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Determination of the Volume of Constant Expenses and Its Applications in Industrial Enterprises

In this paper the authors suggest certain mathematical methods by means of which it is possible to measure exactly the influence of the increase in production volume on the volume of the expenses of a conventionally constant character.

The documents of the IXth Congress of the Romanian Communist Party stress the necessity of improving the planning and management methods as well as the system of indicators used in view of stimulating the enterprises to assume increased tasks regarding production, productivity and the decrease of cost price. This implies the adoption of more advanced economic methods in management, thus giving an incentive to the enterprises and encouraging them to develop more fully their creative initiative with an aim to improve their own activity.

The stimulation of the enterprises along this path requires, among other things, the most accurate determination of the degree in which the factors contributing to the decrease in cost price influence the latter and, in the first place, the factors dependent on the own efffort of the enterprises, and on their contribution to the decrease of production costs. An insufficient knowledge of the factors that influence the level of production expense hinders the enterprises in using their internal reserves to obtain progressive indicators and weakens the application of the principles governing their socialist management.

Among the important factors entering in the decrease of cost price there are : increase in production volume, increase in labour productivity, decrease in specific use of material, decrease in expenses connected with production, decrease in losses through rejects, and improvement in the indices of equipment use.

In spite of the importance of the increase in production volume on the decrease in cost price, the specialty literature does not give it the importance it deserves. Under the conditions of continuous development and at a high rate of production volume, the effects of this factor increase in importance, their character being permanent, similarly to the increase in labour productivity, the decrease in specific consumption, etc.

To be able to compute the degree to which the increase in production volume influences the decrease in cost price a corresponding grouping of production expenses is required.

The production expenses of an enterprise are grouped according to certain points of view:

a) From the standpoint of the importance of their connection with the technological process, production expenses are divided into basic expenses and supplementary expenses;

b) From the standpoint of the distribution of expenses on cost price, they are grouped in direct expenses and indirect expenses;

c) According to their structure they are divided into simple (elementary) expenses and complex expenses;

d) From the standpoint of their connection with the change in production volume, the expenses comprised in the cost price of production are divided into two groups: 1) conventionally variable expenses; 2) conventionally constant (fixed) expenses.

The last group is of particular importance for the aims of this paper and we shall therefore study it to a greater length.

By conventionally variable expenses is understood those expenses whose absolute level or whose total sum changes in proportion with the change in production volume; such are: raw materials and basic materials, simple piecework workers' salaries. The level of these expenses per commodity unit, established by specific consumptions, the salaries dependent on the time standard and the tariff network for the given class of work, etc., are not subjected to changes as a function of the increase or decrease in production volume, so that a proportionality appears between the production volume and the volume of this category of expenses. Obviously, this mathematical ratio, valid for a period of time, has not a long-lasting character since both through the increase in production volume and through other technical and organization steps, possibilities are found to decrease the level of variable expenses per commodity unit, and this will influence the existing proportionality.

The expenses of a conventionally constant (fixed) character comprise those expenses whose absolute sum is not dependent on production volume, i.e. they remain unchanged or they do not change proportionally with respect to the change in production volume. Examples of this sort of expenses are: maintenance, lighting and heating of buildings, current repairs of buildings, salaries of auxiliary workers, salaries of the technical administrative staff of the deprtaments and general expenses of the enterprise.

From the examples of conventionally constant (fixed) expenses there results that this group of expenses, from the standpoint of distribution, is generally included in indirect expenses, while the direct expenses generally comprise conventionally variable expenses. This link between the direct expenses and the conventionally variable expenses, and between the indirect expenses and the conventionally constant expenses, is characteristic without, however, being general. Thus, in the framework of indirect expenses, besides the conventionally constant expenses there stand out a series of expenses of a variable character. For instance, fuel consumption and technological power stand out in most enterprises as an indirect expense (account 130—04), the contributions to social insurance and on direct salaries (account 130—06) are

directly proportional to the volume of direct salaries and thus have the character of a variable expense, and a series of expenses having an indirect character may have either the character of a variable expense or the character of a fixed expense. For example the increase in production volume obtained by an improvement in the indices of intensive use of the equipment will not increase the amortization, while the increase in the indices of extensive use will, in some cases — or when passing over to several shifts — also modify the absolute volume of amortization included in production expenses.

