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On a Model of Allocating the Resources for Achieving the Investments

"The economy of time, as well as, the planned distribution of work time for several production branches, remains the first economic law on the basis of collective production."

K. Marx

The increase in the dynamics and level of economic-social development implies an ever greater complexity expressed also by the increase of the number of investment stocks and of the activities within their framework, by the growing number of connections between the various plan branches and by the ever more marked interdependence between them.

Owing to this growing complexity, it is necessary to use on an ever larger scale economic-mathematical modelling and implicitely the electronic data processing systems for substantiating the investment plans and respectively for ensuring ever better conditions for their implementation. To this end, it is necessary to know ever more analytically the necessary resources for carrying out the investment plans, to determine their best allocation methods, methods for a more efficient control of the carrying out of investments and for the establishment of the best variants for the use of utilizable resources in cases when, because of the exceeding of targets or because of a possible appearance of perturbating factors, the plan must be updated.

Due to the extremely complex and dynamic character of the activities for the carrying out of investments, correlation in the elaboration of plans must be ensured not only at the level of a plan period (annual of five-year plan) which would correspond to a global and static correlation, but also in the dynamics of the evolution of the respective phenomenon up to a monthly, weekly or even daily description in detail, at the level of the doer.

The present requirements of investment planning necessitate the ensurance of a proper correlation of each resource with the necessary activities for carrying out the investment objectives, at the same time, aiming at ensur-

ing an ever higher economic efficiency. This necessity is particularly outlined also in the law on the economic and social development of Romania [1].

The above considerations have led us to the idea of the necessity of an economic-mathematical model of correlating investments with the necessary resources. For the lack of space, we shall not present here all the results obtained but only some aspects we consider to be more representative and which should lead to the formation of an idea about the conception which was at the basis of the elaboration of the model.

At the basis of the conception on the presented model is the fact that the economic system [1]) is a dynamic, cybernetic model with feed-back.

As is only natural, it is considered that the investment plan cannot be conceived isolatedly, independently, but, as a section of the plan of the national economy, of the branch, department, etc., in which the relations of interdependence with the other plan sections are made evident. For this reason, the problem of correlation of the plan with the necessary resources can be considered at the beginning, afterwards particularizing the model for the investment plan and outlining the main aspects deriving, as a result of this particularization.

1. A MATHEMATICAL STATEMENT OF THE ECONOMIC SYSTEM

The mathematical statement of the economic system was done with the aim to serve, in the first place, as a general orientation as regards the presentation of the correlation model and, secondly, to take over some of the already existing results in the field of the theory of dynamic systems and to use them within the tackled problem.

In the following, we shall present the economic system as being a dynamic system Σ expressed in this way [3]:

$$\Sigma = (T, \bar{U}, \Omega, \bar{X}, \bar{Y}, \Gamma, \varphi, \eta) \tag{1}$$

where:

— T is the planning interval; for exemplification, we shall consider that it is equal to a year, that is, we shall refer to the annual plan.

— We consider that the input to the system represents the available resources for the implementation of the plan (Fig. 1). These resources are represented by the input variable $\bar{u}(t)$, $t \in T$, $\bar{u}(t) \in \bar{U}$. Consequently, $\bar{U}$ is the set of values which can be assumed

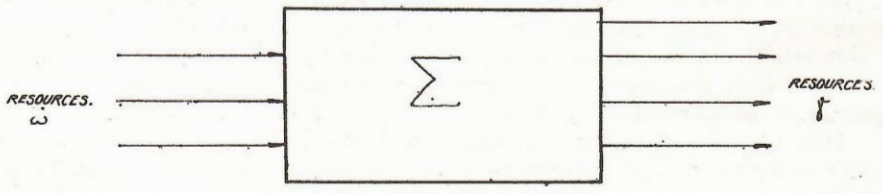


Fig. 1

by the $\bar{u}$ input variable.

[1] Generally, it is considered that the economic system is made up of a multitude of economic units and a multitude of relations outlining the interdependence of the economic units. In order to facilitate the expression and as a generalization of the notion, we mean by economic unit any enterprise, group, etc. supplying or using the resources necessary for the implementation of the plan.

— Ω is a class of functions and is defined as follows:

$$\Omega = \{\omega : T \to \bar{U}\} \tag{2}$$

where, by ω the evolution in time of the resources (of the input variable) admitted by the system is made evident, that is

$$\omega = \{\bar{u}(t)/t \in T, \bar{u}(t) \in \bar{U}\} \tag{3}$$

— The system being dynamic, it will have a succession of conditions outlining the effect of the evolution of the resources determined by ω. A condition will be marked by a variable $\bar{x} \in \bar{X}$ where $\bar{X}$ is the set of values which may be assumed by the conventional variable of condition $\bar{x}$. It is a result of its evolution in a time segment $[t_1, t_2] \subset T$. If we note by τ the initial moment of the plan, the effect of the application of ω up to a moment $t \in T$ is formalized as follows:

$$(t; \tau, \bar{x}, \{\omega(\sigma)/\tau \leqslant \sigma < t\}) \to \gamma(t) \tag{4}$$

where $\gamma(t)$ represents the output of the system at moment t (Fig. 1).

