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Using the Method of Multiple Supply Scheduling

In any kind of problem regarding the stock administration, the main decisions which have to be taken are the answers to two questions:
1) *When to be bought (produced)?*
2) *How much to be bought (produced)?*

Ernest Koenigsberg

One of the main problems of any enterprise is the organization of technical-material supply.

In case of the purchase of a large assortment of products from a supplier, the application of the method of independent optimization*) and of that of global optimization**) does not yield good results, because they have a unilateral character: the first method allows the taking into account of the stocking costs for various products, but, it does not ensure the cumulation of orders for various products, leading to very high total costs for the launching of orders. On the contrary, by applying the method of global optimization, the costs for the placing of orders attain a very low level, in exchange, the stocking costs take into account the total products alone, that is disconsidering the diversity of these costs on products; consequently, the total launching and stocking cost often exceeds the cost determined by the independent optimization of the supply periodicity.

Following the blending of the advantages of the two methods, *the method of multiple resupply scheduling* results. It consists in allocating the products into relatively homogeneous groups in keeping with their stocking cost and

*) Optimization on products .
**) With an optimum period common for all products.

the establishment for each group of the resupply period as a multiple of the basic optimum period resulting from the optimization calculus.

In the mathematical formulation of establishing the optimum system of multiple resupply periods, we confine ourselves to those variants in which at least one of the products is ordered in each basic period T, the latter being a whole number.*)

We introduce the following notations:

K_i — the multiplying coefficient of the basic optimum period T, a coefficient corresponding to product $i (i = 1, 2, ..., N)$;

n_k — the number of products with the resupply period equal to KT;

$\langle K \rangle$ — the set of products with the delivery periodicity equal to $KT (K = 1, 2, 3, ...)$. For some K, the corresponding sets may be void;

μ_i — the demand in time unit for product i;

s_i — the stocking cost of product i in time unit (day);

l_n — launching cost of order;

Δl — the increase of the launching costs determined by the cumulation of order for a product with the orders for other products which should be purchased simultaneously.

The l_n launching cost is made up of two elements; the first element noted with l is invariable in keeping with the number (n) of products**) ordered simultaneously and the second element noted with n. Δl is variable and directly proportional to this very number of products.

Thus said, the launching cost l_n represents a linear function of the number n of products simultaneously ordered from the considered purveyor, that is:

$$l_n = l + n \cdot \Delta l$$

whence

$$l_n = l + n \cdot \lambda l = l(1 + \lambda n)$$

in which n may take the values $1, 2, ... N$, and $\lambda = \frac{\Delta l}{l}$ is a constant size, regardless of the product it refers to.

Solution

We now present the way of solving the problem containing the above elements, mentioning that we refer to the situation when the demand in time unit is known with precision and is stationary for the whole supply period.

We complete the premises of the problem with the condition that the beneficiary cannot interrupt production because of the absence of any raw material.

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*) The limitation to whole numbers for T corresponds to practice and, is also simplifying the optimizing method.

**) By a number of products we mean the number of types of products and the volume of purchases for each type of products we shall call the quantity of product i.

In connection with the establishment of the launching and stocking costs according to the method of multiple scheduling, the products to be supplied are divided into two categories: the first category includes all products for which $K = 1$ and the second category all the other products, namely the products for which $K \geqslant 2$.

We make the following specification: the launching cost l is achieved once for all the products from the set of products $\langle 1 \rangle$, which are n_1; the cost l due to each product from the set $\langle 1 \rangle$, in time unit, will be $\frac{l}{n_1 T}$. Each product included in the resupply lot determines the cost λl, and on time unit, the cost $\frac{\lambda l}{K_i T}$. Obviously, for the first category of products, the cost will be $\frac{\lambda l}{T}$.

Considering acceptable the hypothesis that between two consecutive resupplies, the stock of products develops lineally, the average daily stocking cost will be equal to $\frac{1}{2} \mu_i S_i K_i T$.

Taking into consideration the specifications made, we can write the calculus formulas of the total costs in time unit (day). Thus, if $K = 1$, we shall have:

$$\gamma_i = \frac{2}{1} \mu_i s_i T + \frac{\lambda l}{T} + \frac{l}{n_1 T}, \tag{1a}$$

and for the products for which $K \geqslant 2$, we shall have

$$\gamma_i = \frac{1}{2} \mu_i s_i K_i T \frac{\lambda l}{K_i T}. \tag{1b}$$

The total costs for all products will be:

$$C = \sum_{i=1}^{N} \gamma_i = \sum_{i=1}^{n_1} \frac{1}{2} \mu_i S_i T + n_1 \frac{\lambda l}{T} + n_1 \frac{l}{n_1 T} + \sum_{i=n_1+1}^{N} \frac{1}{2} \mu_i S_i K_i T +$$

$$+ \sum_{i=n_1+1}^{N} \frac{\lambda l}{K_i T} = \frac{1}{2} T \sum_{i=1}^{N} \mu_i S_i K_i + \sum_{i=1}^{N} \frac{\lambda l}{K_i T} + \frac{l}{T} = \tag{2}$$

$$= \frac{1}{2} T \sum_{i=1}^{N} \mu_i S_i K_i + \frac{l}{T} \left(\lambda \sum_{i=1}^{N} \frac{1}{K_i} + 1 \right).$$