Current repairs will, on account of the time employed to carry them out, influence the volume of production by decreasing the working time of the equipment.

Particularly indirect expenses of a complex character, included in the evidence of account 130—"common expenses of the production department" —, although they are made to stand out, in the accounting, in a sufficient number of 16 accounts of the IIIrd degree, cannot be delimited with the required accuracy either in expenses of a constant character or in expenses of a variable character.

Considering both the expenses of a constant character and the expenses of a variable character as known magnitudes, the cost price per commodity unit can be determined by means of a simple relation.

The cost price per commodity unit will be:

$$c = \frac{a}{x} + b \qquad (1)$$

a = unit cost price
b = expenses of a constant character within a period
c = expenses of a variable character per commodity unit
x = quantity of comodities manufactured for a period

and the total production expenses for a period will be:

$$\Sigma x \times c = a + \Sigma x \times b \qquad (2)$$

Let us consider the following hypothetical example: the constant character expenses total 500,000 lei monthly, the variable character expenses are 2,500 lei/commodity unit x. In case a volume of 100, 200,..., 1,100, 1,200 units of commodity x (the only commodity manufactured) is manufactured, the unit cost price will vary according to the following table:

Quantity manufactured monthly	Constant expenses per unit	Variable expenses per unit	Total cost price per unit
x	$a:x$	b	c
100	5,000	2,500	7,500
200	2,500	2,500	5,000
300	1,667	2,500	4,167
400	1,250	2,500	3,750
500	1,000	2,500	3,500
600	833	2,500	3,333
700	714	2,500	3,214
800	625	2,500	3,125
900	556	2,500	3,056
1,000	500	2,500	3,000
1,100	455	2,500	2,955
1,200	417	2,500	2,917

The cost price will vary between the (theoretical) limits of 502,500 le per commodity unit in the exaggerated case of a monthly production of only 1 commodity unit and 2,500.5 lei per commodity unit in the case of a very large production volume (1 million units), assuming in all cases the strict maintenance of the volume of constant expenses.

Plotting the above example we shall have the cost price per commodity unit (see diagram 1), and the total production expenses (see diagram 2).

The decrease in the specific weight of constant character expenses constitutes one of the main sources in view of reducing the cost price of production. The constant character expenses have the peculiarity of being included in the structure of production cost price, in a ratio that changes in a reverse proportion with the magnitude of planned production fulfilment.

Practically, and on basis of the book-keeping data, there is no possibility to determine accurately the absolute volume of constant character expenses, separately from the absolute volume of variable character expenses.

A most accurate determination of the volume of constant character expenses is important not only theoretically, but particularly practically, since in the case of a continuous increase in production volume the effect of the relative decrease of these expenses on the decrease in cost price is continuous. Owing, however, to the different rate of increase of production volume in the various branches of industry and, within each branch, in the various enterprises, the influence exerted embraces a wide range.

In case the effect of inverse proportion between the constant character expenses and the production volume increase, manifested in the decrease of unit cost price, respectively in the total volume of production expenses, is not taken into consideration individually, its result is the establishment of planned tasks for the decrease of cost price and the achievement of a volume of economies that are not stimulating. However, in view of determining this effect it is necessary, besides knowing the increased task of the production plan, to establish most accurately the volume of constant character expenses. These data being known, the change in production cost price, due to the change in specific weight of the constant expenses consequently to an increase in production volume, can be defined as follows:

$$E_a = \left(\frac{a}{\Sigma x_1 \times c_0} - \frac{a}{\Sigma x_0 \times c_0} \right) \Sigma x_1 \times c_0 \qquad (3)$$

where: E_a = economies achieved by changing the specific weight of constant expenses within the total volume of expenses;

a = absolute sum of constant expenses in the basic period;

$x_1 \times c_0$ = production expenses at the level of the basic period for the effective (planned) production;

$x_0 \times c_0$ = production expenses in the basic period for the production in the basic period.

