G. LELESCU, lecturer
A. CARABULEA, Dr., eng.

Aspects Concerning the Profitableness of Railway Electrifying

The development of national economy also implies railway electrification, particularly along portions of high resistance and intensive traffic, both with regard to the transportation of goods and to that of passengers.

1. STATING THE PROBLEM

Since three systems of traction are used at present: steam, Diesel and electric and there is a tendency to almost completely eliminate in the near future steam traction and use intensely electric and Diesel traction it may be useful to show, on basis of technical and economical considerations also turning to account the experience of the countries [1], which are the conditions favouring the maintenance or replacement of one kind of traction by the other. To this end the first part of this paper points to a technical-economical method for comparing Diesel and electric traction on basis of the so-called computed expenses, both in the hypothesis where the investments are effected within a single year and in the case where they are spread out over a longer period.

In the second part a condition is determined from which the required minimum yearly volume of gross tons/km results as a function of the difference in cost price between the various systems of traction considered and evidencing the fact that electric traction shows the highest economic efficiency. Thus, the economic domains for the use of Diesel and electric traction are made evident for the most usual railway resistances and indications are given regarding the choice of the traction system, on basis of the nomograms shown in figures 1 and 2 when the difference in cost price (α) between steam, Diesel and electric traction is known.

2. PREMISES WHEN COMPARING TRACTION SYSTEMS

In view of comparing the steam, Diesel and electric traction systems one may use either the criterion of yearly computation expenses (Z), or the criterion of the term of redeeming (t_r).

$$Z = C + p_n I \text{ (lei/year)} = \text{minimum} \qquad (1)$$

$$t_{rc} = \frac{I_D - I_E}{C_E - C_D} \text{ (years)} = t_{rn} = 8 \ldots 10 \text{ years} \qquad (2)$$

If the investments vary in time, relation *(1)* may be written

$$Z = C + p_n I_r = C + p_n \sum_{k=1}^{t} I_k (1 + p_n)^{t-k} \qquad (3)$$

where

Z = yearly computed expenses;

C = exploitation expenses;

$p_n = \frac{1}{t_{rn}} = 0.1 \text{ (year}^{-1}\text{)}$ = economic efficiency coefficient;

I = one year investments;

I_r = investments reported to the duration t in years required to electrify the railway;

t_{rn} = normed term of redeeming, which is, for power constructions, 8 ... 10 years;

I_D and I_E = equivalent investments for Diesel and, respectively, electric traction, also taking into account the investments in the related branches;

C_D and C_E = equivalent exploitation expenses for Diesel and, respectively, electric traction taking into account transportation capacity in the course of the years considered for the comparison;

k = year when a certain investment is made. For instance, if the work lasts five years, then $t = 5$ and $k = 1; 2; 3; 4; 5$.

Relations *(1)*, *(2)* and *(3)* which refer to the choice of solutions may also be completed by applying relation *(4)*.

$$Z_{t_1} = \sum_{k=1}^{t_1} (I_k + D_k)\ (1 + p_n)^k = \text{minimum} \qquad (4)$$

where

1) Z_t = total expenses in the period t_1;

2) t_1 = may be equal to the life of the installation, to the duration of the normed term of redeeming, or to an unlimited duration when the life of the installations compared differs;

3) D_k= direct exploitation expenses not including amortizations.

To properly apply relation *(4)* Z_{t_1} is computed and the remaining value of the fixed means at the end of the computation period is subtracted.

In the specialty literature [1], [4] only total exploitation expenses without taking investments into account are also used when comparing traction systems. In these cases the results might sometimes lead to less stringent conditions of profitableness.

Hereunder, the elements allowing to determine the yearly investments and expenses in view of comparing the electrical and Diesel solutions will be made to stand out.

2.1. *Determining the yearly computed expenses*

In the case of two solutions A and B when comparing the Diesel *(D)* and electric *(E)* systems, relations *(1)* are written:

$$Z_A^E = C_A^E + p_n I_A^E;\ Z_B^E = C_B^E + p_n I_B^E = p_{c_B}^E \times V_B^{EE} + p_n(I_B^E + I_2^{EF}) \quad (5)$$

$$Z_A^D = C_A^D + p_n I_A^D;\ Z_B^D = C_B^D + p_n I_B^D = p_{c_B}^D \times V_B^{DE} + p_n(I_B^D + I_2^{DE}) \quad (6)$$

The following expression is used to apply relation *(3)*:

$$Z_A^E = C_A^E + p_n \sum_{k=1}^{t} I_A^E(1 + p_n)^{t-k}$$

$$Z_A^D = C_A^D + p_n \sum_{k=1}^{t} I_A^D(1 + p_n)^{t-k}$$

The solution for which the yearly computation expenses are minimuml is adopted.

The meaning of the magnitudes is:

$p_{c_A}^E$ and $^D_{c_A}$ = cost price for the transportation of one gross ton/km for electric traction *(E)* and Diesel *(D)*;

$I_1^?$ and I_A^D = investment in the objective proper;

C_A^E and C_A^D = equivalent exploitation expenses for electric *(E)* and Diesel *(D)* traction.

The equalization is made from the standpoint of the transportation capacity used according to time.

p_n = normed efficiency coefficient for the branch considered.

I_2^{EE} and I_2^{DE} = equalization investment from related branches for electric *(EE)* and Diesel *(DE)* traction.

Choice of the solution is made on basis of the relation

$$\left.\begin{array}{l} Z_A^E < Z_A^D \text{ electric traction is more profitable} \\ Z_B^E > Z_B^D \text{ Diesel traction is more profitable} \end{array}\right\} \qquad (7)$$

The investments are determined by taking into account the investment in the objective proper, to which are added those made in the related branches to ensure the fuel for Diesel traction and the electric power. When determining the investments proper the purchase price of the engine, the investments required to build the lines, as well as the power stations and other investments connected with electrifying, are all taken into account.

To determine the yearly exploitation costs the expenses required for supplying fuel for Diesel and electric traction, the amortization expenses of the installations and the maintenance and repair expenses, etc., are computed.