— $\bar{Y}$ represents the set of values assumed by the $\bar{y}$ output variable and

— Γ is the class of the outputs of the system

$$\Gamma = \{\gamma : T \to \bar{Y}\} \tag{5}$$

As we consider the outputs to be the resources resulting from the implementation of the plan, $\bar{Y}$ will represent these very resources, if the ω input variable, under the action of the changing process ensures the resources (output) envisaged in the plan.

— φ is the transition function of the system and defines the change in the initial condition of resources, under the action of the input variable. It is defined as follows:

$$\varphi(t; \tau, \bar{x}, \omega) = \bar{x}(t) \tag{6}$$

As it is noticed from (6), φ depends on the condition of the system at the initial moment τ and on the previous history of t of the process of transformation of the resources, explained by the introduction of the functional variable $\omega_{(\tau,t)}$

It results from (6) that:

$$\varphi(t; t, \bar{x}, \omega) = \bar{x} \tag{7}$$

that is φ reproduces the initial condition at moment $t \in T$. This property of φ is called consistency [5].

— η expresses the $\bar{y}$ output variable which is a result of the evolution of the condition variable, namely

$$\bar{y}(t) = \eta(t, \bar{x}(t)) \tag{7'}$$

will be called the output variable.

The set $\bar{X}$ is called the space of conditions, the set $T \times \bar{X}$ is called the space of phases, the pair $(t, \bar{x})$ — phase, and the set $\{\varphi(t; \tau, \bar{x}, \omega)/t \geqslant \tau\}$ is called the condition trajectory.

A complete formulation of the notion of dynamic system is to be found in [5].

2 A BRIEF PRESENTATION OF THE MODEL

In order to present some notions used for formularizing the model and some of the results obtained, we shall refer to some entirely particular cases, and in the following we shall also set forth the main generalizations obtained.

We assume that, in the period T, the investment plan envisages a number of investment stocks as well as their terms for commissioning at the planned parameters. We also assume that each investment stock is grouped on objects and in their turn, the latter have specified their structure on activites for each of them. By activity we mean a distinct part of an object which for its fulfilment needs a minimum consumption of resources noted, in the following, by U_i. Obviously, by activity we can also mean a certain object or investment stock, just as we shall agree that an investment stock may be considered an object, at certain levels of aggregation.

It is also considered that the length of fulfilment of each activity is established by assuming that the necessary resources are ensured by a single economic unit which belongs to the set of those which can ensure the necessary resources. We shall not explain the character of resources since they can be different.

The following simplifying hypotheses are supposed:

1. The plan period T envisages n investment objects and each of these objectives O^j, $1 \leqslant j \leqslant n$ is decomposed into the same number m of activities;

2. The activities pertaining to the same object are arranged in time in keeping with the technological process of fulfilment;

3. All objects have the same technological flow of fulfilment;

4. The activities of the same order i, $1 \leqslant i \leqslant m$ are carried out by the same economic unit U_i;

5. An economic unit U_i, $1 \leqslant i \leqslant m$ cannot fulfil more than one activity in the same time;

6. The following notations are considered:.

S_i^j — the activity i of the object O^j to which pertains the resource U_i (we agree to consider that the notion" the economic unit U_i" is equivalent to the notion "resource U_i" because from the point of view of treatment, U_i represents a contractor of resource necessary for the fulfilment of S_i^j. In other words, a subset of the outputs of system U_i becomes an input for system S_i^j and U_i interests us only from the point of view of its outputs which become inputs for S_i^j).

t_i^j — the time for the fulfilment of activity S_i^j, by using resource U_i.

It is considered that two or several activities $S_i^j \in O^j$ can be completely or partly fulfilled in the same time. But, to start activity $S_i^j \in O^j$, $i \geqslant 2$, it is necessary from the technological point of view to carry out a minimum part of $S_{i-1}^j \in O^j$.

We shall note the minimum time necessary for the fulfilment of such a minimum part of the activity $S_i^j \in O^j$ by a_i^j (the necessary minimum time between the starting of activity $S_{i-1}^j \in O^j$ and the starting of activity $S_i^j \in O^j$, $i \geqslant 2$, $i \in M$, $j \in N$, $N = \{1, 2, ..., n\}$, $M = \{1, 2, ..., m\}$).

