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Computation of Elasticity Coefficients on Basis of Data Adjusted by means of Various Mathematical Functions

This paper shows the results obtained in the computation of elasticity coefficients of expenses for goods and services as a function of the income level.

The consumption requirements of the population are influenced by numerous factors of an economic, demographical and other order such as: income structure and level, price level, supply conditions, variety of assortments, abundance of products on the market, degree to which the population is endowed with long-lasting objects, housing conditions, family structure and size, age and sex, customs, preferences, etc.

The order and value of the population's expenses are established in close connection with these factors and particularly with the size of the effective income. A change occuring in income usually also entails a change in the structure of expenses for goods and services.

The sudy of the interdependence between the level of consumption expenses for goods and services, and the income level, is achieved and measured — among other methods — by using various mathematical functions.

Elasticity coefficients are widely used to measure the dependence between expenses and incomes.

In view of characterizing the elasticity of consumption demand as a function of income it is necessary for a few conditions to be fulfilled when computing the respective coefficients. In the first place it is clear that the elasticity coefficients will reflect the trend of consumption demand in conditions where the supply of the population is permanently and uniformly ensured for the respective goods in the distribution network investigated. It is likewise necessary, when computing these coefficients, to eliminate the influence of other factors — besides incomes — which could influence the consumption demand (price changes, etc.).

Elasticity coefficients are generally computed as being the ratio between the relative change of expenses and the relative change of incomes. These coefficients answer the question: if the money income (x) increases by 1 per cent, by what percentage (K) will the cash expenses increase for the consumption of various products (y). Elasticity coefficients so computed may be positive or negative according as there is a direct or reverse correlation with the factor modifying the consumption (demand).

Elasticity coefficients are usually computed on basis of data obtained from selective investigations.

Elasticity coefficients can also be computed on basis of data obtained from total statistical recordings, particularly those regarding the sale of goods.

Both partial elasticity coefficients (for each income group), and the general elasticity coefficient (for all income groups) are to be computed.

Elasticity coefficients are generally computed statistically, since, on basis of selective investigations, it is not possible to assure comparability of the income ranges in their dynamics.

The computation method according to the empiric formula is quite accessible, it requires a reduced volume of work and is therefore preferred in practical computations,

However, in practice, elasticity coefficients determined according to the empiric formula have certain limits of use which therefore prevent them from being employed in more efficient and thorough studies regarding the demands in goods and services in proportion with the factor income.

A series of investigations and practical computations have shown that this method may only be applied to those foodstuffs where the variation of expenses from one income group to another is small. For the other categories of food expenses, for expenses regarding goods other than foodstuffs and services, the elasticity coefficients present, in most cases, variations from one income group to another. This is due to the fact that, on basis of the empiric data used in the computation, this method does not always allow to determine a precise trend of expenses and, respectively, of the elasticity coefficients along the scale of the income groups since, generally, the data observed are influenced by errors or they present occasional variations. On the other hand, this method may be applied without any change to all categories of expenses, although the consumption of some commodities may vary differently with respect to income, and from the standpoint of the technique of computation, by using the same reporting basis (family group with smallest income) the increases considered are not uniform in size and do not ensure the conditions of the mathematical definition for the elasticity coefficients (these increases should be infinitesimal). The fact may also be mentioned that this method does not take into account the relation expense-income for each income size group taken independently.

To make evident more accurately the various trends occurring in the manner the incomes are used along the scale of income groups, the use is recommended, for the whole population or for certain categories of the population, of *analytical adjustments of the empiric data by means of mathematical functions* and then, on this basis, to determine the elasticity coefficients. It is therefore necessary to find for each category of expenses in part (by commo-

dity or groups of commodities) the mathematical function which expresses best the correlation between the value of expenses and the income size of the population.

The choice of the function used for adjusting the empirical data constitutes one of the decisive stages, one on which depends the obtaining of the results that are required to characterize the trend with sufficient accuracy. The plotting of the curve which best approximates the empiric data is an involved operation where the character of the data being adjusted, the time period to which they refer and the variations they manifest from one time period to another, etc. should all be taken into account.