For example, the constant character expenses in the basic year amount to 12 million lei out of the total of 100 million lei production expenses. A production volume increase of 15% was provided in the plan. There results:

$$E_a = \left(\frac{12}{115} - \frac{12}{100} \right) \times 115 = -1.8 \text{ million lei.}$$

As shown before, the constant character expenses cannot be extracted accurately from the book-keeping data since they are recorded in synthetic accounts "Common expenses of production departament" and "General expenses of the enterprise". The application of some mathematical methods is required in view of determining the volume of constant expenses accurately.

If we represent graphically the production volume successively achieved in a sufficiently large number of periods we will notice that, between the time factor *(y)* and the production volume *(x)* there is a close relation of a linear form $y = b \times x$, where the slope of the straight line is determined by the rate of increase of the physical volume of production. The production expenses for these production volumes will appear between two limits; in case all the expenses were variable and not reduced per commodity unit with respect to the basic period, this would show as a regression line parallel to the line representing the evolution of the physical volume having the same slope of parameter b; in case all the expenses were constant the value of b would be zero and the regression line would be parallel to the axis x. Between these two extremes, which do not occur in practice, the regression line of production expenses acquires a slope that is inferior to the regression coefifcient of the lin showing the physical volume of production. This slope is the resultant of the influence of two main causes: the absolute decrease, per commodity unit, of the variable character expenses (decrease in the specific consumption of raw material, decrease of direct salaries per commodity unit, etc.) and the relative decrease of constant character expenses (decrease in specific weight of constant expenses from total production expenses).

The influence of the decrease in variable character expenses on the decrease in production cost price is dependent on a series of factors taken into consideration when making the plan, in the plan of technical organization measures, and they are determined in actual achievements by the manner in which these measures are carried out, by the results obtained consequently to the current of innovations, etc. These influences are dependent on a series of factors which vary from enterprise to enterprise and from period to period.

For this reason it is not possible to have a single mathematical method that can be applied for all the enterprises and all the periods in view of determining the evolution of the level of variable character expense (slope of regression line b from $y = a = + bx$).

In view of determining the constant character expenses included in the manfacturing expenses and to obtain as accurate as possible a result it is necessary to collect the data for at least 6—12 months preceding the period for which the computation is made (the year of the plan). We eliminate from the total manufacturing expenses, the direct expenses — which are, in their great majority, expenses of a variable character. On basis of the series of values representing the physical volume (the value) of production and indirect expenses in the 6—12 months preceding the period analyzed, we can determine the mean magnitude of the constant character expenses for a period.

For this mathematical determination two methods of computation are suggested:

1) Considering that in the relatively short period of 6—12 months no essential changes have occurred in the framework of indirect expenses regarding the relative magnitude of variable character expenses, although fortuitous influences may occur from one month to another, this being the reason for which it is necessary to take into consideration at least 6 months in the computation, we must determine the value of parameter a in the equation $y = a + bx$.

Since the series of values will present, under the influence of fortuitous factors, deviations from the straight line, the data are adjusted by means of the method of the smaller squares. The essence of this method of correlation consists in determining the parameters a and b of the regression function so that the sum of the squares of the actual value deviations of the resultant characteristic y be at a minimum in comparison to the corresponding values of the function y. Setting the condition for $\Sigma (y - Y)^2 =$ minimum, a system of normal equations is obtained which, when solved, yield the values of the parameters a and b, viz.:

Function $f = \Sigma (y - a - bx)^2$ is a minimum if its partial derivatives with respect to a and b:

$$\frac{\delta f}{\delta a} = 2 \Sigma (y - a - bx)$$

$$\frac{\delta f}{\delta b} = 2 \Sigma (Y - a - bx)(-x)$$

are zero.

