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Sequence Checking of the Quality of Goods in Batches Inferior to 5,000 Pieces

It often happens that batches presented for checking contain less than 5,000 pieces. For this reason the authors consider useful to draw up certain graphic schemes for checking the quality of the goods: they suggest a methodology for computing these schemes.

The Directives of the IX-th Congress of the Romanian Communist Party point out as one of the fundamental tasks of the economic plan for 1966—1970 the improving of the goods quality; they set efficient measures in all branches of national economy in view of increasing the products quality. Nicolae Ceauşescu, General Secretary of the Central Committee of the Romanian Communist Party, referring to the measures provided for by the Directives of the IX-th Congress of the Romanian Communist Party, points out that: "A greater attention should be paid to the steady improvement of the quality of the consumer goods, according to the purchaser's taste and exigencies" *.

Concurrently with the efforts in view of obtaining products of a higher quality, it is imperative — as party documents stress it — that checking operations at different stages, where the control occurs, should be improved as well, in order to prevent inferior quality products from reaching the customer.

Improving the quality of goods is a process involving the whole of our people. It has the peculiarity that it cannot be carried out in a general way — that is to say, considering production as a whole; it acts on each particular product, each separate case. In this respect, checking the quality in the goods circulation sphere is of certain importance. It is known that in this field of activity, owing to its very nature, the quality checking is applied to batches of products. However, since goods differ a great deal from one group to another, both in respect of their size and the criteria for estimating

* The IX-th Congress of the Romanian Communist Party, Editura Politică, 1965, page 43

their quality, control methods have to be scientifically substantiated, with the aid of mathematical statistics and the calculus of probability.

Among the checking methods applied to the goods circulation sphere, the sequence extraction proves to be highly efficient. On the main, its characteristic feature consists of deciding, after each product drawn out of the batch has been analysed, whether the respective batch should be accepted or rejected. Admission or rejection of the batch is established by means of a graph where the size n of the selection is represented on the axis of the abscissas, and the number d of deficient pieces on the axis of the ordinates. If the lot consists of a large number of pieces ($N > 5{,}000$), the results obtained after checking the extracted products, noted by x ($i = 1, 2, ..., n$), are independent. When batches forwarded to control are under 5,000 pieces ($N < 5{,}000$), then the results of extract checking are no longer independent and other reasonings are required in calculating them.

Considering that a large number of batches presented for checking have less than 5,000 pieces, the authors of this paper have thought it useful to work out certain graphic schemes for checking the quality of these batches, for the benefit of experts (especially those working in the field of goods circulation) who should have them at hand. In calculating those schemes, the parameters have been considered so as to create the possibility of a continuous improving of the quality checking — according to the customers' increasing requirements and the progress of technological processes. Actually, we are referring to 1,232 schemes for which the necessary parameters * are comprised within the following limits:

$$N = 100 \text{ —} 5{,}000$$
$$p_1 = 0.01 \text{ —} 0.07$$
$$p_2 = 0.04 \text{ —} 0.10$$
$$R_f = 0.10 \text{ —} 0.01$$
$$R_b = 0.10 \text{ —} 0.01$$

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* N: number of products of the batch whose quality we propose to measure;

$p_0 = \frac{D_0}{N}$: deficient ratio admitted (a subunitary number); 100 p_0 is the percentage of a previous agreement between the purveyor and the beneficiary. If it is granted that defects should represent 1 % of the total quantity, then 100 p_0 = 1 % and $p_0 = 0.01$;

$P_1 \frac{D_1}{N}$: deficient ratio of admission (subunitary number) ($p_1 \leqslant p_0$); 100 p_1 the percentage of products judged unadequete at the checking after which the batch has been admitted (100 $p_1 \leqslant 100\ p_0$);

$p_2 = \frac{D_2}{N}$: deficient ratio of the batch above which the quality is considered inadmissible by the beneficiary (subunitary number), superior to p_0; 100 p_2 represents the percentage of unfit products of the rejected lot;

R_f: the purveyor's risk or the probability of a batch for which hypothesis 1 is valid, of being refused. R_f is a subunitary number. 100 R_f represents the refuse percentage of adequate, lots.