The function must also correctly reflect the economic phenomenon and it must be possible to apply it relatively easily to economic analyses and studies. The establishment of these mathematical functions is achieved after numerous tests, confronting of results and mathematical checking of the adjustement curves.

The use of the adjusted expense data by income groups, when compared with the empirical ones, presents some advantages among which we mention: increased possibility of analyzing and making stand out the general trend of expenses along the scale of income groups; it ensures a logical checking of the results obtained by the study of the curves grapuically represented as well as the confrontation of the absolute empirical and adjusted data; it allows for a more efficient use of the elasticity coefficients, in comparison with those obtained on basis of empirical data.

Since the establishment of the law stating the reverse dependence between the weight of expenses for the purchase of foodstuffs and the income sizes, expostulated by E. Engel, until approximately the year 1900, the data obtained by the survey of family budgets, — whose number and content has increased and developed — have been used particularly to determine the computing elements of living standard synthetic indicators (particularly for computing the index of living costs). Concomitantly, researches were initiated — later to develop intensively — meant to establish mathematical methods and procedures capable of being efficiently used in the analysis and interpretation of the data from family budgets, particularly in the spirit of an interpretation of E. Engel's curve.

A.C. Pigou is among the first researchers in this domain. His main concern was to establish methods for comparing elasticity between two commodities. J. Marschak, using the data of family budgets in Germany and grouping the families according to income size, computed elasticity coefficients for some commodities on basis of relation $X = F\left(\frac{Y}{p}\right)$ where X represents expenses, Y family income and p the price of the goods consumed, the ratio $\frac{Y}{p}$ being in fact the actual income. In the specialty literature the fact was stressed that J. Marschak did not afford sufficient attention in determining demand on basis of the empirical data, but he started from "a priori" hypotheses regarding the shape of the curves which, thus, were not based on the investigation of actual facts.

R.G. Allen and A.L. Bowley also used a linear dependence $Y = a + bx$ in their investigation regarding the elasticity of demand.

In one of his works, Colin Clark formulated the opinion that the dependence between expenses for foodstuffs and income is a hyperbolic function, and H. Working interpreted the dependence between expenses and income on basis of a logarithmic hyperbolic function:

$$\frac{F}{T} = a - b \log T$$

where F represents expenses for foodstuffs, T — total expenses, and the parameters a and b are determined by the least squares method.

To determine E. Engel's, curve, J.L. Nicholson tried the parabolic function $y = a + bx + cx^2$, reaching the conclusion that it can only be used in the case of a large number of budgets investigated for each income group.

S.J. Prais considered that the dependence formulated by E. Engel can be expressed by means of the semi-logarithmic function $\frac{Y}{n} = a + b \log \frac{X}{n}$, where Y represents expenses for a certain commodity, X total expenses of the family, n being the number of adult persons in the family, $\frac{Y}{n}$ and $\frac{X}{n}$ being therefore the average expense and income per adult person.

Later, S.J. Prais in collaboration with H.S. Houthakker, trying to establish a hierarchy of functions, found that the linear function is inferior to the semi-logarithmic, excepting expenses for fruit, whose dependence on income is better characterized by the linear function, and that the hyperbolic function, superior to the linear, is inferior to the semi-logarithmic function. The latter gives good results, according to the authors, in representing the dependence of expenses for foodstuffs on income, while the logarithmic function, that of expenses for luxury articles. Later, in a paper written in commemoration of one century since the expostulation of Engel's law, H.S. Houthakker resumes the above ideas and gives arguments showing that E. Engel's law is expressed most correctly by the linear, semi-logarithmic and parabolic functions.

In one of his works, P.P. Maslov, shows that the linear and the parabolic functions are more adequate for studying the dependence between expenses and income on basis of family budgets.

In Hungary, the German Democratic Republic and Czechoslovakia, the linear semi-logarithmic and parabolic function was used with better results for adjustung the data, while the exponential function ($Y = aX^b$) was also used for some groups of expenses.