Equalling the partial derivatives to zero:

$$2 \Sigma (y - a - bx) = 0$$

$$2 \Sigma (y - a - bx)(-x) = 0$$

we find the systems of normal equations:

$$na + b\Sigma x = \Sigma y$$

$$a \Sigma x + b\Sigma x^2 = \Sigma xy$$

Solving this system we obtain:

$$a = \frac{\begin{vmatrix} \Sigma y & \Sigma x \\ \Sigma xy & \Sigma x^2 \end{vmatrix}}{\begin{vmatrix} \Sigma n & \Sigma x \\ \Sigma x & \Sigma x^2 \end{vmatrix}} = \frac{\Sigma y \cdot \Sigma x^2 - \Sigma x \cdot \Sigma x \cdot y}{n \cdot \Sigma x^2 - (\Sigma x)^2} \qquad (4)$$

$$b = \frac{\begin{vmatrix} \Sigma n & \Sigma y \\ \Sigma x & \Sigma xy \end{vmatrix}}{\begin{vmatrix} \Sigma n & \Sigma x \\ \Sigma x & \Sigma x^2 \end{vmatrix}} = \frac{n \cdot \Sigma xy - \Sigma x \cdot \Sigma y}{n \cdot \Sigma x^2 - (\Sigma x)^2} \qquad (5)$$

Considering y = production value, x = indirect expenses, n = number of periods considered, the constant character expenses a will be stablished by applying formula *(4)*.

2) Besides the method of the smallest squares known in statistics, we present a second safer method for prospective computations, by means of which the volume of constant expenses can be determined.

The expenses of a constant character are equal to the difference between the mean indirect expenses and the ratio between the mean square deviation of these expenses and the production volume variation coefficient.

We have:

$$\frac{\frac{n \times \sigma y}{\sigma x}}{\frac{\bar{x}}{n}} = \frac{\Sigma y}{n} - \frac{\sigma y}{\frac{\sigma x}{\bar{x}}} = \bar{y} - \bar{x} \sqrt{\frac{\Sigma (y - \bar{y})^2}{\Sigma (x - \bar{x})^2}} \qquad (6)$$

Here is an example (keeping $y = a + bx$)

Month	Production (thousand lei) x	Indirect expenses (thousand lei) y
1	10,000	5,000
2	10,500	5,200
3	12,000	5,800
4	13,000	6,200
5	14,000	6,600
6	15,500	7,200
Total	75,000	36,000

We compute the necessary values for the formulae *(4)* and *(6)*:

	$x \times y$	x^2
1	50,000,000	100,000,000
2	54,600,000	110,250,000
3	69,600,000	144,000,000
4	80,600,000	169,000,000
5	92,400,000	196,000,000
6	111,600,000	240,250,000
	458,800,000	959,500,000

	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$
1	−2,500	6,250,000	−1,000	1,000,000
2	−2,000	4,000,000	− 800	640,000
3	− 500	250,000	− 200	40,000
4	+ 500	250,000	+ 200	40,000
5	+1,500	2,250,000	+ 600	360,000
6	+3,000	9,000,000	+1,200	1,440,000
	0	22,000,000		3,520,000

There results:

$$a = \frac{\Sigma y \times \Sigma x^2 - \Sigma x \times \Sigma xy}{n\,\Sigma x^2 - (\Sigma x)^2} = \frac{36{,}000 \times 959{,}500{,}000 - 75{,}000 \times 458{,}800{,}000}{6 \times 959{,}500{,}000 - 75{,}000 \times 75{,}000} = 1{,}000 \quad (4)$$

or

$$a = \bar{y} - \bar{x}\sqrt{\frac{\Sigma(y-\bar{y})^2}{\Sigma(x-\bar{x})^2}} = 6{,}000 - 12{,}500\sqrt{\frac{3{,}520}{22{,}000}} =$$

$$= 6{,}000 - 12{,}500\sqrt{0.16} = 1{,}000 \quad (6)$$

In case parameters a and b refer to a linear function without deviations, the two formulae yield the same result.