R_b: the beneficiary's risk or the probability of accepting a lot for which hypothesis 2 is valid. R_b is a subunitary number. 100 R_b represents the acceptance percentage of inadequate lots.

A medium skilled personnel is also able to use them quite easily for a series of groups of goods, such as: footwear, haberdashery, ready-made clothes, household metallic articles, electrotechnical appliances, furniture, various spares and fittings etc., for which as a rule batches do not exceed 5,000 pieces.

In this study we propose to introduce a method for calculating these graphic schemes. An example will show the way to use them.

As already mentioned, if the batches whose quality we intend to measure have less than 5,000 units, we must bear in mind — when proceeding to the sequence extraction — that the results x_i * $(i = 1, 2, ..., n)$ of the extractions are no longer to be considered independent. For that reason the criteria for admission (rejection) of large batches undergo modifications.

The ratio of verisimilarity calculated after each extraction is expressed in this case:

$$V_n = \frac{P_2 \cdot n}{P_1 \cdot n} = \frac{C_{N-D_2}^{n-d} \times C_{D_2}^{d}}{C_{N-D_1}^{n-d} \times C_{D_1}^{d}} \qquad (1)$$

where:

V_n is the ratio of verisimilarity
D — the number of indequate products in the lot of size N
n — the number of products drawn out from the lot under control (size of the selection)
d — the number of faulty products selected
$P_{1 \cdot n}$ — the probability that the selection of size n contain d inadequate products if the batch of size N has D_1 faults $(D_1 \leqslant D_0)$
$P_{2 \cdot n}$ — idem if the batch of size N has D_2 faults $(D_2 > D_0)$

since:

$$P_{1 \cdot n} = \frac{C_{N-D_1}^{n-d} \times C_{D_1}^{d}}{C_N^n \times C_n^d} \qquad (2)$$

and

$$P_{2 \cdot n} = \frac{C_{N-D_2}^{n-d} \times C_{D_2}^{d}}{C_N^n \times C_n^d} . \qquad (3)$$

1. If after extraction n we have

$$V_n \leqslant \frac{R_b}{1 - R_f} \left(\frac{\text{Probability of accepting an inadequate batch}}{\text{Probability of accepting an adequate batch}} \right) \qquad (4)$$

then the batch is admitted (the checking is over). In this case we adopt hypothesis 1 (I_1), which supposes that the checked lot has 100 p_1 faults, hence the lot can be accepted by the beneficiary.

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* If the product drawn out at extraction i is found inadequate, we shall note it by $x_i = 1$, and if adequate by $x_i = 0$.

2. If after extraction n we have:

$$V_n \geqslant \frac{1-R_b}{R_f}\left(\frac{\text{Probability of rejecting an inadequate batch}}{\text{Probability of rejecting an adequate batch}}\right) \tag{5}$$

the lot under control is refused (the checking is over). In this case hypothesis 2 (I_2) is admitted, which supposes that the checked lot has 100 p_2 faults, whereupon it can be rejected.

3. If after extraction n neither the relation *4* nor the relation *5* are fulfilled, that is, if we have:

$$\frac{R_b}{1-R_f} < V_n < \frac{1-R_b}{R_f} \tag{6}$$

then we shall proceed to extraction $n+1$ and to a re-calculation of V_n. Extractions will be carried out untill one of the relations (*4*) or (*5*) is fulfilled. Relation (*4*) represents in the plane $n\ o\ d$ discreet points placed on a straight line which is expressed:

$$n = N\left[\frac{R_b}{1-R_f}\right]^{\frac{1}{D_2-D_1}}\left[1-\left(\frac{D_1}{D_2}\right)^{\frac{1}{D_2-D_1}}\right]\times d + N\left[1-\left(\frac{R_b}{1-R_f}\right)^{\frac{1}{D_2-D_1}}\right] \tag{7}$$

and is called the straight line of admission.