L.M. Goreux brings new ideas in the study of E. Engel's curves, for which he uses a number of eight functions. He computed the elasticity coefficients on basis of the data from 50 surveys carried out in 32 countries, 44 of the surveys being carried out after 1950. He reached the conclusion that none of the functions used deserves any preference and that, theoretically, it is not necessary to use the same function for all the categories of expenses. The author shows that, when choosing the function, it is necessary to continue by a study of graphic adjustment; likewise, a formula should not, in all cases, be preferred to another, since the choice of the curve depends, to the same extent, on the commodity to which it is applied as well as on the country to which it refers.

Among the functions studied by L.M. Goreux, we mention :

$$y = a + bx$$

$$\log y = a + b \log x$$

$$y = a + b \log x$$

$$y = a - \frac{b}{x}$$

$$\log y = a - \frac{b}{x} - c \log x$$

$$y = a + b \log x - c (\log x)^2$$

Some of the functions used were of only an experimental character (parabolic, sygmoid).

In contrast with the dynamic series where analytical adjustments are relatively easy to apply and yield satisfactory results, when the elasticity of expenses for goods and services as a function of income — particularly for short time periods, usually one year — are studied, the use of analytic adjustment requires greater attention since the data employed are mostly obtained on basis of selective investigations which do not always assure sufficiently great stability of the averages for all the categories of expenses for goods and services*.

More and more opinions seem to advocate that the use of the formulae established by L. Törnqvist present some additional advantages — in comparison with other mathematical functions recommended by various authors — in characterizing the dependence between expenses and income, as formulated by E. Engel.

L. Törnqvist**, of Sweden, carrying out more thorough studies on basis of the empirical data and of broader information regarding family budgets, used adjustment methods that are diffetentiated by categories and groups of expenses : one function for goods of first necessity, and another for those less necessary, as well as one for luxury articles :

I. $y = a \frac{x}{x + b}$ to be applied to goods and services considered of first necessity ;

II. $y = a \frac{x - c}{x + b}$ to be applied to goods and services considered less necessary ;

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* L. M. Goreux shows that in view of studying the elasticity of demand, a sample formed of 20,000 households could be used for a double classification : the one according to the size of the household, and the other by social or geographic groups. A simple classification should, normally, be sufficient for a sample of 2,000 households. For samples smaller than 2,000 households no classification should be made.

** His formulae are presented and commented in a series of specialty works among which we mention :

— Herman Wold, *Demand Analysis. A Study in Econometrics*, Almqvist and Wiksell, Stockholm, 1952 ;

— Jan Tinbergen, "*L'économétrie*", Paris, 1954, published by the Centre d'Etudes Economiques — Etudes et mémoires — (French translation from the English edition published by G. Allen and Unwin under the title "Econometrics").

III. $y = ax\frac{x-c}{x+b}$ to be applied to objects considered of a relatively high values and prolonged use,

where:

x = average cash income per person;
y = average cash expenses per person;
a, b, c = parameters evaluated on basis of the empiric data regarding the expenses (y) and the corresponding incomes (x).

The first formula can be used in the case where the specific weight of expenses for goods and services from total incomes show a decrease in proportion with various increasing income levels; the second, which is not applied to law income levels, can be used in case the specific weight of expenses for goods and services from total incomes presents an increase on the scale of income groups, and the third — in the case of an important increase in the weight of expenses from total incomes on various increasing income levels (an unlimited increase of expenses).

However, the curves corresponding to these formulae are applied to those groups of goods and services that cannot replace each other in consumption. For the products that can replace each other in consumption (butter and margarine, etc.) other formulae must be applied. Such a formula, of the type: $y = x\ (a \log x + b)$ was elaborated by A.A. Konius.

A first advantage in using L. Törnqvist's formulae, in comparison with other mathematical functions, is the fact that they apply differentially to one category of expenses or to another. On the other hand, L. Törnqvist has given indications regarding the inclusion of the goods and services in the three groups established, recommending as a basic criterion the specific weight of expenses from the total incomes of each group, completed by a minute analysis of the behaviour of expenses in proportion with the change in income.

