondingly, a new expression is obtained for the index:

$$\bar{\lambda} = \frac{V}{R} 100$$

now showing the relation between the sum of the flows above the diagonal and the sum of those under the diagonal of the triangulate table, but, in essence, its concrete sense is the same as of λ.

It results from the studies made by E. Helmstädter [12], on the basis of the input-output tables of nearly 20 countries (with 10 ... 30 branches) that in the developed economies, the degree of linearity is between 80 ... 85%. Generally, the sum of deliveries exceeds 4—5.5 times the sum of receipts (supplies). These data show a marked linearity and a relatively weak circula-

rity even of the developed economies. The less complex and diversified, the under-developed or developing economies are characterized by an ever greater degree of linearity and by weaker interdependences.

The complement up to 100 of the degree of linearity is called *degree of circularity.*

$$\xi = 100 - \lambda$$

Besides these basic indices, a number of secondary indices of the triangulate table can be built.

The *spread ratio "s"* illustrates the degree of linearity of the inter-branch flows as well. A high value of the coefficient shows a high degree of linearity. Several possibilities are given for calculating the coefficient:

$$S = \frac{\overline{V} - \overline{R}}{T} 100 \quad \left(\text{obviously: } \lambda^* = \frac{S + 100}{2}\right)$$

or

$$S = \frac{D}{T} 100$$

Similarly with the latter, *the spread ratio on branches* are calculated:

$$S_i = \frac{D_i}{T_i} 100$$

where:

$$D_i = \sum_{j>i} | x_{ij} - x_{ji} | \quad \text{and} \quad T_i = \sum_j x_{ij}$$

Finally, we should also mention that as the empiric input-output tables contain so called contradictions, *the contradiction coefficient* is also calculated.

In the triangulate matrix, the branches occupy such a position that the deliveries to the next sectors should be bigger than the supplies coming from the respective branches. It means that the difference of the flows between two branches in general, must be positive:

$$d_{ij} = x_{ij} - x_{ji} > 0 \text{ for } j > i$$

In matrix (d_{ij}), only positive elements should exist above the main diagonal. However, negative differences exist as well. The sum of the absolute values of these elements W related to the sum of the elements of the matrix of differences, D, gives the coefficient:

$$W = \frac{W}{D} 100$$

Of particular importance are the conclusions drawn from the analysis of the triangulate input-output tables.

A first remark which has resulted from the many concrets analyses is that concerning the similitude of the economic structures. W. Leontief conspicuously points out that "expounded in the input-output table, the model of transactions between branches and other main sectors of the system shows that, the more developed an economy is, the more its internal structure resembles that of other developed economies. Moreover, the relations between these internal transactions and the total external activity of the system, real gear connections, to the effect that they are determined, to a great extent, by technology prove to be relatively the same from an economy to another". In exchange, the tables of the developing countries are smaller and incomplete in comparison with those of the developed economies [1, p. 68].

The ample analysis made in 1972 as part of the United Nations MATHECO programme, on the basis of the input-output tables of 17 socialist and capitalist European countries and the U.S.A. [18] have fully confirmed W. Leontief's findings.

Statistical data have confirmed the hypothesis on the relative stability of the hierarchical order of branches. Certain branches in international comparisons occupy a stable place. In 8—10 years, the order of branches in the triangulate balance of the same countries has registered unimportant modifications [25]. Probably, at a certain level of development, essential changes should take place in technology for an overturn of the hierarchical order of branches to occur.

As a matter of fact, the planning of changes in the structure of a developing economy, from the point of view of the basic components of internal consumption and of investments, exports and imports, is possible on the very basis of the input output analysis [1, p. 89].

Generally, the existence of an inverse correlation between the degree of linearity and the degree of aggregation of the balance results from concrete studies. The recent studies made by E. Helmstädter [25] reveal certain data contradicting this thesis. Of course, a definitive answer will be given by subsequent studies based on more solid statistics.

Of course, the problem arises over of the connection between the degree of economic development and the intensity of the inter-dependence of branches. The requirements of the law on the increase of productivity impose the continuous deepening of the division of social labour. This process may lead to the setting up of branches which simultaneously turn out the divided products and which considerably surpass the productive circuit. In exchange, the division of production into several products or distinct processes as part of which the same raw material is processed in succession and the transfer of these products to some specialized production units increase productive consumption and the circulation of products within the sphere of material production. The above-mentioned study [7] has shown that the economic development seems to be accompanied by the intensification of economic interdependence, by the growth of inverse flows in the inter-branch connections. At the same time, however, the upstream indirect interdependences seem to have a diminishing effect on the efficiency of social production and on the index of raw social product per head of the population. Of course, the explanation based on arguments of this surprising situation surpasses the framework of our study.

It should also be mentioned that the triangulation of input—output balances is a valuable instrument in comparative analyses. Its efficiency, however, is greatly conditioned by the methodological problems on the obtaining of normalized tables. Many "perturbations" occur in this operation owing to the differences in elaborating the methodology of the creation of basic balances.

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A PROPOS DES VERTUS DU TABLEAU TRIANGULE DES ÉCHANGES INTERINDUSTRIELS

On définit les liaisons — directe, indirecte et circulaire entre les flux qui existent d'une branche à l'autre, ainsi que la dépendance linéaire, l'interdépendance circulaire et celle directe, en aval et en amont. On met en relief l'essence de la triangulation parfaite, après quoi on présente les principales tentatives d'analyse complexe utilisant certains tableaux triangulés.

L'auteur examine par la suite les différents critères concrets selon lesquels un arrangement adéquat des branches peut être réalisé, relevant tant les vertus de ces critères, que leur limites.

Se référant aux techniques utilisées pour réaliser la forme triangulée du modèle entrée-sortie, l'auteur fait ressortir les succès remportés, ainsi que le stade atteint.

L'auteur présente, également, nombre d'utilisations du modèle triangulé des échanges interindustriels, tout en relevant entre autres, les possibilités d'effectuer des analyses complexes, de calculer des variantes ou de déterminer les effets du changement des différents éléments de la structure, de se servir du modèle triangulé dans les calculs d'optimisation, de caractériser la construction hiérarchique et la structure par branches de l'économie nationale, du „degré d'organisation" de l'économie.

О ХАРАКТЕРИСТИКАХ ТРИАНГУЛЯРНОЙ ТАБЛИЦЫ МЕЖОТРАСЛЕВОГО БАЛАНСА

Определяются прямые, косвенные и круговые связи в потоках между отраслями, а также линейная зависимость, циркулярная взаимозависимость, непосредственная взаимозависимость снизу вверх и сверху вниз. Подчеркивается сущность триангуляции, необходимые критерии для совершенной триангуляции, затем показываются основные попытки комплексного анализа на основе триангулярной таблицы.

Ссылаясь на технику реализации триангулярной формы межотраслевого баланса, отмечаются достигнутые успехи, а также достигнутая стадия.

Затем в работе показывается частичное использование триангулярного межотраслевого баланса, подчеркивая, между прочим, возможность проведения комплексного анализа, возможность подсчета вариантов или определения эффекта изменения различных структурных элементов, использования триангулярного баланса в оптимальном расчете, в характеристике иерархической конструкции и структуры народного хозяйства, «степени» организации» экономики.

25. II. 1974