The relation (*5*) represents other discreet points in the same plane, placed on a straight line whose expression is:

$$n = N\left[\frac{1-R_b}{R_f}\right]^{\frac{1}{D_2-D_1}}\left[1-\left(\frac{D_1}{D_2}\right)^{\frac{1}{D_2-D_1}}\right]\times d + N\left[1-\left(\frac{1-R_b}{R_f}\right)^{\frac{1}{D_2-D_1}}\right] \tag{8}$$

which is called the straight line of refuse. These straight lines meet in a point M whose ordinate is:

$$d = \frac{1}{1-\left(\frac{D_1}{D_2}\right)^{\frac{1}{D_2-D_1}}} \tag{9}$$

or, if we take into account that:

$$D_1 = N\times p_1; \qquad D_2 = N\cdot p_2 \tag{10}$$

relation (*9*) becomes:

$$d = \frac{1}{1-\left(\frac{P_1}{P_2}\right)^{\frac{1}{N(p_2-p_1)}}} \tag{11}$$

a relation showing that the number of inadequate products of the selection depends on the size of the batch.

For $N > 100$ and $p_1 \leqslant 0.1$, $p_2 \leqslant 0.1$ we appreciate that:

$$d \approx \frac{p_2-p_1}{ln\frac{p_2}{p_1}}\,n + 0.5 \tag{12}$$

which means that function $d(N)$ is approximated with its asymptote $[a(n)]$:

$$a(n) = \frac{p_2 - p_1}{ln \frac{p_2}{p_1}} n + 0.5 \qquad (13)$$

representing a straight line crossing the point of abscissa $n = N$ and of ordinate d.

The straight lines of admission and rejection can be represented in the plane $n\,o\,d$ by means of their intersection point $M(d,N)$ and their cuts on axis n.

If in relation (7) — equation of straight line of admission — we make $d = 0$ and take into account relation (10) we find the intersection point of this straight line with axis n, which is:

$$n_a = N\left[1 - \left(\frac{R_b}{1 - R_f}\right)^{\frac{1}{N(p_2 - p_1)}}\right] \qquad (14)$$

the cut of the straight line of admission with axis n.

By proceeding analogically with relation (8) we find

$$n_r = N\left[1 - \left(\frac{1 - R_b}{R_f}\right)^{\frac{1}{N(p_2 - p_1)}}\right] \qquad (15)$$

the cut of the straight line of rejection with axis n.

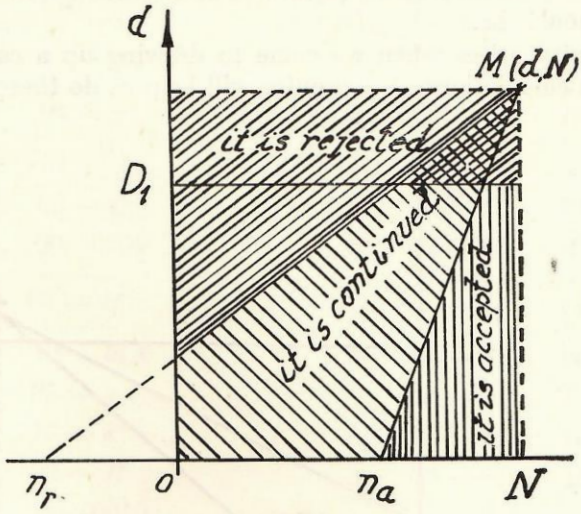

Fig. 1

In order to check up the quality of a batch of size N under the given conditions R_b, R_f, p_1 and p_2, we proceed practically as follows:

a) We draw up the control graph by calculating, with the help of relations (14) and (15), the cuts of the two straight lines (of admission and rejection) which we figure afterwards on plane $n\,o\,d$ on axis n. With the aid of rela-

tion (*12*) we calculate the intersection point M of these two straigh lines and figure it in the same plane. We point out that relation (*12*) is only giving the ordinate of point d. The abscissa is always equal to N. We join the points $n_a =$ and n_r with M, thus obtaining the two straight lines.