If the yearly costs *(Z)* in lei/km are computed for various equivalent specific resistances taking into account traffic volume, a series of guiding values can be determined regarding the load in gross ton/day, to which the Diesel and electric systems are equivalent, the domain of profitableness being thus also determined.

2.2. *Determining the conditions of profitableness for electric traction compared to other systems of traction.*

Generally, the solutions are compared by considering the expenses of the effective steam consumption, respectively the one required by the Diesel engine, and the consumption of electric power is computed, taking into account productivity at the stable supply bases.

In this case, the purchase price of electric power takes into account the required productivity (exemple in *Annex*). In view of showing some economic effects at the level of national economy, regarding fuel-power costs, we shall also compare the solutions of electric traction to the other systems of traction taking into account the productivity of the successive transformations of conventional fuel into electric power. This amounts to take into consideration, in computations, the productivity of the power stations supplying the railway electrified network.

It is considered that the highest yield of a Diesel engine is about 32%, that of the electric engine computed at the station 15% — 20% in the case of thermal stations using conventional fuel as a power agent and reaching up to 50% in the case of hydroelectric stations, while that of steam engines is theoretically 11% — 12% and practically 6% — 8%.

Hereunder, with respect to the expenses for fuel, one will take into account the productivity at the power station supplying the electrified railroad as well as the productivity of: the transportation line, the traction substation,

the contact line and the electric engine *(Annex)*. The computations must also take into account the fact that, in the case of electric traction, maintenance repair and amortization expenses of the contact network and of the traction electric stations also intervene with a considerable share.

In view of correctly determining the ratio between the fuel used up and the mechanical work obtained in the framework of the three traction systems: electrical, Diesel and steam, the productivities shown above will be used.

Under these conditions total productivity in the case of electric traction varies between 0.15 — 0.20. Out of 7,000 kcal/kg. conventional fuel used up at the power station, the electric engine used 1,050 — 1,400 kcal/kg. conventional fuel, and by means of the relation 1 kWh = 860 kcal mechanical work equivalent to 1.2 — 1.6 kWh/kg. conventional fuel or 1.7 — 2.2 HPh/kg. conventional fuel can be obtained *(Annex)*.

In the case of a steam engine, for a productivity of 0.6 — 0.8, the heat used up from one kg. conventional fuel is 420 — 560, and by means of the relation 860 kcal = 1.36 HPh one obtains 0.7 — 0.9 HPh/kg. conventional fuel *(Annex)*.

In the case of Diesel traction, considering the total yield of the Diesel engine as being 0.28 — 0.32 there results that from one kg. conventional fuel used up 1960—2240 Kcal/kg. conventional fuel, or 3.1—3.6 HPh/kg. conventional fuel are used up.

There results that for the same consumption of conventional fuel the electric engine develops 2.5 times more mechanical work; to produce one unit of mechanical work the consumption of the power station is at least three times smaller in fuel than in a steam engine.

In Diesel traction, under the aspect of national economy, specific consumption by HP h stands out as being smaller than for electric traction, while mechanical work is larger. We should here take into account the fact that the weight of hydroelectric stations will keep increasing in our power system, the result being that the total yield of electric traction will at least reach the level of a Diesel engine. On the other hand, if we consider the yield of an electric engine at the first traction substation, i.e. from the moment the current is received, then the yield of an electric engine is much higher; the comparison is made taking into account the cost price of electric power and of fuel oil.

As it is shown hereafter (relations *16* and *17)* electric traction, with a power of 28,000 kgf. at the rim in continuous regime, allows the hauling of loads that are higher by 80 per cent in comparison to steam traction and by more than 40 per cent in comparison to Diesel traction by 2,100 HP engines. These considerations allow the cost price to be reduced by about 14 per cent in the first case and by about 9 per cent in the case of change over from Diesel to electric traction *(7)*.

We shall, further, refer to the manner in which the conditions of profitableness are determined in view of introducing electric traction for certain hauling section as a function of traffic volume expressed in gross tons/km., the solutions being compared on basis of the determination of cost price. The cost price per gross ton/km for various hauling sections, as a function of track

profile, gross tonnage/train, specific fuel consumption and degree to which railway carriages are used, is determined *(7)* by means of relations of the form:

a) For goods traffic:

$$p^m_{c_0} = 2K'_m\left[\frac{0{\cdot}3}{Q_0} + 1.342\times10^{-4}\times q^m_0 + 2.4\times10^{-4}\right] + \frac{2\,K''_m(1+z)}{w+(1+z)w_0} \qquad (8)$$

$$p^m_{c_1} = 2K'_m\left[\frac{0{\cdot}3}{Q_1} + 1.34210^{-4}\times q^m_1\times 2.4\times10^{-4}\right] + \frac{2K''_m(1+z)}{w+(1+z)w_0} + \frac{C^m_a + C^m_i - C^m_e}{U_m}$$

b) For passenger traffic:

$$p^c_{c_0} = 2K'_c\left[\frac{0{\cdot}27}{T_0} + 3.62\times q^c_1\times10^{-4} + 6.9\times10^{-4}\right] + \frac{2K''_c}{\delta}\cdot\frac{W}{T_0} \qquad (9)$$

$$p^c_{c_1} = 2K'_c\left[\frac{0{\cdot}27}{T_1} + 3.62\times q^c_1\times10^{-4} + 6.9\times10^{-4} + \frac{2K''_c}{\delta}\times\frac{W}{T_1}\right] + \frac{C^c_a + C^c_i - C^c_e}{U_c}.$$

where:

$p^m_{c_0}$ and $p^c_{c_0}$ = lei/gross ton km. cost price for goods *(m)* and passengers *(c)* before electrifying;

$p^m_{c_1}$ and $p^c_{c_1}$ = cost price for goods and passengers after electrifying;

K'_m and K''_m = complex mean specific cost per train/km (K'_m) and per axle (K''_m); for passengers the coefficients are (K'_c) and K''_c);

Q_0; T_0 and Q_1; T_1 = gross tons per train before (Q_0) and after electrifying (Q_1) for goods and T_0 and T_1 for passenger traffic;

q^m_0, q^m_1; q^c_0, q^c_1 = specific fuel consumption before (q^m_0; q^c_0) and after electrifying (q^m_1; q^c_1) in kg. conventional fuel/hundreds of gross tons/km;

z = coefficient of distance covered by empty carriages;

w_0 = mean dead weight per axle, and w = actual load per axle in net tons;

C_a; C_i; C_e = amortization costs resulting from electrifying *(a)*; supplementary maintenance costs *(i)* and exploitation costs *(e)* which decrease for goods *(m)* and passengers *(c)*;

δ = mean number of passengers per axle, and w mean number of passengers per train;

U_m and U_c = volume of gross tons/km per day or year for goods traffic (U_m) and for passenger traffic (U_c).