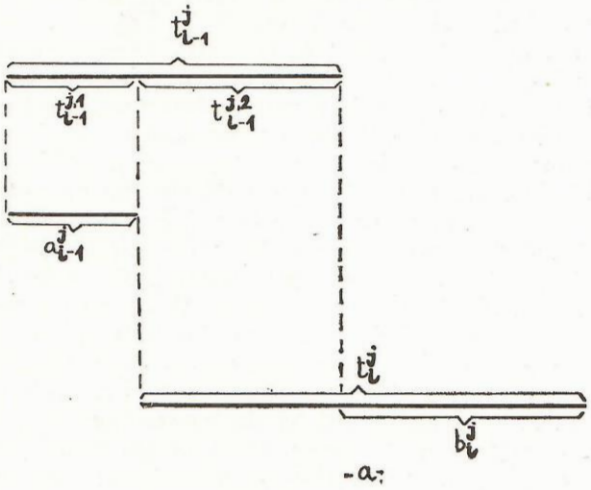
As, generally, we have $t_{i-1}^j \neq t_i^j$, $i \geqslant 2$, for ensuring the continuity (rhythmicity) of the carrying out of activity S_i^j by using resource U_i it is necessary

to start carrying out the activity S_i^j, so that it should end with a time b_i^j (the necessary minimum time, from the technological point of view between the finishing of activity S_{i-1}^j and the finishing of activity S_i^j, $i \geqslant 2$) later than the finishing of S_{i-1}^j (Fig. 2). (Figure 2 outlines the time t corresponding to activities S_i^j).

To ensure the two sizes a_i^j and b_i^j two values $t_i^{j,1}$ and $t_i^{j,2}$ are measured (Fig. 2), as follows:

If $t_i^j - b_i^j \geqslant t_{i-1}^j - a_{i-1}^j$ then

$$\left.\begin{aligned} t_{i-1}^{j,1} &= a_{i-1}^j \\ t_{i-1}^{j,2} &= t_{i-1}^j - t_{i-1}^{j,1} \end{aligned}\right\} \tag{8}$$



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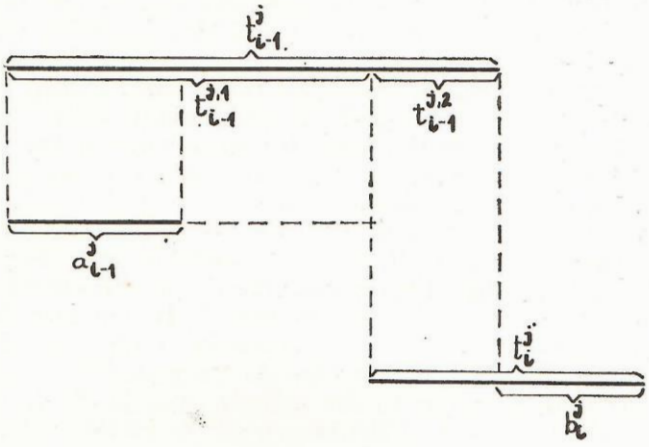

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Fig. 2

If $t_i^j - b_i^j < t_{i-1}^j - a_{i-1}^j$, then

$$\left.\begin{aligned} t_{i-1}^{j,2} &= t_i^j - b_i^j \\ t_{i-1}^{j,1} &= t_{i-1}^j - t_{i-1}^{j,2} \end{aligned}\right\} \tag{9}$$

In other words, we have measured $t_i^{j,1}$ and $t_i^{j,2}$ so that $t_i^j = t_i^{j,1} + t_i^{j,2}$, where the values of $t_i^{j,1}$ and $t_i^{j,2}$, depend on the values of t_{i-1}^j, t_i^j, a_i^j, b_i^j. If a_i^j is replaced by $t_i^{j,1}$, the continuity of work within the activity $S_i^j \in U_i$ is ensured.

The sizes a_i^j and b_i^j depend on the technological process of the fulfilment of each object separately and on other conditions of fulfilment.

Fig. 3 presents an example where $O^1 \div O^3$ are made evident and, for each object O^j, $1 \leqslant j \leqslant 3$, the main activities S_i^j $1 \leqslant i \leqslant 5$, with the times t_i^j of utilization of resources U_i, $i = 1 \div 5$. Times $t_i^{j,1}$, $t_i^{j,2}$ are measured according to relations (8) and (9).

The sizes x_i^j and y_i^j have certain economic interpretations which we shall try to outline when explaining the way of their measuring.

Thus, for example, it is noticed that x_2^2 appearing within the object O^2, activity $S_2^2 \in O^2$, is given by the difference between the moment of starting of activity S_2^2 by U_2 (a moment resulting from the fact that until then U_2 was not available) and the moment when the fulfilment of $S_1^2 \in O^2$ in the time $t_1^{2,1}$ was ensured. Thus, from this moment, activity S_2^2 could have started had U_2 been available. Since U_1 must work rhythmically, it results that S_1^2 advances with the time x_2^2 until U_2 can start the activity for $S_2^2 \cdot x_i^j$ represents the time for the formation of the production stock within activity S_{i-1}^j or the delay time of activity S_i^j. If, we suppose that S_1^2 is the activity of supply with technological equipment and S_2^2 the assemblage activity, then x_2^2 represents the time in which an equipment stock at the beneficiary is created.