We draw up the straight line whose ordinate is D which constitutes the superior limit of number of faults admitted in a selection of size n, which cannot exceed the admitted number of faults of the whole lot (Fig. 1).

b) We start drawing the products out of the batch and analyze thei quality. We figure each extraction by a point in the checking graph. Thir point will have as an abscissa the order number of the extraction, and as an ordinate the number of faults (cumulated) up to the respective extraction $\left(\sum_{i=1}^{n} x_i\right)$.

If, for instance, until extraction 5, 3 faulty products have been found, the point corresponding to this extraction will have the abscissa 5 and the the ordinate 3 (Fig. 2)

The points corresponding to the first extractions are usually situated between the two straight lines of the graph. In this case we check up the relation (*6*) and carry on checking until one of the points which have been thus constructed appear either under (on) the straight line of admission (the batch is accepted), or over (on) the straight line of rejection or the straight line D_1 (the lot is refused).

In case of admission, relation (*4*) should be checked up, in case of rejection relation (*5*) should be.

The difficulty arises when we come to drawing up a control graph involving certain calculations. A computer will help us do these calculations.

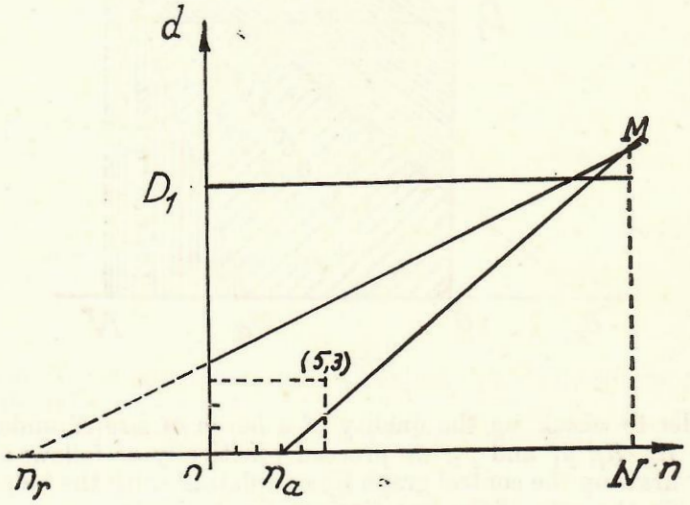

Fig. 2

We shall subsequently present the computation scheme and the symbolic programme for calculating the cuts of the straight lines of admission and their intersection, with the aid of a CIFA-102 computer.

We shall also annex (in tables 1, 2 and 3) the values of n_a, n_r and d that we have obtained for a batch of 2,000 products and the following values of p_1 and p_2:

p_2	p_1
0.10	0.01
0.09	0.06
0.08	0.05
0.07	0.04
0.06	0.03
0.05	0.02
0.04	0.01

Programme for n_a symbolic addresses

A_0	01 — C_{12}	11 — A_6	A_{20}	01 — C_6	14 — A_{12}
A_1	14 — C_2	00—0004	A_{21}	03 — C_1	07 — A_1
A_2	01 — A_{12}	17 — C_7	A_{22}	01 — C_{10}	14 — A_6
A_3	02 — C_{14}	11 — A_{12}	A_{23}	03 — C_2	07 — A_{34}
A_4	01 — A_6	17 — C_7	A_{24}	01 — A_{12}	03 — C_6
A_5	02 — C_{11}	11 — A_6	A_{25}	02 — C_{10}	11 — A_{12}
A_6	01 — N	04 — p	A_{26}	03 — C_3	07 — A_{35}
A_7	11 — q	01 — A_{12}	A_{27}	01 — C_6	14 — C_2
A_{10}	17 — C_5	02 — C_{13}	A_{30}	01 — C_6	14 — A_{17}
A_{11}	11 — A_{12}	00—0004	A_{32}	03 — C_4	07 — A_{36}
A_{12}	01 — R_6	05 — B	A_{31}	01 — C_6	14 — A_6
A_{13}	11 — X	10 — $\ln X$	A_{33}	00 — 0000	00 — 0000
A_{14}	12 — R	05 — q	A_{34}	00 — 0014	10 — A_6
A_{15}	11 — X	10 — e^{-x}	A_{35}	00 — 0014	10 — A_4
A_{16}	01 — C_{15}	03 — R	A_{36}	00 — 0014	10 — A_2
A_{17}	04 — N	00—0010			