In view of determining the number of gross tons/km from which electric traction becomes profitable the fact must be taken into account that approximately 70 per cent of the total yearly exploitation costs are allotted to

goods traffic and 30 per cent to passenger traffic. After determining the unit cost price per gross ton/km for each traffic, the condition is set for a lower cost price to result after electrifying than in the previous situation.

$$p_{c_1} < p_{c_0}$$

$$a)\quad p^m_{c_1} < p^m_{c_0}; \qquad b)\quad p^c_{c_1} < p^c_{c_0} \tag{10}$$

The relations between the yearly exploitation costs are of the form:

$$\begin{aligned} C^m_a + C^m_i - C^m_e &= 0.7\ (C_a + C_i - C_e) \\ C^c_a + C^c_i + C^c_e &= 0.3\ (C_a + C_i - C_e) \end{aligned} \tag{11}$$

From conditions 8 and 9 there results:

$$U_m > \frac{C^m_a + C^m_i - C^m_e}{2K'_m\left[0{\cdot}3\,\frac{Q_1 - Q_0}{Q_1\,Q_0} + 1.342\times10^{-4}\ (q^m_0 - q^m_1)\right]} \tag{12}$$

$$U_c > \frac{C^c_a + C^c_i - C^c_e}{2K'_c\left[0.27\,\frac{T - T_0}{T_1\,T_0} + 3.62\times10^{-4}\ (q^c_0 - q^c_1)\right]}.$$

Adding inequalities *(12)* and then taking into account considerations *(10)* and *(11)* and the fact that the differences in unit cost price of goods traffic (Δp_m) and passenger traffic (Δp_c) are precisely:

$$\Delta p_m = p^m_{c_1} - p^m_{c_0} = 2K'_m\left[0.3\,\frac{Q_1 - Q_0}{Q_1\,Q_0} + 1.342\times10^{-4}\ (q^m_0 - q^m_1)\right]$$

$$\Delta p_c = p^c_{c_1} - p^c_{c_0} = 2K'_e\left[0.27\,\frac{T_1 - T_0}{T_1\,T_0} + 3.62\times10^{-4}\,(q^c_0 - q^c_1)\right]$$

there results:

$$U > (C_a + C_i - C_e)\left(\frac{0.7}{\Delta p_m} + \frac{0.3}{\Delta p_c}\right) \tag{13}$$

where $U = U_m + U_c$.

The gross tonnage by train (Q) as a function of the specific rolling drag (r_i) of the engine — which, to make things simpler, is considered equal for all the traction systems — and that of the carriages (r_v), taking into account the traction power at the rim (F_0) in kgf., the weight of the engine (G_1) in tons and the characteristic resistance (α_1) of the track in $\left(\frac{\text{kgf}}{t}\right)$, can be determined by means of the relations:

$$\left.\begin{aligned} Q_e &= \frac{F^e_0 - G_e(r_1 + \alpha_1)}{(r_v + \alpha_1)} \text{ for electric traction } (e) \\ Q_d &= \frac{F^d_0 - G_d(r_1 + \alpha_1)}{(r_v + \alpha_1)} \text{ for Diesel traction } (d) \\ Q_a &= \frac{F^a_0 - G_a(r_1 + \alpha_1)}{(r_v + \alpha_1)} \text{ for steam traction } (a) \end{aligned}\right\} \tag{14}$$

Taking into account the ratio of the power at the rim between the three engines (electric; Diesel; the most powerful steam engines) and the ratio of their weights, we have:

$$F^e \approx 1.8\ F_0^a; \qquad F_0^e \approx 1.4\ F_0^d; \qquad G_e \approx 0.8\ G_a; \qquad G_p \approx G_d \qquad (15)$$

From relations *(14)* and *(15)* there results:

$$Q_e = 1.8\ |Q_a + G_a \frac{(r_1 + \alpha_1)}{(r_v + \alpha_1)} \qquad (16)$$

$$Q_e = 1.4\ Q_d + 0.4\, G_d \times \frac{(r_1 + \alpha_1)}{(r_v + \alpha_2)} \qquad (17)$$

From these relations it may be seen that electric traction, under the same conditions of exploitation, offers the possibility of hauling tonnages more than 88 per cent larger than steam traction and more than 40 per cent larger than Diesel traction.

Fuel consumption at the power station, reported to the volume of gross tons/km effected by the electric engine can be computed as follows:

$$q_1 = \frac{860 \times (r_v + \alpha_1)}{3.67 \times 7 \times 10^3 \eta} \begin{bmatrix} \text{kg. conventional fuel/hundreds} \\ \text{of gross tons/km.} \end{bmatrix} \qquad (18)$$

where η = productivity of transformations. The other data in relations *(8)* and *(9)* for the steam and Diesel engines are obtained from current exploitation.