It is necessary to choose period T and to make the allocation of all products into the sets corresponding to $K = 1, 2, 3, \ldots$, so that the total costs C should be minimum. For this, we calculate the coefficients of C for the sets $\{K_i\}$ in keeping with T:

$$\frac{dC}{dT} = \frac{1}{2} \sum_{i=1}^{N} \mu_i S_i K_i - \frac{l}{T^2} \left(\lambda \sum_{i=1}^{N} \frac{1}{K_i} + 1 \right),$$

$$\frac{d^2C}{dT^2} = \frac{2l}{T^3} \left(\lambda \sum_{i=1}^{N} \frac{1}{K_i} + 1 \right).$$

The second coefficient being positive for any set $\{K_i\}$, function C admitts a minimum for T taking values in the interval $(0, \infty)$. As we consider that T takes values in the set of natural numbers, function C becomes a discrete function (C_1) whose minimum can no longer be established after the same method, because, as is known, the function with discrete values is not derivable.

At the same time, however, we are not mistaken if we extend the conclusion drawn on the minimum of C to C_1, because the discrete values of T are uniformly distributed on the interval $(0, \infty)$.

The optimum resupply period T^* will be that T to which corresponds the minimum total costs (C_1^*). For this, we establish the values of C_1 corresponding to the possible values*) of T from which we choose C_1^* and, correspondingly, T^*.

To establish the values of C_1 as results from formula (2), we must know the sums $\sum_{i=1}^{N} \mu_i S_i K_i$ and $\sum_{i=1}^{N} \frac{1}{K_i}$, namely to have established, in the first place, the K_i multiplying coefficients.

For classifying the products on multiplying coefficients, we make the following reasoning: each set of products $\langle K \rangle$ should be made up only of the products which through their presence in this set $\langle K \rangle$ contributes to obtaining a maximum of economic effect (minimum C_1 total costs), that is of the products which cannot lead to the obtaining of savings by passing them from set $\langle K \rangle$ to set $\langle K - 1 \rangle$ or to set $\langle K + 1 \rangle$.

Let us formulate mathematically these conditions of classifying the products into multitudes $\langle K \rangle$, $\{K = 1, 2, \ldots\}$.

The condition for classifying the products in set $\langle K \rangle$ and not in set $\langle K - 1 \rangle$, inferred from relation (1b) may be written as follows:

$$\left(\frac{1}{2}\mu_i S_i K_i T + \frac{\lambda l}{K_i T}\right) - \left[\frac{1}{2}\mu_i S_i (K-1)\, T + \frac{\lambda l}{(K_i - 1)\, T}\right] \leqslant 0$$

or

$$\mu_i S_i K_i (K_i - 1)\; T^2 - 2\lambda l \leqslant 0,$$

hence:

$$\mu_i S_i \leqslant \frac{2\lambda l}{K_i (K_i - 1) T^2}. \tag{3}$$

If we put the condition of classifying the products in set $\langle K \rangle$ and not in set $\langle K + 1 \rangle$, it can be formulated mathematically as follows:

$$\left(\frac{1}{2}\mu_i S_i K_i T + \frac{\lambda l}{K_i T}\right) - \left[\frac{1}{2}\mu_i S_i (K_i + 1)\, T + \frac{\lambda l}{(K_i + 1) T}\right] \leqslant 0$$

$$-\mu_i S_i K_i (K_i + 1)\; T^2 + 2\lambda l \leqslant 0$$

where

$$\mu_i S_i \geqslant \frac{2\lambda l}{K_i (K_i + 1)\; T^2}. \tag{4}$$

*) These possible values represent a relatively small sub-set of the set of natural numbers

Thus, we can use as a distribution criterium both formula (3) and formula (4).

We arbitrarily choose (3), and note the right arm with R_k, namely:

$$\frac{2\lambda l}{K_i(K_i - 1)\ T^2} = R_k. \tag{5}$$

For the products for which conditon

$$R_{m+1} \leqslant \mu_i S_i \leqslant R_m \tag{6}$$

is fulfilled, we shall have $K_i = m$.

We shall consider R_k *maximum* (that is R_1) equal to the infinite and R_k *minimum* as being the last value R_k higher to the *minimum* $\{\mu_i S_i\}$.

An Example of Calculus

To exemplify those shown above, we shall present the case of a (wholesale) supply base which has an industrial enterprise among its purveyors. For a number of 25 products forming the object of their relations stipulated by contract, the fulfilment of the condition of stationariness of the demand for the whole supply period (a year) is noticed.