Constants used

C_1	01 — R_{bf+1}	00 — 0000	cycling	programme	for	R_b
C_2	00—0000	04 — p_{f+1}	„	„	„	p
C_3	01 — R_{bf}	05 — B_{f+1}	„	„	„	B
C_4	01 — N_{f+1}	00 — 0000	„	„	„	N

C_5 00 — 0000 17 — 7777 cross out R_b of A_5
C_6 00 — 0001 00 — 0000 cycling of R_b
C_7 17 — 7777 00 — 0000 cross out p of A_6 and B of A_{12}
C_{10} 00 — 0000 00 — 0001 cycling of p
C_{11} 00 — 0000 04 — p_1 initialization of p (3)
C_{12} 01 — N_1 00 — 0000 initialize N_1
C_{13} 01 — R_{b1} 00 — 0000 initialize R_b in A_{12}
C_{14} 00 — 0000 05 — B_1 initialize B in A_{12}
C_{15} 999,999,999,999

Working addresses

X — address for the argument of subroutines
R — address where the result of subroutines is leid down $q = N \times p$

Notations used

ln^X — the subroutine computing $\frac{ln\, X}{100}$
e^{-x} — „ „ „ e^{-x}
R_{b1} — the address of first R_b (Analogus for p_1, N_1)
R_{bf+1} — the address following after the address of last R_b (Analogous for p_{f+1}, N_{f+1})

(Symbolic) Programme for computing d

A_0	01 — C_0	11 — A_{15}	A_{15}	04 — N	02 — C_{14}
A_1	01 — C_7	11 — C_1	A_{16}	00 — 0010	00 — 0004
A_2	01 — C_5	11 — A_{12}	A_{17}	01 — C_6	14 — A_{10}
A_3	01 — C_4	11 — A_{10}	A_{20}	01 — C_6	14 — A_{12}
A_4	01 — A_{10}	17 — C_{13}	A_{21}	03 — C_1	07 — A_{10}
A_5	02 — C_{11}	11 — A_{10}	A_{22}	00 — 0014	00 — 0004
A_6	01 — A_{12}	17 — C_{13}	A_{23}	01 — C_{10}	14 — A_{10}
A_7	02 — C_{12}	11 — A_{12}	A_{24}	01 — C_{10}	14 — A_{12}
A_{10}	01 — p_2	03 — p_1	A_{25}	01 — C_{10}	14 — C_1
A_{11}	11 — p	00 — 0004	A_{26}	03 — C_2	07 — A_4
A_{12}	01 — p_2	05 — p_1	A_{27}	01 — C_{10}	14 — A_{15}
A_{13}	11 — X	10 — lnx	A_{30}	03 — C_3	07 — A_1
A_{14}	01 — p	05 — R	A_{31}	00 — 0000	00 — 0000

Constants for d

C_0	$04 - N_1$	$02 - C_{14}$	C_6	$00 - 0000$	$00 - 0001$
C_1	$01 - p_2$	$05 - p_{1f+1}$	C_7	$01 - p_{21}$	$05 - p_{f+1}$
C_2	$01 - p_{2f+1}$	$00 - 0000$	C_{10}	$00 - 0001$	$00 - 0000$
C_3	$04 - N_{f+1}$	$00 - 0000$	C_{11}	$00 - 0000$	$03 - p_{11}$
C_4	$01 - p_{21}$	$03 - p_{11}$	C_{12}	$00 - 0000$	$05 - p_{11}$
C_5	$01 - p_{21}$	$05 - p_{11}$	C_{13}	$17 - 7777$	$00 - 0000$
			C_{14}	5	