We called size x_i^j — the delay time of activity S_i^j because it outlines the delay of the starting of the fulfilment of S_i^j as compared to the moment when the stage of fulfilment of S_{i-1}^j could have allowed it to start. This delay is caused by the limited capacity of the economic unit U_i (or, in other words, resource U_i is insufficient) which, because of this limitation and because of its occupation within the fulfilment of S_i^{j-1}, the activity S_i^j could not start at the moment established within the object O^j. It can be easily checked that, if we permutate O^j by O^{j-1}, we shall, generally obtain other values for x_i^j.

Size $y_i^j \geqslant 0$ represents the time difference between the moment when activity $S_i^j \in O^j$ can start, a moment imposed by the necessity of observing the technological order of fulfilment of activity $S_i^j \in O^j$, $i = 1, \div m$ and the moment when U_i becomes available for the starting of this activity. If this difference is negative, that is U_i becomes available later than the moment when activity S_i^j should start, then we can take $y_i^j = 0$. Sizes y_i^j represent the expectation time of resources U_i in the interval T. It is noticed that these sizes just as u_i^j are measured by the limited resources in the time -U_i and by the way in which objects O^j are arranged within the planning of their fulfilment.

Assuming that resources U_i are entirely available *) during the whole interval T for the fulfilment of the targets of the investment plan, in other

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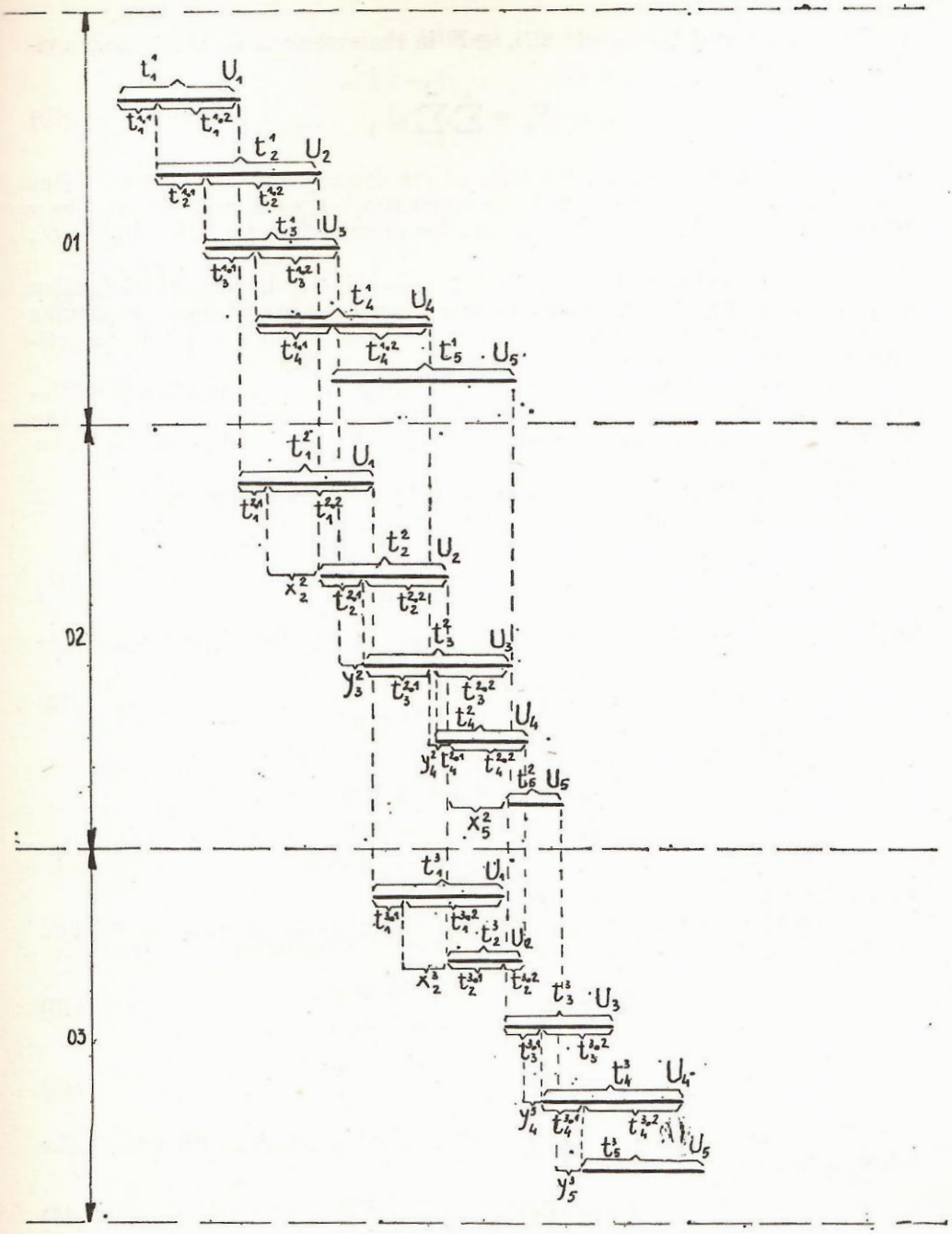
* If U_i are not entirely available, certain percentages are taken into consideration, in keeping with the availability.