Computation of the difference in yearly exploitation expenses is carried out taking into account the amortization rate previously shown as well as that for maintenance and exploitation proper. For instance, for a 111 km length track section having a resistance of 0 — 28 kgf/t, in course of electrifying, three traction substations, two sharing points, four parallel connection points, two maintenance sectors for the contact line and one point for electric supply, are used, which require a total investment of 128.5×10^6 lei. The $(C_a + C_i)$ expenses, when also taking into account the electric engine, are equal to 15.707×10^6 lei. The computation of expenses (C_e) is made taking into account the detail that — due to the fact that they have an approximately 200 km/day longer daily run and a larger transportation capacity—ten electric engines replace 23 steam engines. If the purchasing price of a steam engine is 1.5×10^6 lei, and the amortization rate is 3.7 per cent, there results $C'_e = 1.362 \times 10^6$ lei, and $C''_e = 1.45 \times 1.362 \times 10^3 = 1.98 \times 10^6$ lei, which shows that the amortization (C'_e) and maintenance (C''_e) expenses of the steam traction installations being replaced by the electric ones have a value $C_e = 3.342 \times 10^6$ lei. Hence $V = C_a + C_i - C_e = 12.365 \times 10^6$ lei. For the values considered and $\eta = 0.2$ there results, at the level of national economy: $\Delta p_m^{q-a} = 0.03635$ (lei/gross ton/km.). $\times\ \Delta p_c^{q-e} = 0.0575$ (lei/gross ton/km). When comparing Diesel traction to electric traction, considering for the electric engine the yield when receiving the current ($\eta = 0.8$), and, of course, taking

into consideration the cost of electric power when receiving the current, there results $\Delta p_m^{d-a} = 0.0057132$ lei/gross ton/km ; $\Delta p_c^{d-a} = 0.005864$ lei/gross ton/km.

If it is assumed that one electric engine is equivalent to 1.5 Diesel engine, then the expenses in this case acquire the value $V = C_a + C_i - C_e = (15.707—2.960) \times 10^6 = 12.747 \times 10^6$ lei.

Taking into consideration the fact that for the portion of electrified line approximately 20 electric engines would be necessary to ensure a prospective traffic of 2,250 × 10 gross tons/year and using condition *(13)*, the yearly volume for which electric traction becomes profitable is deduced. Thus, in comparison to steam traction, the profitableness of electric traction occurs at a yearly volume of about 350×10^6 gross tons/km for the case $\eta = 0.2$, i.e. figured from the standpoint of national economy. Taking into account the purchase price of electric power when the current is received and the total yield of the electric engine, a necessary level close to the one thus established will be reached.

When compared to Diesel traction there results that electric traction with the yield ($\eta = 0.8$) when the current is received, becomes profitable for a yearly volume larger than 2,250 × 10 gross ton/km.

In order to make available to the study of design and maintenance a graphic computation to be used as a guide in view of immediately determining the profitableness of a traction system in relation to one being replaced, the variation curves

$$U = f(\alpha\,;\, V) \text{ where } \alpha = \min(\Delta p_m\,;\, \Delta p_c), \text{ respectively, } V = \alpha U \qquad (19)$$

have been plotted in figure 1.

Figure 1 constitutes a lattice nomogram.

V and U being known one can deduce the minimum difference of the cost price per gross ton/km., for which electric traction is profitable. This nomogram can also be used in case the investments for electrifying are taken into consideration. Taking into account the fact that $V = C_a + C_i - C_e$ represent about 40 per cent of the investments, one may write ;

$$V = 0.04\ I \text{ and } I = 25\ \text{V} \qquad (20)$$

To translate relation *(19)* into a nomogram one can also use a system of nomogram on an ellipse skeleton and secant on the principle of aligned points, which gives more clarity. To this end, according to the general theory of nomograms, equation *(19)* is written in the Soreau form, viz :

$$\begin{vmatrix} 1 \ldots \dfrac{H \times m \times U}{1 + m^2 U^2} \ldots \dfrac{L \times m^2 \times U^2}{1 + m^2 \times U^2} \\ 1 \ldots \dfrac{H \times n \times \alpha}{1 + n^2 \alpha^2} \ldots \dfrac{L \times n^2 \alpha^2}{1 + n^2 \alpha^2} \\ 1 \qquad 0 \qquad -\dfrac{L \times m \times n \times V}{1 - m \times n \times V} \end{vmatrix} = 0 \qquad (21)$$

Where H ; L ; m ; n ; are scale factors which determine the size and placement of the figure. The factors $H = L = 200$ mm. represent the length of the reduced skeleton ellipse axis, and $m + 6 \times 10^{-10}$ and $n = -10$ de-