Working addresses

$$p = p_2 - p_1$$

$$X = \frac{p_2}{p_1}$$

$$R = lnX$$

Table 1 (n_a)

R_f	$1 - R_f$	$p_2 - p_1$	R_b			
			0.10	0.05	0.02	0.01
0.10	0.90	0.03	74	96	127	150
		0.04	56	72	95	112
		0.05	44	58	76	90
		0.06	36	48	64	75
		0.07	32	41	54	65
		0.08	27	36	48	56
		0.09	24	32	42	50
0.05	0.95	0.03	75	98	139	152
		0.04	56	75	96	114
		0.05	45	60	77	91
		0.06	38	49	64	76
		0.07	32	42	56	65
		0.08	28	37	48	57
		0.09	25	32	42	50
0.02	0.98	0.03	70	99	129	153
		0.04	57	75	97	115
		0.05	48	60	78	92
		0.06	38	50	65	76
		0.07	33	42	56	66
		0.08	28	37	48	57
		0.09	25	33	43	51
0.01	0.99	0.03	77	100	130	153
		0.04	57	76	88	115
		0.05	46	60	78	92
		0.06	38	50	66	76
		0.07	32	43	56	66
		0.08	28	38	48	57
		0.09	26	32	44	51

Table 2 (n_r)

R_f	$p_2 - p_1$	R_b			
		0.10	0.05	0.02	0.01
		$1 - R_b$			
		0.90	0.95	0.98	0.99
0.10	0.03	73	75	76	76
	0.04	55	56	57	57
	0.05	44	45	46	46
	0.06	36	37	38	38
	0.07	31	32	32	33
	0.08	27	28	28	29
	0.09	24	25	25	25
0.05	0.03	96	98	99	99
	0.04	72	73	74	75
	0.05	58	59	59	60
	0.06	48	49	49	50
	0.07	41	42	42	42
	0.08	36	37	37	37
	0.09	32	33	33	33
0.02	0.03	127	129	130	130
	0 04	95	96	97	97
	0.05	76	77	78	78
	0.06	63	64	65	65
	0.07	54	55	55	56
	0.08	47	48	49	49
	0.09	42	42	43	43
0.01	0.03	150	152	153	153
	0.04	112	114	114	115
	0.05	90	91	92	92
	0.06	75	76	76	76
	0.07	64	65	65	66
	0.08	56	57	57	57
	0.09	49	50	51	51

Table 3 (d)

p_2	p_1						
	0.07	0.06	0.05	0.04	0.03	0.02	0.01
0.1	166.5	158.5	144.5	130.5	116.5	98.5	78.5
0.09	—	146.5	134.5	122.5	108.5	92.5	72.5
0.08	—	—	126 5	114.5	102.5	86.5	66.5
0.07	—	—	—	106.5	94.5	88.5	62.5
0.06	—	—	—	—	86.5	72.5	54.5
0.05	—	—	—	—	—	64.5	48.5
0.04	—	—	—	—	—	—	42.5

SCHEME FOR COMPUTING THE CUT OF STRAIGHT LINE OF ADMISSION

$$n_a = \left[1 - \left(\frac{Rb}{1-Rf}\right)^{\frac{1}{N(P_2-P_1)}}\right] = N(1-A); \text{ where } A = \left(\frac{Rb}{1-Rf}\right)^{\frac{1}{N(P_2-P_1)}} = e^{\frac{1}{N(P_2-P_1)} \ln \frac{Rb}{1-Rf}}$$

$$A = e^{\frac{1}{Np} \ln \frac{Rb}{B}} \quad \text{where } p = P_2 - P_1 \quad B = 1 - R_f$$