- GANTT DIAGRAM -

Fig. 3

words, representing the inputs $\bar{u}(t)$, $t \in T$ in the system Σ we shall make evident the sum:

$$Y_p = \sum_{j=1}^{n} \sum_{i=1}^{m} y_i^j \tag{10}$$

which represent the expectation time of resources, from the moment τ (the beginning of the plan) and until the moment when each resource no longer represents an input size in the system, for permutation p of the objects O^j, $j = 1 \div n$.

For (10) to have a sense for $j = 1 \div n$, $i = 1 \div m$, by using the definition of y_i^j given above, it is necessary to introduce a fictitious object O°, having the same number of activities — m, but with $t_i^0 = 0$, for $i = 1 \div m$. An activity S_0^j, $j = 0 \div n$, is also introduced for each object O^j.

These completions are necessary for taking into consideration the difference of time between the beginning of the use of the first resource U_i for the activity $S_1^1 \in O_1$ and the beginning of the use of resources U_i, $i = 2 \div m$ for activities $S_i^1 \in O_1$.

We shall thus have $n + 1$ objects with $m + 1$ activities each.

Emphasizing matrix

$$T = [t_i^j] \qquad \begin{matrix} i = 1 \div m \\ j = 1 \div n \end{matrix} \tag{11}$$

whose elements are time t_i^j attached to activities S^j, the elements of matrix

$$\bar{T} = [\bar{t}_i^j] \qquad \begin{matrix} i = 0 \div m \\ j = 0 \div n \end{matrix} \tag{12}$$

where:

$$\bar{t}_0^j = 0, \text{ for } j = 0 \div n$$
$$\bar{t}_i^0 = 0, \text{ for } i = 1 \div m$$
$$\bar{t}_i^j = t_i^j, \; i = 1 \div m, \; j = 1 \div n$$

are used for measuring Y_p in (10).

From the elements of matrix (11), with the aid of relations (8) and (9), the elements of matrices are deduced:

$$T'^1 = [t_1^{j,1}] \qquad \begin{matrix} i = 1 \div m \\ j = 1 \div n \end{matrix} \tag{13}$$

$$T'^2 = [t_i^{j,2}] \qquad \begin{matrix} i = 1 \div m \\ j = 1 \div n \end{matrix} \tag{14}$$

From the elements of matrix (12) and using relations (8) and (9), the following matrices are deduced:

$$\bar{T}'^1 = [\bar{t}_i^{j,1}] \qquad \begin{matrix} i = 0 \div m \\ j = 0 \div n \end{matrix} \tag{15}$$

$$\bar{T}'^2 = [\bar{t}_i^{j,2}] \qquad \begin{matrix} i = 0 \div m \\ j = 0 \div n \end{matrix} \tag{16}$$

whose elements are obtained as follows:

$$\begin{aligned} \bar{t}_0^{j,1} &= \bar{t}_0^{j,2} = 0, && j = 0 \div m \\ \bar{t}_i^{0,1} &= \bar{t}_i^{0,2} = 0, && i = 0 \div n \\ \bar{t}_i^{j,1} &= \bar{t}_i^{j,1}, && i = 1 \div m, \quad j = 1 \div n \\ \bar{t}_i^{j,2} &= \bar{t}_i^{j,2}, && i = 1 \div m, \quad j = 1 \div n. \end{aligned} \tag{17}$$

It is noticed from Fig. 3 that the values of $x_i^j, y_i^j, i = 1 \div m, j = 1 \div n$, can be measured, in keeping with $\bar{t}_i^j, \bar{t}_i^{j,1}, \bar{t}_i^{j,2}$ (the elements of matrices (12), (15), (16), after the following relations, whose way of determination is presented in [7].