Table 1 presents the purchasing prices per measure unit of the products which should be purchased (column 1), the daily stocking costs (column 2),

Table 1

i	Price on measure unit	s_i	μ_i	$\mu_i s_i$
1	1 584	0.792	46	36.432
2	1 350	0.675	95	64.125
3	577	0.288	283	81.504
4	346	0.173	691	119.543
5	218.50	0.109	620	67.580
6	127	0.063	586	36.918
7	109	0.054	46	2.484
8	107.40	0.054	95	5.130
9	85.70	0.043	30	1.290
10	79	0.039	116	4.524
11	73.40	0.037	38	1.406
12	60.70	0.030	105	3.150
13	59.50	0.030	50	1.500
14	49.50	0.025	384	9.600
15	35.30	0.018	214	3.852
16	35	0.017	597	10.149
17	33.60	0.017	511	8.687
18	32.85	0.016	150	2.400
19	30.60	0.015	120	1.800
20	30.40	0.015	430	6.450
21	27.85	0.014	891	12.474
22	18.20	0.009	150	1.350
23	17.20	0.009	320	2.880
24	12	0.006	421	2.526
25	12	0.006	280	1.680

the daily demand (column 3) as well as the stocking cost of the entire quantity necessary for a day (column 4).

The 1 order launching cost is of 5 500 lei (regardless of the number of ordered products) and $\lambda = 0.369$.

We shall establish, in the first place, the possible values of T, in fact T_{min} and T_{max}.

T minimum possible is considered here to be the size of T which ensures at least the product to which corresponds $\mu_i S_i$ *maximum* ($\mu_1 S_1$, in table 2) belongs to the group of products for which $K_i = 1$.

We consider $R_1 = 119.543$, thus $K = 1$ and from formula (5) we obtain the value $T = 4.102$.

T minimum possible will be the first whole number bigger than T obtained from condition $R_1 = max\ \{\mu_i S_i\} = 119.543$, that is $T_{min} = 5$.

Based on experience and on the concrete restrictive conditions imposed on the respective supply, it is considered that $T max = 20$. Obviously, any whole number bigger than 20 will not change the final result on the optimum resupply periodicity, in exchange, it implies additional calculations.

For all possible values of T, that is for $T = 5, 6, 7, ..., 20$, the corresponding sizes of R_k are calculated after formula (5).

The results obtained are presented in table 2.

With the aid of values R_k the allocation of the products to sets $\{K_i\}$ in keeping with relation (6) is achieved. (For allocation see annex).

Table 2

The Calculated Values of R_k

T \ K	1	2	3	4	5	6	7	8	9	10	11
5	81.180	27.060	13.530	8.118	5.412	3.866	2.899	2.255	1.804	1.476	1.230
6	56.375	18.792	9.396	5.637	3.758	2.684	2.013	1.566	1.253	1.025	
7	41.418	13.806	6.903	4.142	2.761	1.972	1.479	1.150			
8	31.711	10.570	5.285	3.171	2.114	1.510	1.132				
9	25.055	8.352	4.176	2.505	1.670	1.193					
10	20.295	6.765	3.382	2.029	1.353	0.966					
11	16.773	5.591	2.795	1.677	1.118						
12	14.094	4.698	2.349	1.409	0.938						
13	12.009	4.003	2.001	1.201							
14	10.354	3.451	1.726	1.035							
15	9.020	3.007	1.503	0.902							
16	7.928	2.487	1.321	0.793							
17	7.022	2.341	1.170								
18	6.264	2.088	1.044								
19	5.622	1.874	0.936								
20	5.074	1.691	0.846								
21	4.602	1.534	0.767								
22	4.193	1.398	0.699								

Further, the total costs corresponding to each T possible are established after formula:

$$C = \frac{1}{2}\left(ST + \frac{L}{T}\right) \tag{7}$$

where:

$$S = \sum_{i=1}^{N} \mu_i S_i K_i$$

$$L = 2l\lambda \sum_{i=1}^{N} \frac{1}{K_i} + 2l = 4059 \times \sum_{i=1}^{25} \frac{1}{K_i} + 1\,1000.$$

The calculations are organized according to the annex and the results are presented in table 3.

Table 3

T	S	L	$\frac{2}{1} \cdot \left(T \cdot S + \frac{L}{T}\right)$
5	1 040.811	38 978.687	64 99.896
6	870.003	45 729.804	6 420.742
7	827.696	47 478.233	6 288.238
8	716.634	53 570.792	6 214.710
9	660.962	57 544.553	6 171.248
10	649.320	58 555.244	6 174.362
11	630.720	60 580.685	6 222.627
12	616.680	62 372.179	6 298.924
13	595.636	64 788.382	6 363.495
14	589.984	66 803.132	6 515.714
15	565.405	71 876.882	6 634.433
16	547.056	76 272.779	6 759.977
17	540.882	77 965.382	6 890.596
18	534.432	78 994.882	7 031.918
19	534.432	79 994.882	7 031.918
20	527.502	82 702.235	7 342.576

Thus, the best periodicity is given by the basic T equal to 9 days, which corresponds to the minimum total cost C_1^* in 4-th column of table 3.

The optimum programme for supplying the beneficiary is presented in table 4.

As the optimum basic period is of 9 days, 40 orders are done annually. Their calendar scheduling can be achieved in various ways. A first schedule is presented in column 2 of table 5.

The uniformization of the size of orders is necessary for ensuring the rational use of the means of transport. In a first form, it can be achieved in keeping with the number of sets $\langle K \rangle$ of an order. According to this criterion, the programme presented in column 3 of table 5 was obtained.