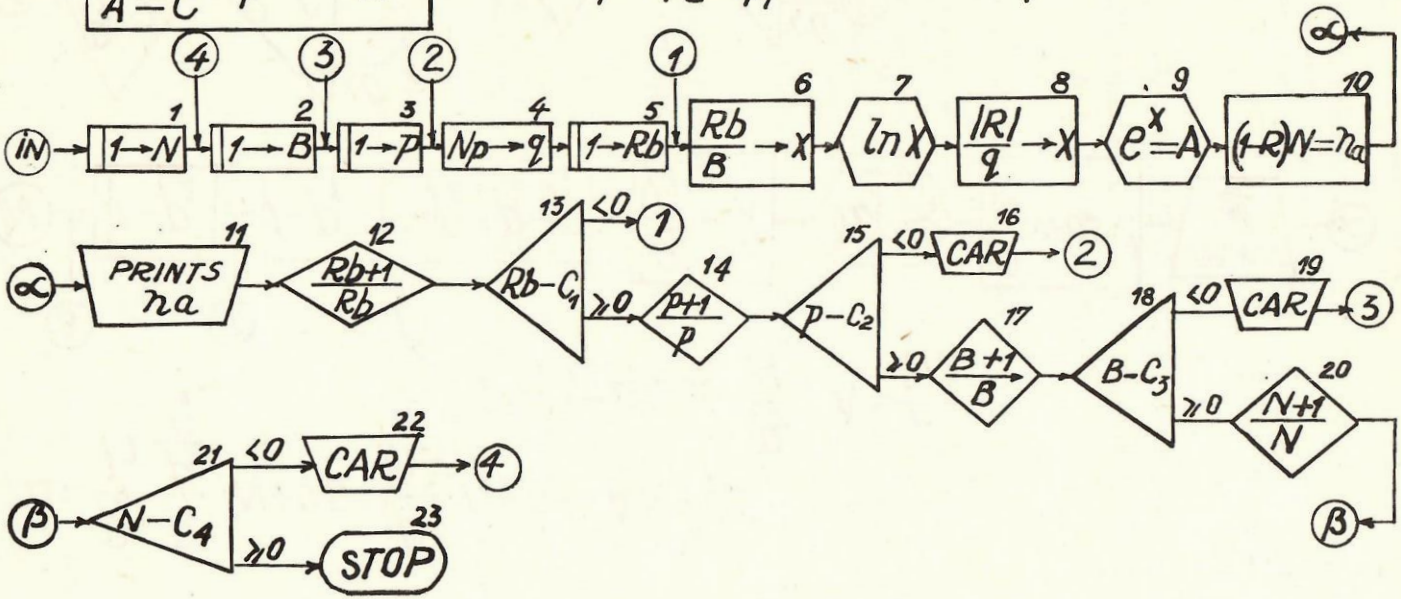

Fig. 3

Note: For cutting the straight line of rejection, the same scheme can be used, with some slight modifications

SCHEME FOR COMPUTING d

$$d = \frac{P_2 - P_1}{\ln \frac{P_2}{P_1}} N + 0{,}5 = \frac{P}{\ln X} N + 0{,}5 = A \cdot N + 0{,}5$$