$$x_i^1 = 0, \quad i = 1 \div m; \quad x_i^2 = 0$$

$$x_2^2 = max\,(\bar{t}_2^1 - \bar{t}_1^{1,2} - \bar{t}_1^{2,1}, 0)$$

$$x_k^2 = max\left(\bar{t}_k^1 + \sum_{i=2}^{k-1} \bar{t}_i^{1,1} - \sum_{i=1}^{k-1} \bar{t}_i^{2,1} - \bar{t}_i^{1,2} - \sum_{i=2}^{k-1} x_i^2; 0\right)$$

$$k = 3 \div m \tag{18}$$

$$x_1^j = 0$$

$$x_2^j = max\,(\bar{t}_2^{j,1} + x_2^{j-1} - \bar{t}_1^{j-1,2} - \bar{t}_1^{j,1}; 0)$$

$$x_k^j = max\left[\bar{t}_k^{j-1} + \sum_{j=2}^{k-1} \bar{t}_i^{j-1,1} + \sum_{i=2}^{k} x_i^{j-1} - \left(\sum_{i=1}^{k-1} \bar{t}_i^{j,1} + \bar{t}_1^{j-1,2} + \sum_{i=2}^{k-1} x_i^j\right); 0\right],$$

$$k = 3 \div m$$

$$j = 3 \div n.$$

Similarly, the following relations are obtained:

$$\begin{aligned} y_i^0 &= 0 \\ y_1^1 &= 0 \\ y_2^1 &= \bar{t}_1^{1,1} \\ y_3^1 &= \bar{t}_k^{1,1} \end{aligned} \tag{19}$$

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$$y_i^1 = \sum_{k=1}^{i-1} \bar{t}_k^{1,1}, \quad i = 4 \div m$$

$$y_1^2 = 0$$

$$y_2^2 = \bar{t}_1^{1,2} + \bar{t}_1^{2,1} + x_2^2 - \bar{t}_2^1 \tag{20}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$y_k^2 = \bar{t}_1^{1,2} + \sum_{i=1}^{k-1} \bar{t}_i^{2,1} + \sum_{i=2}^{k} x_i^2 - \left(\sum_{i=2}^{k-1} \bar{t}_i^{1,1} + \bar{t}_k^1 \right),$$

$$k = 3 \div m$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$y_1^j = 0$$

$$y_2^1 = \bar{t}_1^{j-1,2} + \bar{t}_1^{j,1} + x_2^j - (\bar{t}_2^{j-1} + x_2^{j-1}) \tag{21}$$

$$y_k^j = \bar{t}_1^{j-1,2} + \sum_{i=1}^{k-1} \bar{t}_i^{j,1} + \sum_{i=2}^{k} x_i^j - \left(\bar{t}_k^{j-1} + \sum_{i=2}^{k-1} \bar{t}_i^{j-1,1} + \sum_{i=2}^{k} x_i^{j-1} \right)$$

By using Figure 3, we can make evident a size- the time limit- $M\,J(i,j)$, $i = 1 \div m$, $j = 1 \div n$ of the economic unit U_i whose value represents the time interval with which it can delay the completion of activity S_i^j, without affecting the continuity (rhythmicity) of the fulfilment of the other activities. For example, in Fig. 3 $S_1^2(t_1^2)$ may have a delay equal to x_2^2 without affecting the starting of activity $S_2^2(t_2^2)$. If we consider $S_1^3(t_1^3)$, which also uses U_i, its start can be delayed by x_2^2, without affecting the beginning of activity $S_2^3(t_2^3)$.

U_1 can thus have a time reserve equal to min (x_2^2, x_2^3), if we consider that it is used only for S_1^j, $j = 1 \div 3$.

Generally, we shall have:

$$MJ(i,j) = min\,(x_{i+1}^k)$$

$$k = j, n \tag{22}$$

$$i = 1 \div m - 1, \qquad j = 2 \div n$$

For $j = 1$, $\qquad MJ(i,1) = 0$ (23)

We also note:

$TP(j)$ — the term for putting into operation, at the projected parameters, object j, established by the plan, $1 \leqslant j \leqslant n$

$TO(j)$ — the term for putting into operation, at the projected parameters, object j, established with the aid of relation

$$TO(j) = \sum_{i=1}^{m-1} \bar{t}_i^{j,1} + \sum_{i=2}^{m} x_i^j + \bar{t}_m^j \tag{24}$$

We shall also have

$$MJ(m,j) = \min_{k=l+1 \div n} (y_m^k) +$$

$$min[TP(l) - TO(l) - min(y_m^k)] \tag{25}$$

$$l = j \div n \qquad k = j + 1 \div n$$

It can be noticed that we may have $MJ(m, j) < 0$. We shall note

$$MT(j) = \sum_{i=1}^{m} MJ(i, j) \tag{26}$$

$$j = 1 \div n$$

and we shall say that MT is the internal time reserve of system or, that it represents the transition function φ of system Σ.

It is noticed that in order to have the desired outputs of the system that is, for the terms of the putting into operation of the objects at the projected parameters to be observed, we must have $MT(1) \geqslant 0$.

Relations (22), (23), (24) represent an instrument for establishing the time reserve for each U_i, $i = 1 \div m$.