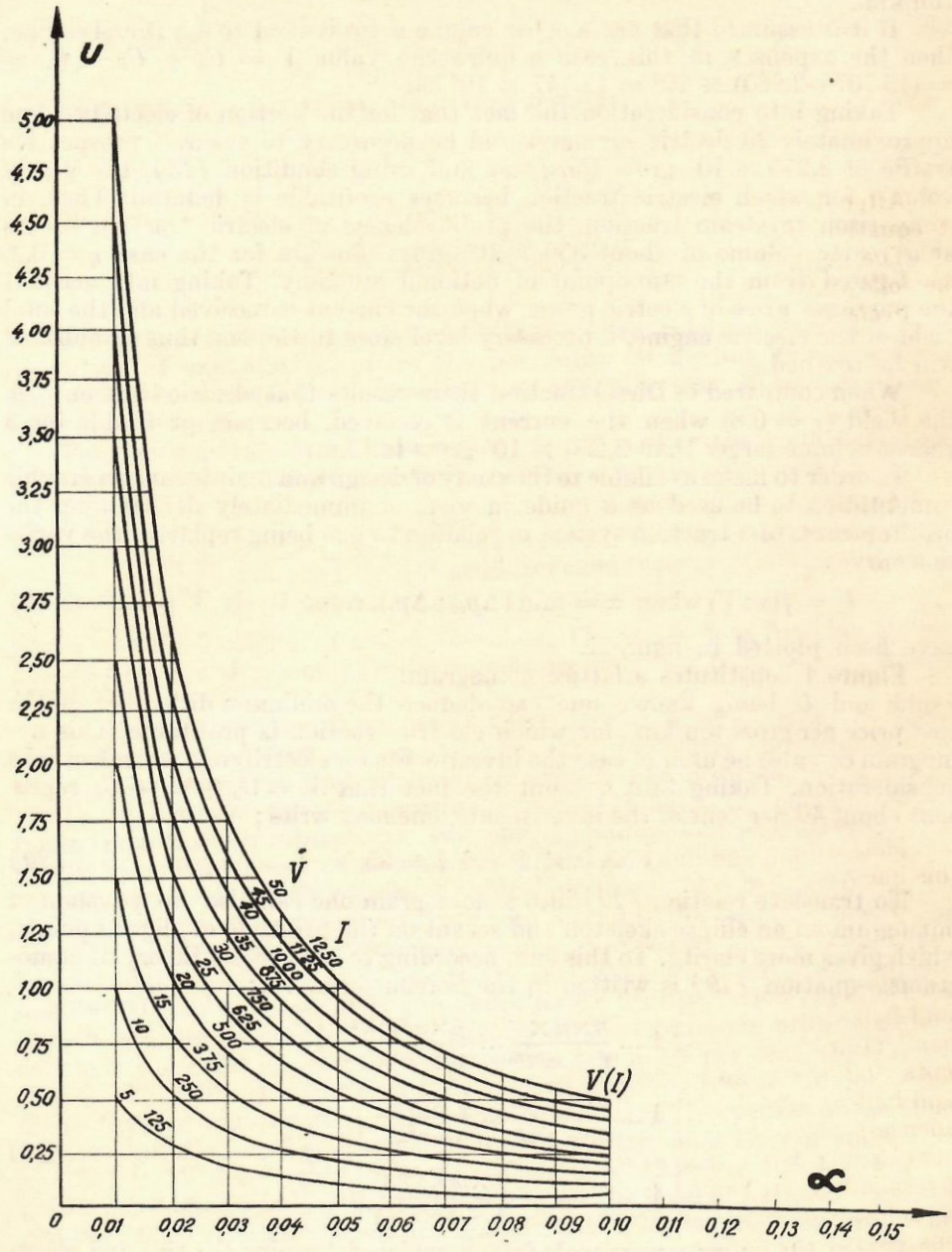

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Fig. 1

termine the extension and contraction of the scales. Substituting these data in *(21)* Soreau's equation is obtained in the form:

$$\begin{vmatrix} 1 & \frac{12 \times 10^{-8} \times U}{1 + 36 \times 10^{-20} \times U^2} & \frac{\times 10^{18} \times U^2}{1 + 36 \times 10^{-20} \times U^2} \\ 1 & \frac{2 \times 10^3 \alpha}{1 + 10^2 \alpha^2} & \frac{2 \times 10^4 \times \alpha^2}{1 + 10^2 \times \alpha^2} \\ 1 & 0 & \frac{12 \times 10^{-7} \times V}{1 + 6 \times 10^{-9} \times V} \end{vmatrix} = 0 \qquad (22)$$

After developing one notices that equation *(22)* contains the solutions of equation *(19)*.

The geometric loci of the points marked by the variables U; α; V; are the following:

Scale U. The parametric equations:

$$x = \frac{12 \times 10^{-18} U}{1 + 36 \times 10^{-20} U^2}; \qquad y = \frac{72 \times 10^{-18} \times U^2}{1 + 36^{-20} U^2} \qquad (23)$$

If U is eliminated, one obtains:

$$x^2 + y^2 - 200\ y = 0 \qquad (24)$$

A 100 mm. radius circle is obtained, tangent at the origin.

For $U > 0$ there results $x > 0$ and the right hand half circle is used.

Scale α. The parametric equations:

$$x = -\frac{2 \times 10^3 \alpha}{1 + 10^2 \alpha^2}; \qquad y = \frac{2 \times 10^4 \times \alpha^2}{1 + 10^2 \times \alpha^2}. \qquad (25)$$

Eliminating (α) equation *(24)* is again obtained. Since ($\alpha > 0$) there results ($x < 0$), hence the left hand half circle is used.

Scale V (I). The parametric equations:

$$x = 0; \qquad y = \frac{12 \times 10^{-7} \times V}{1 + 6 \times 10^{-9} \times V}. \qquad (26)$$

The Cartesian equation of the scale support is precisely axis (oy) i.e. $x=0$. This line constitutes the secant of the ellipse skeleton.

The scales were graduated by considering the billion as a measuring unit for the volume of gross tons/km, the variation limits being $0.25 \times 10^9 \leqslant U \leqslant 5 \times 10^9$. The values for U varied every 0.25 and, by means of relation *(26)*, one obtains the values of y required to plot the points on the right hand half circle. For the scale α (difference in cost price by gross ton/km) the "ban" (100 bani = 1 leu) was taken as a measuring unit, with the variation limits $0.01 \leqslant \alpha \leqslant 0.1$. By using relation *(25)*, the points on the left hand half circle are determined. For the *scale V(I)* (difference in exploitation expenses) the million was taken as a unit and the variation limits are comprised between $5 \times 10^6 \leqslant V \leqslant 100 \times 10^6$. By means of relation *(26)* the points corresponding to the values of magnitude V are found. On the *scale V* the values corresponding to the investments thus deduced ($I = 25\ V$) in millions, were also plotted.

With respect to the practical use of the diagram it may be shown that, when V and α are known, U is deduced, i.e. the limit volume of gross

