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A Linear Model for the Territorial Economic Development Planning

1. GENERAL CONSIDERATIONS

It is known that one of the main directions of action of the Party as mentioned in the Program of the Romanian Communist Party for the building of the manysidedly developed socialist society and advancing of Romania toward communism is the permanent care for a rational distribution of productive forces on the whole territory of the country. For the realization of this general orientation it has been established for the period 1976—1980 a minimal level of the global industrial production for every district that implies for some less developed from industrial view point districts a much more rapid growth than the average rate for the whole country. It is stated that each district has to realize a global industrial production of at least 10 milliard leis; for some districts such as Bistriţa-Năsăud, Sălaj, Tulcea, Vaslui, Covasna, Botoşani, Vrancea, Gorj, Ialomiţa, this orientation implies a growth rate ranging from 15 to 25% and even more, hence $1 \cdot 7 - 2 \cdot 8$ times more than the average rate for the country (11.2%). At the same time, such orientation leads to a more rational distribution of labour force among industry and agriculture corresponding to the world tendencies, especially to the exprience of the industrial developed countries. The directions of development established for Romania up to the end of the century show that these orientations will be actual for a rather long time from now. It is very interesting to try a corresponding mathematical model that should give the most rational way of realizing the proposed goals, taking into account the economic starting level of different territorial units, their natural and labour resources and some other restrictions. In such approach to the problem of planning and decision making for the territorial development on the first plane occurs the relationship between the territorial and central levels; this relationship is realized mainly through restrictions that the central levels introduce in

the territorial models and on the other side through outputs of the territorial models that are transmitted to the central level to be compared to the global resources and objectives in order to generate new restrictions for the territorial level.

Such a procedure of planning and decision making appears thus in several steps; in the first step the central level imposes to the territorial ones a rather small number of restrictions, expressed in very synthetic indices: the analysis of the obtained output allows the central level to be more specific in some restrictions and after that the problem is reconsidered at the territorial level, the procedure is repeated untill one gets a good global harmonization in the conditions of a rational and efficient development at the territorial level. Corresponding to the above approach one obtains mathematical programming models, generally dynamic and nonlinear. The nonlinearities appear because of different production functions. Considering the limited role of these functions in the model and the necessity of getting effective computational results it is highly desirable to use linear production functions with coefficients that may characterize different industrial branches, development stages and strategies.

We may try optimization at territorial level with respect to different performance indices. In the general conception we described above, it seems reasonable that we have to ask for the way of realizing the objectives imposed by the central level with a minimum of investment. These local optimum problems have then to be associated with different optimizations at the global level, balance computations and so on.

There is a possibility to use such models also for special programs of territorial development. The central level imposes certain objectives for the development of a given territory (that gives a set of restrictions for the corresponding territorial model) indicates the material and labour aid that can be provided from the center and the model can furnish the smallest amount of investments that allows to realize the objectives considering the starting level and the resources.

It must be said that the model is not able to consider all aspects; if some indices of the quality of life and of incomes and consumption of the population could be included somehow in the model most of the qualitative aspects of the results of development have to be the object of specific analysis.

2. THE TERRITORIAL MODEL

We point out that what we are describing is a model, that is a simplified representation which tries to obtain information concerning the most essential aspects; hence we shall take into consideration in the model for each territorial unit only some of the economic branches or activities that seem to be essential for the given territorial unit as well from the local view point (their weight in the economy of the territorial units, the concrete possibilities of implantation and development) as from the view point of the national economy as a whole. That allows to use physical, natural indices which are essential in considering restrictions for natural resources or materials for which there are strong restrictions at the level of the national economy. Such restrictions are

more important today under the circumstances of a very heavy accent on economic and rational utilization of energy and other resources.

For restrictions concerning synthetic indices as global production, commodities production, labour productivity a.s.o., we may use corresponding prices to get the value expression. We shall note by K_n^i the volume of productive funds existing at the beginning of the year n in the branch or activity i; L_n^i will be the corresponding amount of labour force.

Denote by Q_n^i the volume in natural units, of the production in the year n, of the branch or activity i. For industrial activities we shall consider relations of the form

$$Q_n^i = f_n(\alpha_n^i K_n^i + \beta_n^i K_{n+1}^i), \quad \alpha_n^i + \beta_n^i = 1$$

expresssing dependence of the production on an average volume of productive funds; we shall take $L_n^i = K_n^i k_n^i$ to describe the labour force necessary to make the funds produce. Here α_n^i and β_n^i are weights depending on the rates of entering the activity of the new funds; if this is a uniform one $\alpha_n^i = \beta_n^i = 0.5$. Parameters f_n^i and γ_n^i represent the efficiency of funds.

We shall use for agricultural activities a linear in L_n^i and K_n^i function that allows to point out mutations in the labour force as a result of the growth of funds.