We mention that the scheduling can be worked out in a more exacting way, by taking into consideration also the quantities, not only the list includ-

Table 4

$T^* = 9$

i_{new}	K_i	$T_i = T^* \cdot K_i$	μ_i	$q_i = T_i \cdot \mu_i$
1	1	9	691	6 291
2	1	9	283	2 547
3	1	9	620	5 580
4	1	9	95	855
5	1	9	586	5 274
6	1	9	46	414
7	2	18	891	16 038
8	2	18	597	10 746
9	2	18	384	6 912
10	2	18	511	9 198
11	3	27	430	11 610
12	3	27	95	2 565
13	3	27	116	3 132
14	4	36	214	7 704
15	4	36	105	3 780
16	4	36	320	11 520
17	4	36	421	15 156
18	5	45	46	2 070
19	5	45	150	6 750
20	5	45	120	5 400
21	5	45	280	12 600
22	6	54	50	2 700
23	6	54	38	2 052
24	6	54	150	8 100
25	6	54	30	1 620

ed in each order as well as the incompatibility among various products as regards their transport.

The solving of these problems is not examined here since this item does not propose to approach it.

Comparison with Other Methods

As it was shown at the beginning of this study, other methods applied are the method of independent optimization (on each product) and that of global optimization (by establishing an optimum period for all products).

We shall further underline the results to which leads the application of these methods for our problem; it will also be pointed out in what situation a method leads to results higher to those obtained through the application of other methods. The criterion according to which this appreciation will be made is represented by the total stocking and launching costs in the conditons of the complete satisfaction of the demand for the products to be supplied.

In the event of the optimization on products, the exaggerated "crumbling" of the lots is taking lace, considerably increasing the supply costs and the cumulation of the lots of various products has an accidental character. But, such a cumulation which is leading to the possibility of a supply on groups

Table 5

Number of order	$<K>$ Variant 1	$<K>$ Variant 2
1	2	3
1	<1>	<1> <4> <5>
2	<1> <2>	<1> <2>
3	<1> <3>	<1> <6>
4	<1> <2> <4>	<1> <2> <3>
5	<1> <5>	<1> <4>
6	<1> <2> <3> <6>	<1> <2> <5>
7	<1>	<1> <3>
8	<1> <2> <4>	<1> <2>
9	<1> <3>	<1> <4> <6>
10	<1> <2> <5>	<1> <2> <3>
11	<1>	<1> <5>
12	<1> <2> <3> <4> <6>	<1> <2>
13	<1>	<1> <3> <4>
14	<1> <2>	<1> <2>
15	<1> <3> <5>	<1> <6>
16	<1> <2> <4>	<1> <2> <3> <5>
17	<1>	<1> <4>
18	<1> <2> <3> <6>	<1> <2>
19	<1>	<1> <3>
20	<1> <2> <4> <5>	<1> <2>
21	<1> <3>	<1> <4> <5> <6>
22	<1> <2>	<1> <2> <3>
23	<1>	<1>
24	<1> <2> <3> <4> <6>	<1> <2>

Table 5 (continued)

1	2	3
25	<1> <5>	<1> <3> <4>
26	<1> <2>	<1> <2> <5>
27	<1> <3>	<1> <6>
28	<1> <2> <4>	<1> <2> <3>
29	<1>	<1> <4>
30	<1> <2> <3> <5> <6>	<1> <2>
31	<1>	<1> <3> <5>
32	<1> <2> <4>	<1> <2>
33	<1> <3>	<1> <4> <6>
34	<1> <2>	<1> <2> <3>
35	<1> <5>	<1>
36	<1> <2> <3> <4> <6>	<1> <2> <5>
37	<1>	<1> <3> <4>
38	<1> <2>	<1> <2>
39	<1> <3>	<1> <6>
40	<1> <2> <4> <5>	<1> <2> <3>

of products can be achieved, to a great extent, by applying the method of global optimization. In this case, however, the obtained periodicity of orders will differ from the optimal periodicity on each of its components (in case of an independent supply and optimization), a fact which leads to some increase of costs. On the other hand, this method allows the obtaining of savings for the orders. The combined effect of these two opposed tendencies determines the advantage or disadvantage of this method compared to the method of independent optimization on products. It is supposed that the same periodicity for all products is more advantageous in the case of most homogeneous products as regards their total stocking costs $\mu_i \ S_i$ per time unit; otherwise, in case the products to be supplied from one and the same purveyor give great dispersion to the values $\mu_i S_i$, the method of independent optimization is preferred.

We note by A^* the minimum total cost obtained through the application of the method of independent optimization and by B^* the minimum total cost obtained through the application of the method of global optimization.