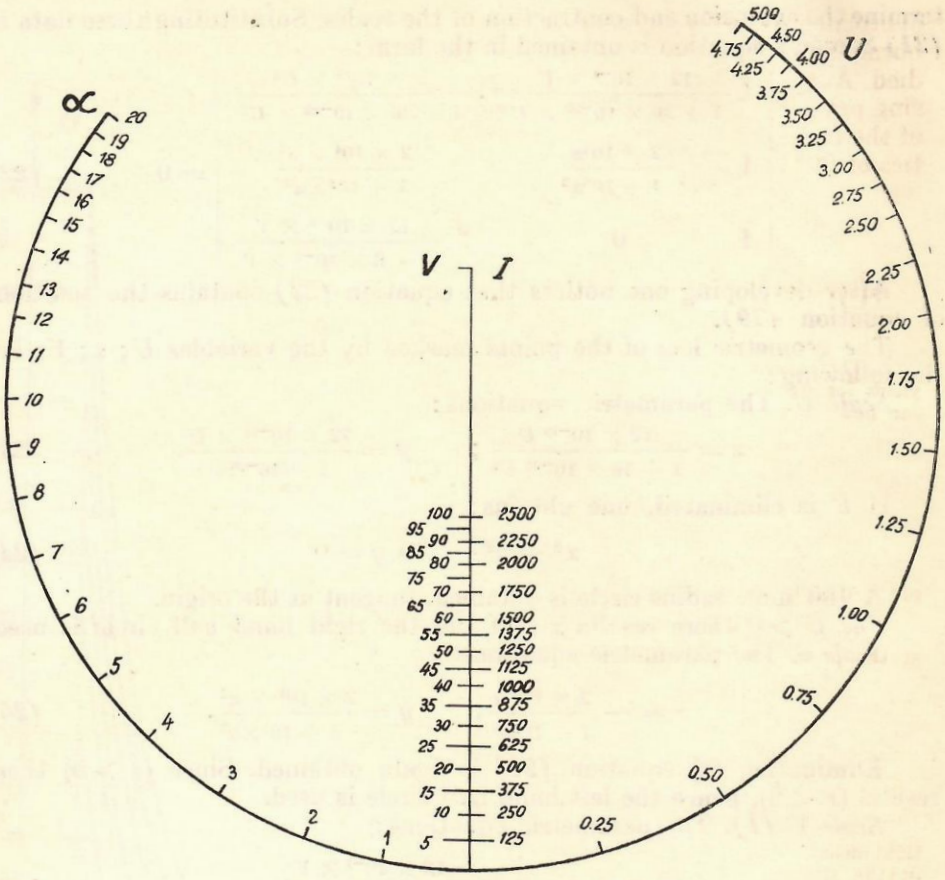


Fig. 2

tons/km required for electrifying to be profitable. The values of α and V are joined by a resolvent. At the intersection of this resolvent with the scale of variable U one finds the value for the number of gross tons/km.

The nomogram is shown in figure 2. As an illustration one considers $V = 60 \times 10$ lei; $\alpha = 0.04$ lei/gross ton/km. and one finds $U = 1.5 \times 10^9$ gross ton/km, which is also confirmed by computation. The graduations in figure 2 are only demonstrative but they allow to determine with sufficient accuracy the conditions for which a traction system is profitable, in comparison with the one to be replaced.

3. CONCLUSIONS

Obviously, this problem also presents other aspects, rather varied and complex, which might be taken into consideration in order that the degree of approximation should be as good as possible when establishing the conditions of profitableness. Thus, an aspect which the authors intend to study in view

of broadening the scope of treatment in this problem is to introduce the influence borne by the intensity of traffic on the cost price, in the hauling sections studied. Another aspect is that of the general correlation between the unit purchasing price of electric power at the point where the current is received, and that of the fuel used for other traction systems, as it results from the various varieties of fuel.

ANNEX

TURNING OF FUEL CONSUMPTION INTO ELECTRIC POWER

There results from studies carried out by the railways of various countries that the yields for electric power, starting from the power station (at the level of national economy) can be:

At the steam-power electric station:	$0.25 \leqslant \eta a \leqslant 0.27$
At the transportation line:	$0.90 \leqslant \eta t \leqslant 0.95$
At the traction substations:	$0.90 \leqslant \eta s \leqslant 0.95$
At the contact line:	$0.90 \leqslant \eta c \leqslant 0.95$
At the electric engine:	$0.80 \leqslant \eta e \leqslant 0.85$
Total yield:	$\eta = \eta_a \eta_t \eta_s \eta_c \eta_e$

or: $0.15 \leqslant \eta \leqslant 0.20$

There results that out of the 7,000 kcal contained by 1 kg conventional fuel used up at the power station, a quantity (Q_e) can be used at the electric engine:

$$1050 \leqslant Q_e \leqslant 1400 \left[\frac{\text{Kcal}}{\text{kg. conventional fuel}} \right].$$

Taking into account the transforming relation

$$1 \text{ kWh} = 860 \text{ Kcal}$$

this means that 1 kg. conventional fuel used up at the thermal power station produces at the electric engine, a mechanical work (L_e).

$$1.2 \leqslant L_e \leqslant 1.6 \left[\frac{\text{kWh}}{\text{kg. conventional fuel}} \right]$$

or that, for an electric power of 1 kWh developed by the electric engine, a quantity (q_e) of conventional fuel is necessary

$$0.6 \leqslant q_e \leqslant 0.8 \left[\frac{\text{kg. conventional fuel}}{\text{kWh}} \right].$$

If the transformation

$$1 \text{ kWh} = 1.36 \text{ HPh}$$

is made and the specific consumption $\left[\frac{\text{kg. conventional fuel}}{\text{HPh}} \right]$ is noted by q_m there results

$$0.45 \leqslant q_m \leqslant 0.60 \left[\frac{\text{kg. conventional fuel}}{\text{HPh}} \right]$$

or

$$1.7 \leqslant L_m \leqslant 2.2 \left[\frac{\text{HPh}}{\text{kg. conventional fuel}} \right].$$

For the most powerful steam engines the total yield can be practically estimated between the limits

$$0.06 < \eta_a < 0.08$$

Making the calculations and using notations similar to those for electric traction, the following analysis results:

$$420 \leqslant q_a \leqslant 560 \left[\frac{\text{Kcal}}{\text{kg. conventional fuel}} \right]$$

$$1.1 \leqslant q_a \leqslant 1.5 \left[\frac{\text{kg. conventional fuel}}{\text{HPh}} \right]$$

$$0.7 \leqslant L_a \leqslant 0.9 \left[\frac{\text{HPh}}{\text{kg. conventional fuel}} \right].$$

The ratios of consumptions, respectively the specific mechanical work are:

$$\frac{q_m}{q_a} = \frac{0.45}{1.1} = \frac{0.6}{1.5} = 0.4 = \frac{2}{5}$$

$$\frac{L_m}{L_a} \approx \frac{1.7}{0.7} = \frac{2.2}{0.9} = 2.5 .$$

If the same comparison is made for Diesel traction where the total yield (η_d) is:

$$0.28 < \eta_d < 0.32$$

and making the calculations and respective notations there results the analysis:

$$1960 \leqslant Q_d \leqslant 2240 \left[\frac{\text{Kcal}}{\text{kg. conventional fuel}} \right]$$

$$0.28 \leqslant q_d \leqslant 0.23 \left[\frac{\text{kg. conventional fuel}}{\text{HP h}} \right]$$

$$3.1 \leqslant L_d \leqslant 3.6 \left[\frac{\text{kg. conventional fuel}}{\text{Hph}} \right].$$

Consumption of conventional fuel at the thermal power station as against the volume of transportation effected.