The following example (with deviations from the linear function) is closer to cases which occur in practice:

Month	Production (thousand lei) x	Indirect expenses (thousand lei) y	$x \times y$	x^2
1	10	5.5	55	100
2	10.5	5.6	58.6	110.25
3	12	5.8	69.6	144
4	13	6.1	79.3	169
5	14	6.4	89.6	196
6	15.5	6.6	102.3	240.25
Total	75.0	36.0	454.6	959.5
Average	12.5	6.0		

Month	$(x - \bar{x})$	$(x - \bar{x})^2$	$(y - \bar{y})$	$(y - \bar{y})^2$
1	−2.5	6.25	−0.5	0.25
2	−2.0	4	−0.4	0.16
3	−0.5	0.25	−0.2	0.04
4	−0.5	0.25	+0.1	0.01
5	+1.5	2.25	+0.4	0.16
6	+3.0	9	+0.6	0.36
		22.0		0.98

$$a = \frac{\Sigma y \times \Sigma x^2 - \Sigma xy \times \Sigma x}{n \Sigma x^2 - (\Sigma x)^2} = \frac{36{,}959 \times 5 - 75{,}454 \times 6}{6{,}959 \times 5 - 75 \times 75} = \frac{447}{132} = 3.386$$

$$a = \bar{y} - \bar{x}\sqrt{\frac{(y-y)^2}{(x-x)^2}} = 6 - 12.5 \times \sqrt{\frac{0.98}{22}} = 6 - 12.5 \times \sqrt{0.044545} =$$

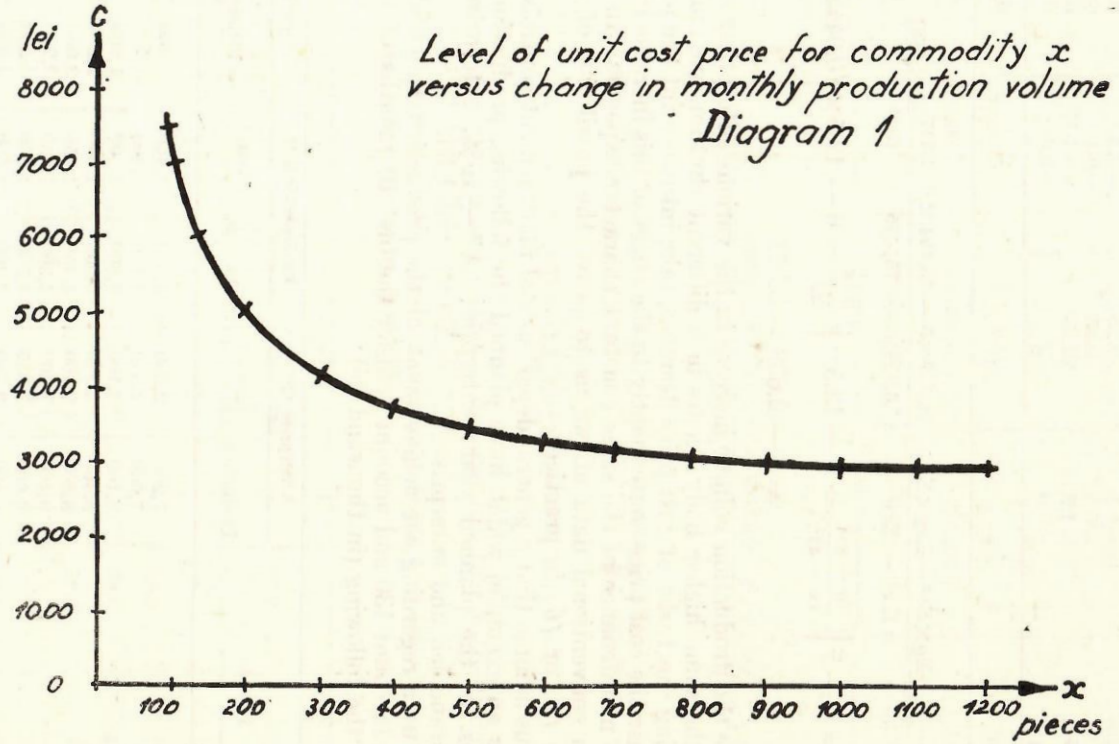
$$= 6 - 2.638 = 3.362$$

Since the production volume increase in the various enterprises subordinated to the same higher body occurs in a different rhythm, one must, when establishing the tasks of cost price decrease, take into account the possibility to decrease the cost price consequently to the effect of this increase in parallel with the maintenance of the same constant character expenses. An example based on conventional data allows us to prove the possibility of applying formulae *(4)* or *(6)* in practice:

Let us assume that a general departmental management controls 3 enterprises (as an example) which have planned the following production volume increasses for the planned year: enterprise "A" — 10%, enterprise "B" — 500 thousand lei, and enterprise "C" — 200 thousand lei.