The essential of the dynamics is expressed by the dynamics of productive funds.

$$K_{n+1}^i = (1 - \mu_n^i)\, K_n^i + \sum_{\tau=0}^{\theta_i} \lambda_{n,\, n-\tau}^i \cdot Z_{n-\tau}^i$$

where μ_n^i is a coefficient of devaluation of funds, θ_i is the maximum lag, $\lambda_{n,\, n-\tau}^i$ the weight of investments made in year $n - \tau$ that enter the activity in year n (for the activity of branch i). The main decision variables is Z_n^i expressing the amount of investment for the year n in the branch or activity i.

We have the following restrictions:

a. Restrictions Concerning Occupation of Labour Force

Since different branches or activities use in different proportions male and female labour force the restrictions will be written foreach of these types of labour force. We have:

$$\sum_{i \in I} \eta_n^{i,\, s} L_n^i = \hat{L}_n^s \quad s = 1.2,$$

where $\hat{L}_n^s$ is the volume of labour force of type s that must be occupied in the year n; $\eta_n^{i,\, s}$ is a coefficient expressing the weight of the labour force of type s that has to be achieved in the year n for activity $i(\eta_n^{i,\, 1} + \eta_n^{i,\, 2} = 1)$, I is the set of all activities considered in the model.

The model assumes a preliminary forecast of the evolution of labour force on the territory.

b. Restrictions Concerning Resources

We assume that on the territory r limitative resources have been identified. The structure and volume of production have to satisfy restrictions imposed by the limitative character of these resources.
Such restrictions have the form:

$$\sum_{i \in I} \delta_n^{i,j} \cdot Q_n^i \leqslant R_n^j \qquad (j = 1, 2 \ldots r)$$

where: $\delta_n^{i,j}$ is the specific consumption in year n for the resource j in activity i, and R_n^j the available for year n amount of the resource j.

c. Restrictions Concerning Synthetic Indices

We must have:

$$\sum_{i \in I_a} X_n^i \geqslant \hat{X}_n^a, \quad \sum_{i \in I_b} X_n^i \geqslant \hat{X}_n^b$$

where $X_n^i = p_n^i Q_n^i$ is the global (or commodities) production in year n for activity i, $\hat{X}_n^a$, $\hat{X}_n^b$ the bounds established for the territory by the central level in industry and agriculture respectively, p_n^i are prices.

A second group of restrictions is:

$$\sum_{i \in I_a} X_n^i \geqslant \hat{W}_n^a \sum_{i \in I_a} L_n^i, \quad \sum_{i \in I_b} X_n^i \geqslant \hat{W}_n^b \sum_{i \in I_b} L_n^i$$

representing lower bounds for labour productivity, also established by the central level. It is also important a restriction of the form

$$\sum_{i \in I} v_n^i L_n^i \geqslant \hat{v}_n \sum_{i \in I} L_n^i$$

where v_n^i is the average income in activity i, in year n and v_n is a lower bound for the average income in year n for the territory.

We see that restrictions given so far correspond to the first step, when the central level imposes only limitations on the global production, productivity and minimal income. In the following steps new restrictions for sizes allowable for production and consumption of materials in different branches and activities are added by the central level.

The unknowns are Z_n^i, the investment in year n, for activity i, and the cost function is $\sum_{i,n} Z_n^i$ that has to be minimized; hence we want the objectives to be realized by satisfying all restrictions and with a minimum investment effort.

Since all restrictions and the dependences are linear, we obtained mathematically a linear dynamic programming problem; to solve it we may use linear programming packages. Even if planning refers to 15—20 years, since at territorial level the number of branches is not too large (we take into account only the essential ones) the size of the problem seems to be not prohibitive.

The general description of the procedure is: in the first step we have a small amount of limitations imposed by the central level; in the second step after the analysis at the central level of the outputs of territorial models new restrictions concerning production and consumption of limitative resources and materials are added.

Since these new restrictions may be too strong or the investment required too high, a new analysis at the central level will change some of the restrictions and the optimization is repeated.

In this way the interconnection between central and territorial level is expressed not only in restrictions the central level gives to the territorial model, but also by the outputs the territorial level sends to the central one after solving its optimization problem.

These outputs are $Z_n = \sum_i Z_n^i$ the amount of investments for year n; $Z_n^i = \sum_i \omega_n^i Z_n^i$ the amount of investment in year n, assuming import; $A_{nk} = \sum_i a_{n,k}^i Q_n^i$ consumption of limitative, centrally distributed materials, $B_{n,k} = \sum_i b_{n,k}^i Q_n^i$, consumption of imported materials for the material balance Q_n^i have also to be communicated. Since we have a lag of entering the activity of investment, we need also restrictions for the level of some funds after the planning horizon considered. Considering that the model is essentially based on linear programming we may not expect computational difficulties; the entrance data also are rather close to the ones used in current planning practice.