It is interesting to determine the specific fuel consumption at the thermal power station, as against the volume of (hundred) gross tons/kilometres effected by the electric engine.

To this end one must pass from fuel consumption per HPh to the consumption per hundred gross tons/km.

The consumption per HPh must therefore be transformed into consumption per tfkm.

1. $[\text{HPh}] = 75\ [\text{kgfm}] \times 3600\ [\text{S}] = 10^{-6} \times 75 \times 3600\ [\text{tfkm}] = 10^{-2} \times 27\ [\text{tfkm}]$

$$1\ \text{kWh} = 1.36\ \text{HPh}$$

$$1\ [\text{kWh}] = 10^{-2} \times 27 \times 1.36\ [\text{tfkm}] = 10^{-3} \times 367\ [\text{tfkm}]$$

$$\frac{860 \left[\frac{\text{kcal}}{\text{kWh}} \right]}{7000 \left[\frac{\text{kcal}}{\text{kg. conv. fuel}} \right]} = \frac{860}{7000\,\eta} \left[\frac{\text{kg. conventional fuel}}{\text{kWh}} \right]$$

$$[\rho] = \frac{860}{10^{-3} \times 367 \times 7000\,\eta} \left[\frac{\text{kg. conventional fuel}}{\text{tfkm}} \right]$$

To pass on from the consumption per tfkm., we shall take into account the relation of gross tonnage by train.

$$Q = \frac{F_0 - (r_e + \alpha_1)\, G}{r_v + \alpha_1}$$

where

Q is the gross tonnage per train $\left[\frac{\text{gross tonnage}}{\text{train}}\right]$
F_c — tangential power at rim [kgf]
G — engine weight [tons]
r_e — specific rolling resistance of engine
r_v — specific rolling resistance of carriages
l — characteristic resistance of line $\left[\frac{\text{kgf}}{\text{t}}\right]$

The difference:

$$F_c = F_0 - (r_e + \alpha_1)G\,,$$

is the draw-hook tractive power.

From the above relation there results:

$$r_v + \alpha_1 = \frac{F_c}{Q}\left[\frac{\text{kgf}}{\text{t}}\right]$$

or

$$10^{-1}\,(r_v + \alpha_1) = \frac{10^{-3} \times F_c\ [\text{ft}]}{10^{-2} \times Q\ [\text{hundreds gross tons}]}$$

Hence:

$$\mu = \frac{10^{-1} \times (r_v + \alpha_1) \times 860}{10^{-3} \times 367\ 7000\ \eta}\left[\frac{\text{kg. conv. fuel}}{\text{tfkm}}\right]\left[\frac{\text{tf}}{\text{hundreds gross tons}}\right]$$

$$\text{or: } \mu = \frac{860\ (r_v + \alpha)}{3.67 \times 7000\,\eta}\left[\frac{\text{kg. conv. fuel}}{\text{hundreds gross tons}}\right].$$

For average network conditions one considers $\alpha = 0$. If $\alpha \neq 0$, this is the case of a certain hauling section.

For example, should we consider a hauling section of chatacteristic resistance $\alpha_1 = 24$ kgf/t and considering $r_v = 3.0$ kgf/t and $\eta = 0.20$, there would result:

$$\mu = \frac{860 \times 27}{3.67 \times 1400} = 4.55 \text{ kg. conventional fuel/hundreds gross tons km.}$$

Thus, 4.55 kg. conventional fuel are used up at the electric power station for every hundred gross tons km.

For steam traction the direct specific consumption under these conditions is 8.6 kg. conventional fuel/hundred gross tons/km, hence in a ratio of 0.52 in comparison to the equivalent for electric traction.

Consumption of electric power as against the volume of transportation effected

It is very important to also determine the consumption of electric power per hundred gross tons/km.

To this end, starting from the relation by which the engine draw-hook tractive power is determined for an engine considered as drawing on a level and aligned track, we have:

$$P_c = \frac{r_v \times Q \times v}{270} \qquad (1)$$

where

P_c is the draw-hook tractive power in [HP]
r_v — specific rolling resistance of the carriage assembly in [kgf/t]
Q — gross tonnage of train in [tons]
v — traffic speed in $\left[\frac{\text{km}}{\text{h}}\right]$.

On the other hand the power at the rim of the driving wheel can be expressed by:

$$P_t = (1 + a)P_c \tag{2}$$

or

$$P_t = \eta(1 - \beta)P_c \tag{3}$$

where:

P_t — the power at the rim in [HP]
a — relative increase of the draw-hook tractive power required for self-hauling;
η — total yield of engine, starting from the power station of from the traction substation where the current is received;
P_e — effective engine power;
β — relative decrease of effective power required for internal services (lighting, lubricating, heating, etc.)

Taking relations (*1*), (*2*), (*3*), into account, one may write:

$$P_e = \frac{P_t}{\eta(1 - \beta)} = \frac{(1 + a)P_e}{\eta(1 - \beta)} - \frac{(1 + a)r_v Q_v}{270\,\eta(1 - \beta)}\,[\text{HP}] \tag{4}$$

Likewise:

1 kWh = 1.36 HP

1 HPh = 0.736 kWh

Noting the hourly power consumption $\left[\frac{\text{kWh}}{\text{h}}\right]$ by K, we have, dimensionnally:

$$K = 0.736\left[\frac{\text{kWh}}{\text{HPh}}\right][\text{HP}] = 0.736\left[\frac{\text{kWh}}{\text{h}}\right], \text{ i.e.}$$

$K = 0.736\ P_e$ or

$$K = \frac{0.736\,(1 + a)\ r_v Q_v}{270\ \eta\,(1 - \beta)}. \tag{5}$$

On the other hand the hourly volume per hundred gross tons km, which we note by H is:

$$H = 10^{-2}\,Qv \tag{6}$$

$H = [10^{-2}Q]\,[v] = [\text{hundred gross tons}]\left[\frac{\text{km}}{\text{h}}\right] = [\text{hundred gross tons km g}]$.

