Yoshisade Murotsu, Fuminori Ohba, Hiroshi Itoh

ON A SOLUTION OF STOCHASTIC NONLINEAR PROGRAMMING PROBLEMS

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The authors consider the following problems:
Problem 1. Under the chance-constraints:

$$\text{Prob.} \quad [g,\ (x,\ a) \geqslant 0] \geqslant P_i \tag{1}$$

find x to minimize the expected value of the objective function Z.
Problem 2. Under the constraints (1) and an additional chance-constraint on the deviation of the objective function

$$\text{Prob:} \quad [Z \geqslant \beta \bar{Z}] \leqslant P_z \tag{2}$$

find x^* to minimize $\bar{Z}$.

Where

- $P_i (i = 1 \ldots m)$, β and P_z are given constants;
- $x = (x_i) = n$ — dimensional control vector;
- $a = (a_i) = q$ — dimensional coefficient vector
- $c = (c_i) = r$ — dimensional coefficient vector
- $g_i(\ldots)$ and $f(\ldots)$ are real valued functions with sufficient smoothness.

Using the mean and the variance of $g_i(x, a)$, the authors transform the chance-constraint (1) into the deterministic nonlinear constraint:

$$\overline{g_i(x,\ a)} - \lambda_i \sigma_{gi} \geqslant 0 \tag{3}$$

where λ_i is the undetermined multiplier

- $\overline{(\cdot)}$ mean value of $(\cdot)$
- $\sigma_{(\cdot)}$ standard deviation of $(\cdot)$

Similary, the chance-constraint on the objective function is transformed into

$$(\beta - 1)\ \bar{Z} - \lambda_z \sigma_z \geqslant 0 \tag{4}$$

where

$$\bar{Z} = f(x, i) + \frac{1}{2} \sum_{j,k=1}^{r} \frac{\partial^2 f}{\partial C_j \partial C_k} \overline{\Delta C_j \Delta C_k} +$$

$$+ \frac{1}{6} \sum_{j,k,l=1}^{r} \frac{\partial^3 f}{\partial C_j \partial C_k \partial C_l} \Delta C_j \Delta C_k \Delta C_l +$$

$$+ \frac{1}{24} \sum_{j,k,l,m=1}^{r} \frac{\partial^4 f}{\partial C_j \partial C_k \partial C_l \partial C_m} \overline{\Delta C_j \Delta C_k \Delta C_l \Delta C_m} + \ldots$$

$$\sigma_z^2 = \sum_{j,k=1}^{r} \left(\frac{\partial f}{\partial C_j}\right)\left(\frac{\partial f}{\partial C_j}\right) \overline{\Delta C_j \Delta C_k} +$$

$$+ \sum_{j,k,l=1}^{r} \left(\frac{\partial f}{\partial C_j}\right)\left(\frac{\partial^3 f}{\partial C_k \partial C_l}\right) \overline{\Delta C_j \Delta C_k \Delta C_l} +$$

$$+ \frac{1}{12} \sum_{j,k,l,m=1}^{r} \left[3\left(\frac{\partial^2 f}{\partial C_j \partial C_k}\right)\left(\frac{\partial^2 f}{\partial C_l \partial C_m}\right) + \right.$$

$$\left. + 4\left(\frac{\partial t}{\partial_j C}\right)\left(\frac{\partial^3 f}{\partial C_k \partial C_l \partial C_m}\right)\right] \overline{\Delta C_j \Delta C_k \Delta C_l \Delta C_m} +$$

$$+ \frac{1}{4} \sum_{j,k,l,m=1}^{r} \left(\frac{\partial^2 f}{\partial C_j \partial C_k}\right)\left(\frac{\partial^2 f}{\partial C_l \partial C_m}\right) \Delta C_j \Delta C_k \Delta C_l \Delta C_m + \ldots$$

An algorithm to sistematically solve the deterministic nonlinear programming problems is presented. Also, the numerical examples are provided to ilustrate the given method.

M. Stoica