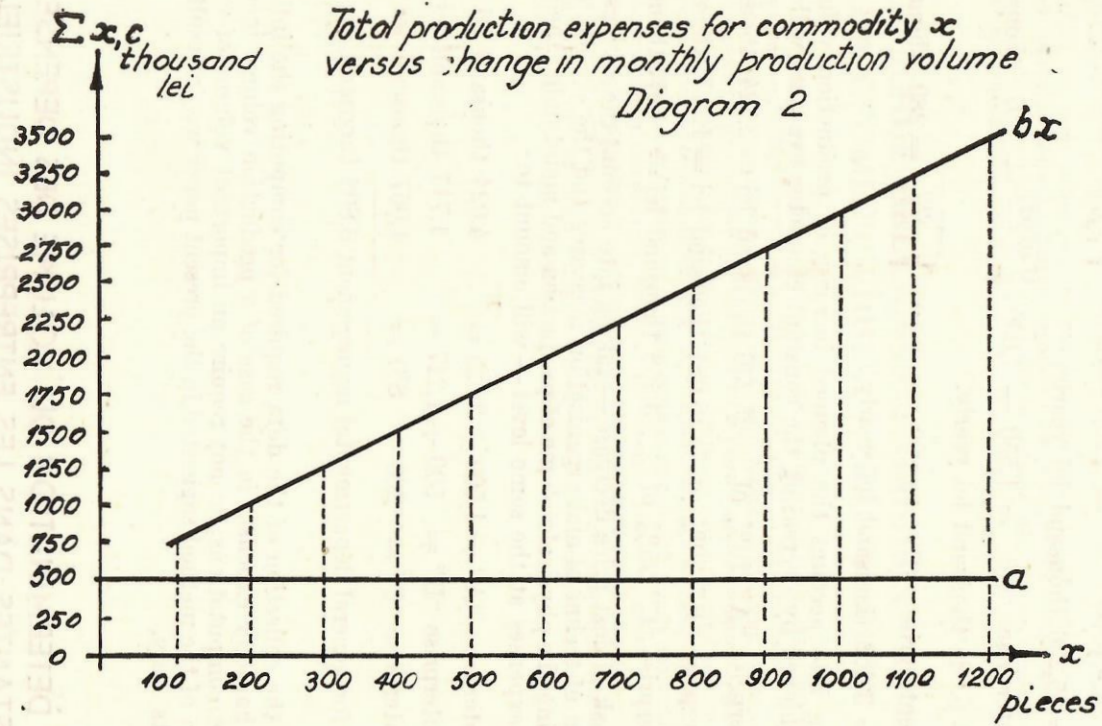
The data regarding the achievement of the production plan and indirect expenses (account 130 and account 134) for the first 10 months of the basic year are the following (in thousand lei);

Period	Enterprise "A"		Enterprise "B"		Entreprise "C"	
	Production	Account 130 + 134	Production	Account 130 + 134	Production	Account 130 + 134
I	7,200	2,600	3,500	1,400	800	400
II	7,600	2,850	3,400	1,400	1,200	550
III	8,100	3,100	3,800	1,500	1,600	650
IV	8,300	3,200	3,900	1,550	2,200	800
V	8,800	3,400	4,400	1,650	3,000	950
VI	9,400	3,600	4,300	1,650	3,200	1,000
VII	9,600	3,650	4,500	1,700	3,500	1,050
VIII	9,800	3,700	4,400	1,600	4,500	1,100
IX	8,600	3,400	4,000	1,550	3,200	900
X	8,100	3,000	3,800	1,500	1,800	600
Total 10 months	85,500	32,500	40,000	15,500	25,000	8,000



Level of unit cost price for commodity x versus change in monthly production volume

Diagram 1



Total production expenses for commodity x versus change in monthly production volume

Diogram 2

What is the volume of savings that can be achieved by each enterprise in part?