We note:

A_i^j — The moment for starting activity S_i^j

B_i^j — The moment for completing activity S_i^j

$$C_i^j \triangleq A_i^j + t_i^{j,1}$$

It results from Fig. 3 that:

$$\begin{aligned} A_1^1 &= \tau \\ A_i^1 &= C_{i-1}^1, \qquad i = 2 \div m \\ B_i^1 &= A_i^1 + t_i^1 \quad i = 1 \div m \\ A_i^j &= B_i^{j-1} + y_i^j \\ B_i^j &= A_i^j + t_i^j \quad i = 1 \div m \end{aligned} \tag{27}$$

Considering that in the interval A_i^j, B_i^j for the fulfilment of activity S_i^j, function $\omega_i(t)$ (which outlines the evolution of resource U_i) is constant, it can be defined as follows:

$$\omega_i(t) = \begin{cases} 0, \text{ for } t = A_1^1 \\ \dfrac{U_1}{t_1^j}, \text{ for } A_1^j < t < B_1^j, \quad j = 1 \div n \end{cases} \tag{28}$$

$$\omega_i(t) = \begin{cases} 0, \text{ if } \begin{cases} \tau \leqslant t \leqslant C_{i-1}^1 \\ B_2^{j-1} < t \leqslant A_2^j \end{cases} \quad \text{or} \\ \dfrac{U_i}{t_i^j} \text{ if } A_i^j < t \leqslant B_i^j, \quad j = 2 \div n \end{cases} \tag{29}$$

Relations (28) and (29) cannot, however, be satisfactory, because, for $t \in (A_i^j, B_i^j)$, for j constant, $\omega_i(t)$ is not continuous as results from (28) and (29). If the economic unit U_i is considered, in its turn, to be a dynamic system, one can apply a model of dynamic correlations of its resources with the internal plan which is determined in keeping with the fulfilment of S_i^j and, impli-

citely, the way in which its resources are used in the considered period, results. To facilitate the expression, when referring to $\omega_i(t)$ we shall quote relations (28) and (29).

Recapitulating those shown above, we can note that within the dynamic system:

$$\Sigma = (T, \overline{U}, \Omega, \overline{X}, \overline{Y}, \Gamma, \varphi, \eta)$$

1. T — the plan period, equal to a year, for exemplification
2. $\overline{U}$ — the set of values of the input variable $\bar{u}(t)$ obtained with the aid of relations (28) and (29), if we consider that $\omega_i(t) = \bar{u}(t)$.
3. Ω — the class of functions $\omega_i(t)$, $i = 1 \div m$, defined by relations (28) and (29).
4. It can be noticed that the value of $MT(j)$ given by expression (26) has the property of defining the change in the initial condition and resources, a change depending on t, on τ, on the initial condition, namely at moment τ and on the evolution of the input variable (resources) in the interval (τ, t).

This can be noticed, if we follow the way in which $MT(j)$ was deducted with the aid of relations (22) and (26). If we consider $MT(1) = \sum_{i=}^{m} MJ(i, 1)$, we shall have the initial form of the condition of the system, that is $\bar{x}(\tau) = \sum_{i=1}^{m} MJ(i, 1)$. If we want to know condition $\bar{x}(t)$ at moment t, we must know the evolution of the input variable ω in the interval (τ, t), recalculate x_i^j, y_i^j if, due to some perturbating factors the system failed to develop according to the pre-established condition and apply relations (22) and (26) for the number of objects and activities still to be carried out.

In conclusion, by condition $\bar{x}(t)$ of the system, we mean the possibility at the moment $\bar{t}$ to obtain the outputs γ of the system Σ we have proposed (a possibility conditioned by the moment t; τ, $\bar{x}(\tau)$ and by a certain evolution of its resources in the interval (τ, t) and established with the aid of the size MT, calculated with the aid of relation (26).

5. $\overline{Y}$ represents the set of values of the resources as a result of the carrying out of activities S_i^j, $i = 1 \div m$.

6. Since it is supposed that in the (A_i^j, B_i^j) interval, activity S_i^j is fulfilled by using resource U_i, its completion is outlined by indicating the moment B_i^j. We can consider $\gamma_i^j : (A_i^j, B_i^j) \to \overline{y}_i^j$ to be a monotonously growing function and that

$$\gamma_i^j(t) = \begin{cases} 0, \text{ if } t \in T \setminus (A_i^j, B_i^j) \\ \overline{y}_i^j(t) \text{ if } t \in (A_i^j, B_i^j) \end{cases} \tag{30}$$

where $\overline{y}_i^j$ is a certain value of the resource resulting from the carrying out of part of S_i^j, consuming part of resource U_i in the interval (A_i^j, B_i^j). Consequently:

$$\Gamma = \{\gamma_i^j : T \to \overline{Y} / i \in M, j \in N\} \tag{31}$$

We cannot specify more about the behaviour of function γ_i^j in the interval (A_i^j, B_i^j), in this stage. However, we can affirm that the evolution of the

fulfilment of activity S_i^j depends on the respective technological process and on the way in which the economic unit U_i is fulfilling its tasks stipulated by contract in the interval (A_i^j, B_i^j). To abide by the targets stipulated by contract, unit U_i which, in its turn, is considered to be a dynamic and cybernetic system, may apply the same model of aggregation of resources with the internal plan.