Noting by λ the kWh consumption per hundred gross tons km we have:

$$\lambda = \frac{K}{H}$$

$$[\lambda] = \frac{\frac{\text{kWh}}{\text{h}}}{[\text{hundred gross tons km}]} = \frac{\text{kWh}}{\text{hundred gross tons km}}$$

Hence, taking into account the expressions for K and H in (5) and (6):

$$\lambda = \frac{0.736\,(1 + a)\,r_v}{2.7\,\eta\,(1 - \beta)}\,. \tag{7}$$

For electric engines one can take:

$$a \approx 0.04\,; \qquad \beta = 0.02$$

In this case:

$$0.736 \times 1.04 = 0.765$$

$$2.7 \times 0.98 = 2.65$$

Hence: $\lambda = \frac{0.765\,r_v}{2.65\,\eta} = \frac{r_v}{3.46\,\eta} = \frac{r_v}{3.5\,\eta}$

$$\lambda = \frac{r_v}{3.5\,\eta}\,. \tag{8}$$

Relation (8) is valid for average computations along the network.
For a concrete hauling section of characteristic resistance α_1:

$$\lambda = \frac{r_v + \alpha_1}{3.5\,\eta}\,. \tag{9}$$

The notion of virtual resistance is used in hauling computations. It contains both the specific rolling resistance of the carriages and the track characteristic resistance.

From computations made so far there results that for a virtual resistance of 1 kgf/t, 0.343 kWh/hundred gross tons/km is used up in the hypothesis of a yield of about 0.8 considered from the substation where the current is received.

By making in relation (9), $r_v = \alpha_1 = 1$ and with $\eta = 0.8$, one obtains:

$$\lambda = \frac{1}{3.5 \times 0.8} = \frac{1}{2.8} = 0.357 \left[\frac{\text{kWh}}{\text{hundreds gross tons/km}}\right]$$

i.e. a value which is close to the one resulting from practical traction computations.

An example of the computation made to find the difference in the cost of electric power and of fuel for Diesel traction

Assuming a network having a maximum specific resistance $\alpha = 10$ kgf/t; the yield of the electric engine after receiving the current $\eta_e = 0.8$; and knowing that the purchase price of one kWh is 0.20 lei/kWh and that of one kg. fuel oil 0.85 lei/kg; let the saving resulting from electric traction when replacing Diesel traction be determined for a volume $U = 3{,}109$ gross tons kilometre.

The specific rolling resistance of the carriages will be considered $w'' = 5$ kgf/t.

Specific electric power consumption is:

$$\lambda = \frac{w'' + \alpha}{3.5\,\eta} = \frac{15}{2.8} = 5.35\ \frac{\text{kWh}}{\text{hundred tons h.km.}}$$

$$E = 10^{-2} \times \lambda\, U = 10^{-2} \times 5.35 \times 3 \times 10^9 = 160.5 \times 10^6\ \text{kWh}$$

The power expenses will be:

$$C_e = 160.5 \times 10^6 \times 0.2 = 32.1 \times 10^6\ \text{lei.}$$

Specific fuel oil consumption for Diesel traction is computed (*10*) by means of the relation:

$$\lambda_d = 0.091\ (w'' + \alpha) + 0.091 \times 15 = 1.365\ \frac{\text{kg. fuel oil}}{\text{hundred gross t/km}}.$$

Total fuel oil consumption is:

$$M = \lambda_d \times 10^{-2}\ U = 1.365 \times 3 \times 10^7 = 40.95 \times 10^6 \text{ kg}$$

Expenses for fuel oil are:

$$C_m = 40.95 \times 10^6 \times 0.85 = 34.8 \text{ lei}$$

Saving achieved (at the level of the railways) is:

$$(34.8 - 32.1) \times 10^6 = 2.7 \times 10^6 \text{ lei}$$

If the computation was made at the level of national economy, for the above elements, considering the yield of electric traction $\eta = 0.2$, from the electric power station, and in the hypothesis that the purchase price of the conventional fuel used is 0.35 lei/kg. conventional fuel, there results:

$$\mu_e = \frac{860\ (w_0'' + \alpha)}{3.67 \times 7000\,\eta} = \frac{860 \times 15}{3.67 \times 1400} = 2.5 \text{ kg. conventional fuel/hundred gross t/km}$$

Absolute consumption of fuel at thermal power station is

$$K_e = 10^{-2} \times 3 \times 10^9 \times 2.5 = 75 \times 10^6 \text{ kg. conventional fuel.}$$

Expenses for this fuel are:

$$C_e = 0.35 \times 75 \times 10^6 = 26.25 \times 10^6 \text{ lei.}$$

The difference in fuel expenses at the level of national economy between Diesel and electric traction is:

$$(34.80 - 26.25) \times 10^6 = 8.55 \times 10^6 \text{ lei.}$$

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ASPECTS CONCERNANT LA RENTABILITE DE L'ELECTRIFICATION DES CHEMINS DE FER

Le problème de la détermination des conditions d'efficience économique, respectivement de rentabilité de la traction électrique, a préoccupé et continue à préoccuper les chercheurs spécialisés dans les chemins de fer des pays où ce problème requiert des études.

Parmi les différents critériums qui déterminent la rentabilité de la traction électrique, celui qui est presque toujours pris en considération est le volume du traffic mesuré en tonnes brut kilomètres.

On a étudié dans cet article certains aspects de l'efficience économique de l'électrisication des chemins de fer, en tenant compte du volume de tonnes brut kilomètres nécesfaires et les économies sur les frais d'exploitation respectifs.

ПО ПОВОДУ РЕНТАБЕЛЬНОСТИ ЭЛЕКТРИФИКАЦИИ ЖЕЛЕЗНЫХ ДОРОГ

Проблема определения условий экономической эффективности, — а именно рентабельности—электрической тяги занимала и занимает в настоящее время исследователей, специалистов железных дорог в странах, где необходимо изучение этой проблемы.

Среди различных критериев, которые обусловливают рентабельность электрической тяги, объем перевозок, измеряемый в тоннах брутто километрах, почти всегда имеется в виду.

В представленной работе исследованы некоторые стороны экономической эффективности электрификации железных дорог с учетом количества необходимых тонн брутто километров и соответствующей экономии на эксплуатационных расходах.