In the case of independent optimization (on products) of the periodicity of supply, the total Γ_i launching and stocking cost per product i will be established as follows:

$$\Gamma_i = \Gamma_i^{(l)} + \Gamma_i^{(s)} = \frac{Q_i l(1+\lambda)}{q_i} + \frac{\theta s_i}{2} q_i$$

where:

$\Gamma_i^{(l)}$ represents the launching cost for product i;

$\Gamma_i^{(s)}$ — the stocking cost for product i in the whole period of time θ;

Q_i — the necessary quantity of product i for the whole period θ;

q_i — the necessary quantity of product i for the period between the two consecutive supplies;

In the case of the optimum choice of the lot q_i, namely:

$$q_i^* = \sqrt{2 \frac{Q_i l(1+\lambda)}{\theta \cdot S_i}}$$

the total cost Γ_i^* will be:

$$\Gamma_i^* = \sqrt{2\,\theta l(1+\lambda)\, Q_i S_i}$$

and, on the time unit:

$$\gamma_i^* = \frac{\Gamma_i^*}{\theta} = \frac{\sqrt{2\theta l(1+\lambda)\, Q_i S_i}}{\theta} = \sqrt{2l(1+\lambda)\, \mu_i S_i}$$

where:

$$\mu_i = \frac{Q_i}{\theta}.$$

The launching and stocking costs in the time unit for all the N products will be:

$$A^* = \sum_{i=1}^{N} \gamma_i^* = \sum_{i=1}^{N} \sqrt{2l(1+\lambda)\, \mu_i S_i} = \sqrt{2l(1+\lambda)} \cdot \sum_{i=1}^{N} \sqrt{\mu_i S_i} \qquad (8)$$

In the case of global optimization (the same periodicity for all products) the total costs for launching orders and for stocking products, in the time unit, will be:

$$B = \sum_{i=1}^{N} \frac{\Gamma_i}{\theta} \sum_{i=1}^{N} \left(\frac{Q_i l(1+\lambda N)}{q_i} + \frac{\theta s_i}{2} q_i \right) \times \frac{1}{\theta}.$$

As $\frac{Q}{q} = \frac{\theta}{T}$ from where $q = \frac{Q\,T}{\theta}$, we shall have:

$$B = \sum_{i=1}^{N} \left(\frac{Q_i l(1+\lambda N)}{\frac{Q_i T}{\theta}} + \frac{\theta s_i}{2} \times \frac{Q_i T}{\theta}, \right) \times \frac{1}{\theta},$$

where:

T represents the resupplying period.

Taking into consideration that $\frac{Q_i}{\theta} = \mu_i$ we obtain:

$$B = \frac{l(1 + \lambda N)}{T} + \frac{T}{2} \sum_{i=1}^{N} \mu_i S_i$$

The minimum of this expression is obtained when the component sizes are equal between them. We have:

$$\frac{l(1 + \lambda N)}{T} \times \frac{T}{2} \sum_{i=1}^{N} \mu_i S_i = \frac{1}{2} l(1 + \lambda N) \sum_{i=1}^{N} \mu_i S_i = const.$$

Thus, from the relation

$$\frac{l(1 + \lambda N)}{T} = \frac{T}{2} \sum_{i=1}^{N} \mu_i S_i$$

we shall obtain the optimum size of T, namely

$$T^* = \sqrt{2l(1 + \lambda N)} \times \frac{1}{\sqrt{\sum_{i=1}^{N} \mu_i S_i}}$$

Corresponding to T^*, we shall have the minimum total costs

$$B^* = \sqrt{2l(1 + \lambda N)} \times \sqrt{\sum_{i=1}^{N} \mu_i S_i}. \tag{9}$$

Obviously, for a certain concrete situation, we shall consider that the second method is more efficient than the first one if condition $B^* < A^*$ is fulfilled, that is if the relation

$$\sqrt{2l(1 + \lambda N)} \cdot \sqrt{\sum_{i=1}^{N} \mu_i S_i} < \sqrt{2l(1 + \lambda)} \cdot \sum_{i=1}^{N} \sqrt{\mu_i S_i}$$

is verified

or

$$\frac{\sum_{i=1}^{N} \mu_i S_i}{\left(\sum_{i=1}^{N} \sqrt{\mu_i S_i}\right)^2} < \frac{1 + \lambda}{1 + \lambda N}.$$

Noting by u the left arm of this inequality, we have:

$$u < \frac{1 + \lambda}{1 + \lambda N}. \tag{10}$$

The size u represents a variation index of the characteristic $\mu_i S_i$. The maximum value of u is the unit in case that a single product $\mu_i S_i$ is positive and, the minimum value is attained in case that $\sum_{i=1}^{N} \sqrt{\mu_i S_i}$ is maximum, that is in the limit situation when all values $\mu_i S_i$ are equal between them, so, when all products are perfectly homogeneous according to the characteristic which interests us, namely according to the values $\mu_i S_i$.

In practice the index u takes values comprised between the limits shown and for decinding on the opportuneness of the utilization of one of the two methods presented above, we compare this index to the relation between the cost for launching the order for a single product and the cost for the simultaneous launching of orders for all products according to relation (10). This relation gives a precise form to the conception on the opinion expressed before, according to which the more heterogenous the products after the characteristic $\mu_i S_i$ the less one can establish the same periodicity for their purchasing and vice versa. Thus said, to use the same periodicity for the purchase of products, the values of the characteristic $\mu_i S_i$ must not have a dispersion surpassing a certain limit. From relation (10) this very limit results, which is $\frac{1+\lambda}{1+\lambda N}$.