$$P = P_2 - P_1 ; \; X = \frac{P_2}{P_1} ; \; A = \frac{P}{\ln X}$$

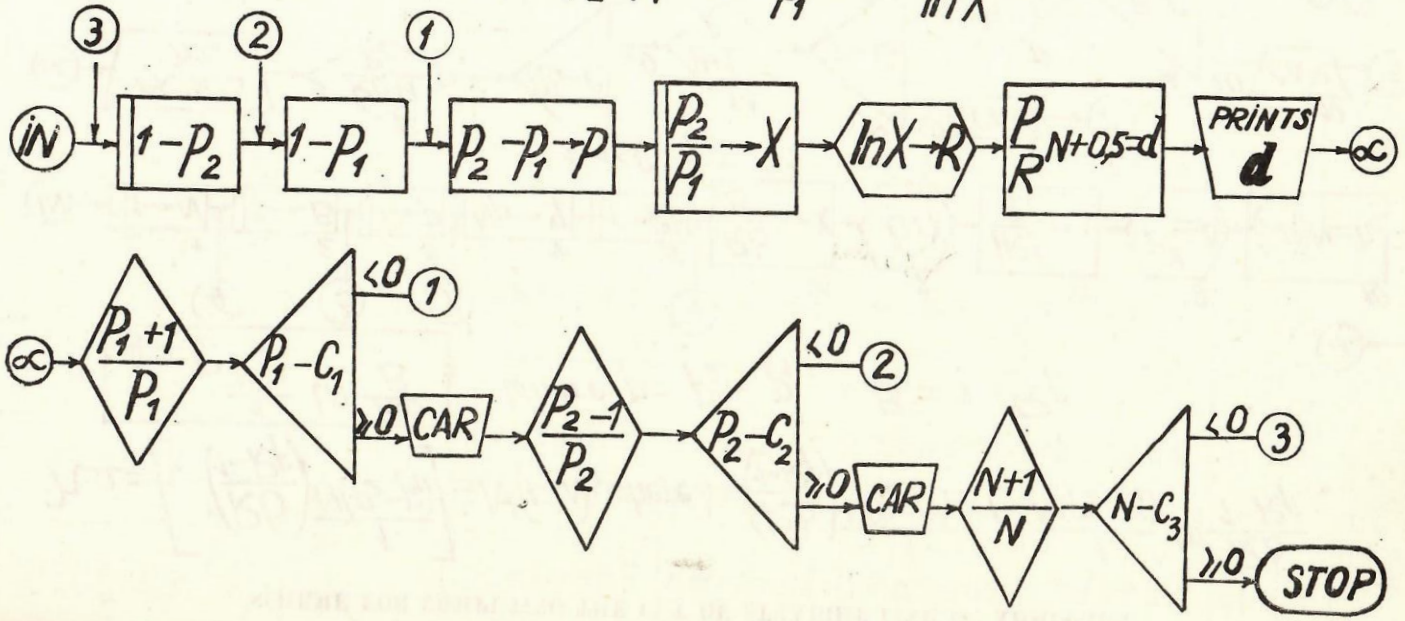

Fig. 4

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LE CONTROLE EN SEQUENCE DE LA QUALITE DES MARCHANDISES POUR DES LOTS INFERIEURS A 5000 PIECES

L'ouvrage présente une méthode de contrôle en séquence de la qualité des marchandises en lots ne dépassant pas 5000 pièces.

Dans la première partie, les auteurs établissent les bases mathématiques de l'étude et indiquent la méthode pratique de travail ainsi que l'utilité du graphique de contrôle présenté.

On présente également les programmes de calcul de la machine à calculer ECONOMIST CIFA 102 pour les coupures des deux données du graphique de contrôle et de leur point d'intersection. Ces programmes peuvent servir au calcul des trois paramètres, pour toute valeur imposée par la pratique aux variables N, P_1, P_2, R_b et R_f.

On présente aussi des schémas de calcul logiques pour ces deux programmes, ainsi que les valeurs obtenues par les auteurs avec ces programmes pour des lots de 2000 pièces.

ВЫБОРОЧНЫЙ КОНТРОЛЬ КАЧЕСТВА ТОВАРОВ ДЛЯ ПАРТИЙ ДО 5000 ШТУК

Статья показывает метод выборочного контроля качества товаров в партиях, имеющих меньше 5000 штук.

В первой части даются математические основы работы, указывается метод практической работы и польза представленной схемы контроля.

Далее представлены расчетные программы отрезков двух данных контрольной схемы и точки их пересечения для счетной машины «ECONOMIST CIFA 102». Эти программы могут служить для расчета трёх параметров для любого заданного практикой значения переменных N, P_1, P_2, R_b şi R_f.

Показаны также и логические схемы расчета для этих двух программ, также и значения, полученные авторами по этим программам, для партий товаров, имеющих 2000 штук.