7. The transition function φ of system Σ is given by $MT(j), j = 1 \div n$, which is made evident by relation (26). It can be noticed that:

— if $MT(j) = 0$ it results that the investment plan is fulfilled at the commission terms envisaged in the plan if each resource is used at the terms established by the plan;

— if $MT(j) < 0$, it signifies the impossibility of fulfilling the investment plan, namely, at least object 0^j will have a delay equal to $- MT(j)$

— if $MT(j) > 0$, we shall have an advance of objects $0^j, j \div n$, at least by a time interval equal to $MT(j)$.

8. By using the transition function $\varphi(t; \tau, \bar{x}, \omega)$ as an index of the possibility of fulfilling the investment plan, $\eta(t, \bar{x}(t))$ may be considered as a function for establishing resources $y_i^j(t)$ whose stages of implementation are revealed by the value of this index.

We shall not indicate the establishment of resources $y_2^j(t)$ with the aid of $\eta(t, \bar{x}(t))$ at least in this study. We prefer to outline their values through relation (30) and to check the stage of their fulfilment with the aid of relations (22), (27), that is, in the end with $\varphi(t, \tau, x, \omega)$.

The whole construction of the model was achieved so far by considering a certain permutation $p \in P$ of objects $0^j, j = 1 \div n$. It can be easily noticed that if we consider another permutation $p' \neq p$, $p' \in P$, where P is the set of permutations of the objects 0^j, $j \in N$, we shall obtain, in general, other values for the sizes outlined in the model. This remark leads to the determination of optimization algorithms (minimizing, maximizing) of some of the variables which interest us. For example, a mathematical algorithm can be built for minimizing the expectation time y_i^j, that is

$$Z = \min_{p \in P} \sum_{j=1}^{n} \sum_{i=1}^{m} y_{i,p}^j \tag{32}$$

by considering the abidance by the terms for the commissioning of the investment stocks.

For the lack of space, we shall not present here the general model in which all simplifying hypotheses are eliminated. We shall enumerate only part of the hypotheses which have been eliminated. Thus, instead of hypothesis 2, it is considered that each object $0^j, j = 1, m$, is decomposed into at most m activities (it cannot assume a certain whole value; in the event of using the system for simulating various plan variants, the increase of m does not raise the calculation time too much, as it happens when we want to use optimization criteria).

Instead of hypothesis 3, it is considered that the technological flow for each object may be any, etc.

The model is conceived so as to take into consideration both the wear of the fixed assets and the new commissionings of the production units.

3. CONCLUSIONS

The attained mathematical model can be used for obtaining results outlining the spacing out in time of investments and the real possibilities of their fulfilment, at the terms envisaged in the plan and at the projected parameters.

The model may be applied without taking into consideration optimization criteria. For example, it can be used within the drawing up of the investment plan for certain branches of the national economy and within the correlation of resources with it. The possible deficiences concerning the correlation of resources with the investment targets and the observance of the commission terms which might appear as a result of the impossibility of solving in a short time and with the "classical" techniques the voluminous analytical calculations required by the dynamic correlation of the investment plans may be eliminated in this way.

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A PROPOS D'UN MODELE D'ALLOCATION DES RESSOURCES POUR LA REALISATION DES INVESTISSEMENTS

Le modèle présenté est conçu en vue de son utilisation pour l'obtention de l'allocation dynamique des ressources destinées à réaliser les investissements.

L'ordonnancement des ressources à la multitude des investissements est effectué en tenant compte de certaines fonctions objectives, telles que, par exemple, $z = \min \sum\sum Y_{ij,p}$ $p \in P_{ij}$ où $Y_{ij,p}$ représente le temps d'atente de la ressource U_i, dans l'investissements O^j, et P est la multitude des permutations des investissements O^j, $j = 1 \div N$.

ОБ ОДНОЙ МОДЕЛИ ВЫДЕЛЕНИЯ РЕСУРСОВ ДЛЯ ОСУЩЕСТВЛЕНИЯ КАПИТАЛЬНЫХ ВЛОЖЕНИЙ

Представляемая модель разработана в целях использования для осуществления динамического выделения ресурсов для выполнения капитальных вложений.

Выделение ресурсов по множеству капитальных вложений производится с учетом целевых функций, как, например :

$$Z = \min_{p \in P} \sum_{i} \sum_{j} Y_{ij,p},$$

где $Y_{ij,p}$ представляет собой время ожидания ресурса U_i в рамках капитального вложения O^j, а P является множеством перестановок капитальных вложений O^j, $j = 1 \div N$.

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