Applying for our numerical example formulas (8) and (9), we obtain:

$$A^* = \sqrt{2 \cdot 5\,500(1 + 0.369) \cdot 85 \cdot 493} = 10\,491 \cdot 273$$

$$B^* = \sqrt{2 \cdot 5\,500(1 + 0 \cdot 369 \cdot 25)} \cdot \sqrt{489 \cdot 434} = 7\,419 \cdot 457.$$

The best result obtained through the application of the method of global optimization was to be expected, since index u of the dispersion of the values of the characteristic $\mu_i S_i$ has a lower value than that necessary for making optimization on products preferable.

Namely:

$$u = \frac{489.434}{(85.493)^2} = 0.067$$

$$u = 0 \cdot 067 < \frac{1+\lambda}{1+\lambda N} = \frac{1 + 0.369}{1 + 0.369 \times 25} = 0 \cdot 134.$$

An important problem in applying the mathematical models in the administration of stocks as well as in any other field is the establishment of the extent of the stability of the results obtained in comparison with the errors accepted in establishing the parameters of these models.

Generally, the determination of the value of the parameters of the mathematical models of stock administration is not characterized by great precision, mainly due to the fact that the information system on which it is based is not adequate for carrying out this objective.

In many cases, however, the results obtained through the application of these methods are not very sensitive to the errors accepted in establishing the value of their parameters, a fact which strengthens the conviction about the possibility of efficiently using the mathematical models in the rationalization of supply and in the optimization of the stocks of various products.

Annex		$T = 5$			$T = 6$			$T = 7$			$T = 8$		
i (new)	$\mu_i s_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$
1	119.543	1	1	119.543	1	1	119.543	1	1	119.543	1	1	119.543
2	81.504	1	1	81.504	1	1	81.504	1	1	81.504	1	1	81.504
3	67.580	2	0.5	135.160	1	1	67.580	1	1	67.580	1	1	67.580
4	64.125	2	0.5	128.250	1	1	64.125	1	1	64.125	1	1	64.125
5	36.918	2	0.5	73.836	2	0.5	73.836	2	0.5	73.836	1	1	36.918
6	36.432	2	0.5	72.864	2	0.5	72.864	2	0.5	72.864	1	1	36.432
7	12.475	4	0.25	49.896	3	0.333	37.422	3	0.333	37.422	2	0.5	24.948
8	10.149	4	0.25	40.596	3	0.333	30.447	3	0.333	30.447	3	0.333	30.447
9	9.600	4	0.25	38.400	3	0.333	28.800	3	0.333	28.800	3	0.333	28.800
10	8.687	4	0.25	34.748	4	0.25	34.748	3	0.333	26.061	3	0.333	26.061
11	6.450	5	0.200	32.250	4	0.25	25.800	4	0.25	25.800	3	0.333	19.350
12	5.130	6	0.167	30.780	5	0.2	25.650	4	0.25	20.520	4	0.25	20.520
13	4.524	6	0.167	27.144	5	0.2	22.620	4	0.25	18.096	4	0.25	18.096
14	3.852	7	0.143	26.964	5	0.2	19.260	5	0.2	19.260	4	0.25	15.408
15	3.150	7	0.143	22.050	6	0.167	18.900	5	0.2	15.750	5	0.2	15.750
16	2.880	8	0.125	23.040	6	0.167	17.280	5	0.2	14.400	5	0.2	14.400
17	2.526	8	0.125	20.208	7	0.143	17.682	6	0.167	15.156	5	0.2	12.630
18	2.484	8	0.125	19.872	7	0.143	17.388	6	0.167	14.904	5	0.2	12.420
19	2.400	8	0.125	19.200	7	0.143	16.700	6	0.167	14.400	5	0.2	12.000
20	1.800	10	0.1	18.000	8	0.125	14.400	7	0.143	12.600	6	0.167	10.800
21	1.680	10	0.1	16.800	8	0.125	13.440	7	0.143	11.760	6	0.167	10.080
22	1.500	10	0.1	15.000	9	0.111	13.500	7	0.143	10.500	7	0.143	10.500
23	1.406	11	0.091	15.466	9	0.111	12.654	8	0.125	11.248	7	0.143	9.842
24	1.350	11	0.091	14.850	9	0.111	12.150	8	0.125	10.800	7	0.143	9.450
25	1.290	11	0.091	14.190	9	0.111	11.610	8	0.125	10.320	7	0.143	9.030
		—	6.893	1040.811	—	8.556	870.003	—	8.987	827.696	—	10.488	716.634