APPENDIX

The Economic-Mathematical Model for Territorial Development Planning
Notations

I — the set of branches in which the economy of the territory was classified;
I_a — the set of industrial branches;
I_b — the set of agricultural branches;
T — the planning horizon;
m — the number of raw materials considered in the model;
r — the number of limitative resources considered in the model;
K_n^i — the volume of productive funds existing at the beginning of year n in the branch i;
L_n^i — the labour force alloted to branch i in the year n;
Q_n^i — the volume, in natural units, of the production of the branch i in the year n;
f_n^i — the efficency coefficient of fixed funds in branch i in the year n;
α_n^i, β_n^i — weights, expressing the rate of entering the activity of new funds in branch i;
γ_n^i — the inverse of the funds per capita ratio in branch i in the year n;
μ_n^i — the coefficient of phisycal and moral wear and tear of fixed funds;

$\lambda^i_{n,\,n-\tau}$ — the "lag-coefficient" which expresses the weight of investment made in branch i in the year n and entering the activity in year n;

Q_i — the maximum lag for branch i;

Z^i_n — the volume of investment alloted to branch i in year n;

$\hat{L}^1_n$, $\hat{L}^2_n$ — the volume of labour force (men and women, respectively) existing in year n;

$\eta^{i,1}_n$ $\eta^{i,2}_n$ — the weights of male and female labour force, respectively which have to be occupied in the year n for the branch i;

$\delta^{i,j}_n$ — the specific consumption for the resource j in branch i, in year n;

R^j_n — the amount of resource j, available in year n;

X^i_n — the value of the global production of branch i in the year n;

$\hat{X}^a_n$, $\hat{X}^b_n$ — lower bounds established for the production of the territory by the central level in industry and agriculture, respectively;

$\hat{W}^a_n$, $\hat{W}^b_n$ — lower bounds established for labour productivity in industry and agriculture, — *respectively, by the central level;*

$\hat{v}^i_n$ — the average income per worker in branch i in year n;

$\hat{v}_n$ — *lower bound for the average income in year n on the territory;*

Z_n — the total amount of investment which is necessary for the territory in year n;

Z^1_n — the total amount of investment in year n, which assumes import;

ω^i_n — the weight of imported investment in branch i in year n;

$A_{n,k}$ — the total amount of centrally distributed raw materials of type k, needed by the territory in year n;

$B_{n,k}$ — the total amount of imported materials of type k, needed by the territory in year n;

$a^i_{n,k}$, $b^i_{n,k}$, — specific consumptions of internally produced and imported materials of type k, respectively, for branch i in year n.

Model

$$Q^i_n = f^i_n \cdot (\alpha^i_n K^i_n + \beta^i_n K^i_{n+1}) \qquad i \in I_a;\ n \in T$$

$$Q^i_n = f^i_n \cdot (\alpha^i_n K^i_n + \beta^i_n K^i_{n+1}) + g^i_n L^i_n \qquad i \in I_b;\ n \in T$$

$$L^i_n = \gamma^i_n K^i_n \qquad i \in I_a;\ n \in T$$

$$K^i_{n+1} = (1 - \mu^i_n)\, K^i_n + \sum_{\tau=0}^{\theta i} \lambda^i_{n,\,n-\tau} \cdot Z^i_n \qquad i \in I;\ n \in T$$

$$\sum_{i \in I} \eta^{i,\,s}_n \cdot L^i_n = \hat{L}^s_n \qquad s = 1, 2;\ n \in T$$

$$\sum_{i \in I} \delta^{i,j}_n Q^i_n \leqslant R^j_n \qquad j = 1, 2 \ldots r;\ n \in T$$

$$\sum_{i \in I_a} X^i_n \geqslant \hat{X}^a_n \qquad n \in T$$

$$\sum_{i \in I_b} X^i_n \geqslant \hat{X}^b_n \qquad n \in T$$

$$X^i_n = p^i_n \cdot Q^i_n \qquad i \in I;\ n \in T$$

$$\sum_{i \in I_a} X^i_n \geqslant \hat{w}^n_a \sum_{i \in I_1} L^i_n \qquad n \in T$$

$$\sum_{i \in i_b} X_n^i \geqslant \hat{w}_n^b \sum_{i \in I_2} L_n^i \qquad n \in T$$

$$\sum_{i \in I} v_n^i L_n^i \geqslant \hat{v}_n \sum_{i \in I} L_n^i \qquad n \in T$$

Output Variables Which are Transmitted to the Central Level

$$Z_n = \sum_{i \in I} Z_n^i \qquad n \in T$$

$$Z_n^1 = \sum_{i \in I} \omega_n^i \cdot Z_n^i \qquad n \in T$$

$$A_{n,k} = \sum_{i \in I} a_{n,k}^i \cdot Q_n^i \qquad k = 1, 2 \ldots m;\ n \in T$$

$$B_{n,k} = \sum_{i \in I} b_{n,k}^i \cdot Q_n^i \qquad k = 1, 2 \ldots m;\ n \in T$$

$$Q_n^i = f_n^i \cdot (\alpha_n^i K_n^i + \beta_n^i K_{n+1}^i) \qquad i \in I_a;\ n \in T$$

$$Q_n^i = f_n^i \cdot (\alpha_n^i K_n^i + \beta_n^i K_{n+1}^i) + g_n^i \cdot L_n^i \qquad i \in I_b;\ n \in T$$