Applying formula *(6)*, we obtain the magnitude of the constant character expenses:

For enterprise "A" = 3,250 — 8,550 $\sqrt{\frac{1.2}{6.665}}$ = 2,103 thousand lei monthly = 25,236 thousand lei yearly.

For enterprise "B" = 1,550 — 4,000 $\sqrt{\frac{0.0925}{1.36}}$ = 507 thousand lei monthly = 6,084 thousand lei yearly.

For enterprise "C" = 800 — 2,500 $\sqrt{\frac{510}{122,000}}$ = 289 thousand lei monthly = 3,468 thousand lei yearly.

Taking into account the planned increase of production volume, the savings achieved by decreasing the constant character expenses will be:

— for enterprise "A" 10% of 25,236 thousand lei = 2,524 thousand lei
— for enterprise "B" 20% of 6,084 thousand lei = 1,217 thousand lei
— for enterprise "C" 25% of 3,468 thousand lei = 867 thousand lei

and the task of cost price decrease — taking into account the savings forecast in the plan of technical and organization measures and the savings achieved by increasing the physical volume of production and maintaining the constant character expenses at the same level — will amount to:

for enterprise "A" = 1,500 + 2,524 = 4,024 thousand lei
for enterprise "B" = 500 + 1,217 = 1,717 thousand lei
for enterprise "C" = 200 + 867 = 1,067 thousand lei

Total for general departmental management 6,808 thousand lei

Since the collection of the data required for computing the influence of constant character expenses in the case of a production volume increase, as well as the computations, do not require an important volume of work, the application of the method suggested in the present paper would result in positive effects only.

LA DETERMINATION DU VOLUME DES DÉPENSES CONSTANTES DANS LES ENTREPRISES INDUSTRIELLES

Les rythmes différents d'accroissement du volume de la production des entreprises dans les différentes branches de l'industrie, et dans le cadre de ces branches, déterminent la réduction en proportions différentes du prix de revient, comme con-

séquence de la réduction des dépenses avec un caractère conventionnel-constant par produit. Si le rapport inversement proportionnel entre la réduction des dépenses avec un caractère conventionnel-constant et l'accroissement du volume de la production, n'est pas pris en considération d'une manière différenciée, comme c'est le cas de la réduction du prix de revient par unité de produit, il s'en suit l'établissement de tâches de plan non efficaces Afin d'éviter de telles déficiences, il est nécessaire non seulement de connaître les tâches. d'accroissement de la production, mais aussi d'établir avec précision les dépenses conventionnelles-constantes.

En partant de cette nécessité, les auteurs proposent et donnent comme exemples des méthodes mathématiques au moyen desquelles on peut mesurer exactement l'influence de l'accroissement du volume de la production sur le volume des dépenses avec un caractère conventionnel et constant.

ОПРЕДЕЛЕНИЕ ОБЪЕМА ПОСТОЯННЫХ РАСХОДОВ НА ПРОМЫШЛЕННЫХ ПРЕДПРИЯТИЯХ

Различные темпы роста объема производства в отдельных отраслях промышленности и — в их пределах — различные темпы входящих в эти отрасли предприятий обусловливают снижение себестоимости в различных пропорциях в результате сокращения расходов условно-постоянного характера на продукцию. Непринятие в расчет дифференцированным образом обратно пропорционального соотношения между сокращением расходов условно-постоянного характера и ростом объема производства, проявляющегося в снижении себестоимости на единицу продукции, может привести к установлению немобилизующих плановых заданий. Во избежание подобного недостатка необходимо, помимо знания заданий по росту производства, как можно более точное установление условно-постоянных расходов.

Исходя из этой необходимости, авторы предлагают и дают примеры отдельных мате матических методов, при помощи которых можно точно измерять влияние роста объема производства на объем расходов условно-постоянного характера.