Annex		$T = 9$			$T = 10$			$T = 11$			$T = 12$		
i (new)	$\mu_i s_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$
1	119.543	1	1	119.543	1	1	119.543	1	1	119.543	1	1	119.543
2	81.504	1	1	81.504	1	1	81.504	1	1	81.504	1	1	81.504
3	67.580	1	1	67.580	1	1	67.580	1	1	67.580	1	1	67.580
4	64.125	1	1	64.125	1	1	64.125	1	1	64.125	1	1	64.125
5	36.918	1	1	36.918	1	1	36.918	1	1	36.918	1	1	36.918
6	36.432	1	1	36.432	1	1	36.432	1	1	36.432	1	1	36.432
7	12.474	2	0.5	24.948	2	0.5	24.948	2	0.5	24.948	2	0.5	24.948
8	10.149	2	0.5	20.298	2	0.5	20.298	2	0.5	20.298	2	0.5	20.298
9	9.600	2	0.5	19.200	2	0.5	19.200	2	0.5	19.200	2	0.5	19.200
10	8.687	2	0.5	17.374	2	0.5	17.374	2	0.5	17.374	2	0.5	17.374
11	6.450	3	0.333	19.350	3	0.333	19.350	2	0.5	12.900	2	0.5	12.900
12	5.130	3	0.333	15.390	3	0.333	15.390	3	0.333	15.390	2	0.5	10.260
13	4.524	3	0.333	13.575	3	0.333	13.572	3	0.333	13.572	3	0.333	13.572
14	3.852	4	0.25	15.408	3	0.333	11.556	3	0.333	11.556	3	0.333	11.556
15	3.150	4	0.25	12.600	4	0.25	12.600	3	0.333	9.450	3	0.333	9.450
16	2.880	4	0.25	11.520	4	0.25	11.520	3	0.333	8.640	3	0.333	8.640
17	2.526	4	0.25	10.104	4	0.25	10.104	4	0.25	10.104	3	0.333	7.578
18	2.484	5	0.2	12.420	4	0.25	9.936	4	0.25	9.936	3	0.333	7.452
19	2.400	5	0.2	12.000	4	0.25	9.600	4	0.25	9.600	3	0.333	7.200
20	1.800	5	0.2	9.000	5	0.2	9.000	4	0.25	7.200	4	0.25	7.200
21	1.680	5	0.2	8.400	5	0.2	8.400	4	0.25	6.720	4	0.25	6.720
22	1.500	6	0.167	9.000	5	0.2	7.500	5	0.2	7.500	4	0.25	6.000
23	1.406	6	0.167	8.436	5	0.2	7.030	5	0.2	7.030	5	0.2	7.030
24	1.350	6	0.167	8.100	6	0.167	8.100	5	0.2	6.750	5	0.2	6.750
25	1.290	6	0.167	7.740	6	0.167	7.740	5	0.2	6.450	5	0.2	6.450
		—	11.467	660.962	—	11.716	649.320	—	12.215	630.720	—	12.681	616.680

Annex		$T = 13$			$T = 14$			$T = 15$			$T = 16$		
i (new)	$\mu_i s_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$
1	119.543	1	1	119.543	1	1	119.543	1	1	119.543	1	1	119.543
2	81.504	1	1	81.504	1	1	81.504	1	1	81.504	1	1	81.504
3	67.580	1	1	67.580	1	1	67.580	1	1	67.580	1	1	67.580
4	64.125	1	1	64.125	1	1	64.125	1	1	64.125	1	1	64.125
5	36.918	1	1	36.918	1	1	36.918	1	1	36.918	1	1	36.918
6	36.432	1	1	36.432	1	1	36.432	1	1	36.432	1	1	36.432
7	12.474	1	1	12.474	1	1	12.474	1	1	12.474	1	1	12.474
8	10.149	2	0.5	20.298	2	0.5	20.298	1	1	10.149	1	1	10.149
9	9.600	2	0.5	19.200	2	0.5	19.200	1	1	9.600	1	1	9.600
10	8.687	2	0.5	17.374	2	0.5	17.374	2	0.5	17.374	1	1	8.687
11	6.450	2	0.5	12.900	2	0.5	12.900	2	0.5	12.900	2	0.5	12.900
12	5.130	2	0.5	10.260	2	0.5	10.260	2	0.5	10.260	2	0.5	10.260
13	4.524	2	0.5	9.048	2	0.5	9.048	2	0.5	9.048	2	0.5	9.048
14	3.852	3	0.333	11.556	2	0.5	7.704	2	0.5	7.704	2	0.5	7.704
15	3.150	3	0.333	9.450	3	0.333	9.450	2	0.5	6.300	2	0.5	6.300
16	2.880	3	0.333	8.640	3	0.333	8.640	3	0.333	8.640	2	0.5	5.760
17	2.526	3	0.333	7.578	3	0.333	7.578	3	0.333	7.578	2	0.5	5.052
18	2.484	3	0.333	7.452	3	0.333	7.452	3	0.333	7.452	3	0.333	7.452
19	2.400	3	0.333	7.200	3	0.333	7.200	3	0.333	7.200	3	0.333	7.200
20	1.800	4	0.25	7.200	3	0.333	5.400	3	0.333	5.400	3	0.333	5.400
21	1.680	4	0.25	6.720	4	0.25	6.720	3	0.333	5.040	3	0.333	5.040
22	1.500	4	0.25	6.000	4	0.25	6.000	4	0.25	6.000	3	0.333	4.500
23	1.406	4	0.25	5.624	4	0.25	5.624	4	0.25	5.624	3	0.333	4.218
24	1.350	4	0.25	5.400	4	0.25	5.400	4	0.25	5.400	3	0.333	4.050
25	1.290	4	0.25	5.160	4	0.25	5.160	4	0.25	5.160	4	0.25	5.160
		—	13.498	595.636	—	13.748	589.984	—	14.998	565.405	—	16.081	547.056

Annex		$T = 17$			$T = 18$			$T = 19$			$T = 20$		
i (new)	$s_i\mu_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i K_i$	K_i	$\frac{1}{K_i}$	$\mu_i s_i k_i$
1	119.543	1	1	119.543	1	1	119.543	1	1	119.543	1	1	119.543
2	81.504	1	1	81.504	1	1	81.504	1	1	81.504	1	1	81.504
3	67.580	1	1	67.580	1	1	67.580	1	1	67.580	1	1	67.580
4	64.125	1	1	64.125	1	1	64.125	1	1	64.125	1	1	64.125
5	36.918	1	1	36.918	1	1	36.918	1	1	36.918	1	1	36.918
6	36.432	1	1	36.432	1	1	36.432	1	1	36.432	1	1	36.432
7	12.474	1	1	12.474	1	1	12.474	1	1	12.474	1	1	12.474
8	10.149	1	1	10.149	1	1	10.149	1	1	10.149	1	1	10.149
9	9.600	1	1	9.600	1	1	9.600	1	1	9.600	1	1	9.600
10	8.687	1	1	8.687	1	1	8.687	1	1	8.687	1	1	8.687
11	6.450	2	0.5	12.900	1	1	6.450	1	1	6.450	1	1	6.450
12	5.130	2	0.5	10.260	2	0.5	10.260	2	0.5	10.260	1	1	5.130
13	4.524	2	0.5	9.048	2	0.5	9.048	2	0.5	9.048	2	0.5	9.048
14	3.852	2	0.5	7.704	2	0.5	7.704	2	0.5	7.704	1	0.5	7.704
15	3.150	2	0.5	6.300	2	0.5	6.300	2	0.5	6.300	2	0.5	6.300
16	2.880	2	0.5	5.760	2	0.5	5.760	2	0.5	5.760	2	0.5	5.760
17	2.526	2	0.5	5.052	2	0.5	5.052	2	0.5	5.052	2	0.5	5.052
18	2.484	2	0.5	4.968	2	0.5	4.968	2	0.5	4.968	2	0.5	4.968
19	2.400	2	0.5	4.800	2	0.5	4.800	2	0.5	4.800	2	0.5	4.800
20	1.800	3	0.333	5.400	3	0.333	5.400	3	0.333	5.400	2	0.5	3.600
21	1.680	3	0.333	5.040	3	0.333	5.040	3	0.333	5.040	3	0.333	5.040
22	1.500	3	0.333	4.500	3	0.333	4.500	3	0.333	4.500	3	0.333	4.500
23	1.406	3	0.333	4.218	3	0.333	4.218	3	0.333	4.218	3	0.333	4.218
24	1.350	3	0.333	4.050	3	0.333	4.050	3	0.333	4.050	3	0.333	4.050
25	1.290	3	0.333	3.870	3	0.333	3.870	3	0.333	3.870	3	0.333	3.870
		—	16.498	540.882	—	16.998	534.432	—	16.998	534.432	—	17.665	527.502

REFERENCES

1. DZIELINSKI B. P. GOMORY R. E.: *Optimal Programming of Lot Sizes, Inventory and Labour Allocations.* Management Science, no. 9, 1965.
2. EILON S.: *Economic Batch-Size Determination for Multiproduct Scheduling.* Operation. Research Quarterly, no. 4, 1959.
3. HOLT C. C.: *Decision Rules for Allocating Inventory to Lots and Cost Functions for Making Aggregate Inventory Decisions.* Journal of Industrial Engineering, no. 1, 1958.
4. KAUFMANN A.: *Metode și modele ale cercetării operationale.* Vol. 1 (Operations Research Methods and Models). Vol. 1. Editura științifică (translation from French), Bucharest, 1968.
5. RIJIKOV IU.: *Upravlenie zapasami.* Editura Nauka, Moscow, 1969.

EMPLOI DE LA METHODE DES PERIODES MULTIPLES D'APPROVISIONNEMENT

L'application de la méthode des périodes multiples de réapprovisionnement conduit à l'établissement d'un approvisionnement à périodicité unique par groupes de produits. Ceci permet d'obtenir de grandes économies lors des commandes. De même, il est de toute importance de faire différencier la gestion des stocks par groupes de produits, de sorte que l'on assure un intervalle de temps de stockage des produits d'autan plus réduit que le coût de ce stockage est plus grand. La réalisation d'un tel désideratum, ainsi que le cumul maximum des commandes sont possibles par l'application de la méthode des périodes multiples de réapprovisionnement.

ПРИМЕНЕНИЕ МЕТОДА КРАТНЫХ ПЕРИОДОВ СНАБЖЕНИЯ

Применение метода кратных периодов снабжения ведет к установлению снабжения с единой периодичностью по группам видов продукции. Это позволяет получать большую экономию при заказах. Является важным также дифференцирование расходования запасов по группам видов продукции таким образом, чтобы обеспечить образование запасов продукции в тем меньший период времени, чем больше стоимость требуемого образования запасов продукции. Осуществление этого последнего пожелания и наибольшего совмещения заказов представляется возможным путем применения метода кратных периодов снабжения.

5.IV.1974