To compute the elasticity coefficients by means of L. Törnqvist' functions it is necessary, for each of L. Törnqvist's formulae, to substitute the corresponding values of y for the derivative $f'(x)$ in the general formula of elasticity

$$K_e = \frac{x}{y} f'(x)$$

Thus, in the case of L. Törnqvist's formula (I), the elasticity coefficient is determined as follows:

$$K_e = \frac{x}{y}\left(a\frac{x}{x+b}\right)' = \frac{x}{y} \times \frac{a(x+b-x)}{(x+b)^2} = \frac{x}{y} \times \frac{ab}{(x+b)^2}$$

In view of adjusting the empiridal data, the value of the parameters (a) and (b) is determined by means the least squares method, from a system of two equations, after L. Törnqvist's function (I) has first been brought to its linear form:

$$a = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2}$$

and

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

where:

x = incomes;
y = expenses;
n = number of income groups.

In case of L. Törnqvist's formula (II) the elasticity coefficients are determined as follows:

$$K_e = \frac{x}{y}\left[a\frac{(x-c)}{(x+b)}\right]' = \frac{x}{y} \times \frac{a(x+b-x+c)}{(x+b)^2} = \frac{x}{y} \times \frac{a(b+c)}{(x+b)^2}$$

The value of the constants a, b, c is determined, in the case of this formula, by the method of the chosen points. Out of the 9 groups of income for which the computations have been made, three points were chosen, corresponding to income groups III, VI and IX.

Thus, on basis of L. Törnqvist's formula (II), three equations are formed:

$$y_3 = a\frac{x_3 - c}{x_3 + b}$$

$$y_6 = a\frac{x_6 - c}{x_6 + b}$$

$$y_9 = a\frac{x_9 - c}{x_9 + b}$$

The value of a and b results from the solving of this system. The value of c is obtained by substituting a and b in one of the three above formulae:

$$a = \frac{(y_6 x_6 - y_3 x_3)(y_6 - y_9) + (y_6 - y_3)(y_9 x_9 - y_6 x_6)}{(x_6 - x_3)(y_6 - y_9) + (y_6 - y_3)(x_9 - x_6)}$$

$$b = \frac{(x_6 - x_3)(y_9 x_9 - y_6 x_6) + (x_9 - x_6)(y_3 x_3 - y_6 x_6)}{(x_6 - x_3)(y_6 - y_9) + (y_6 - y_3)(x_9 - x_6)}$$

The elasticity coefficient is computed on basis of. L. Törnqvist's function (III), as follows:

$$K_e = \frac{x}{y}\left(ax\frac{x-c}{x+b}\right)' = \frac{x}{y} \times a\left(\frac{x-c}{x+b} + x\,\frac{x+b-x+c}{(x+b)^2}\right) =$$

$$= \frac{x}{y} \times a\frac{(x-c)(x+b) + x(b+c)}{(x+b)^2}$$

The value of the parameters a, b, c, is determined like in the case of formula (II), by the method of the chosen points.

In Romania, in the last few years, L. Törnqvist's formulae have been tested as adjusting methods of the empiric data and, respectively, to compute the elasticity coefficients. Likewise, adjustment computations of the empiric data and computations of the elasticity coefficients have been carried out by using the linear semi-logarithmic, parabolic and exponential functions. The computations evidenced the fact that L. Törnqvist's formulae effectively present some advantages when compared with other mathematical functions and the use of these formulae will have to be studied still more thoroughly in the future.

Various computations and tests were carried out on basis of the data obtained from the investigation of the incomes and expenses of a certain collectivity of wage-earners' families in view of determining elasticity coefficients capable of expressing as accurately as possible the trend of the population's consumption demands as a function of the income level.

The results of these tests have led to important conclusions regarding the mode of computation of elasticity coefficients, out of which we mention:

— the necessity to compute the elasticity coefficients on basis of the series of adjusted data, in view of making the trend of consumption demand stand out clearly;

— the computation of the elasticity coefficients on basis of the adjusted data should be carried out differentially by products and groups of products by means of those mathematical functions which best express the dependence between demand and income level.

We show hereunder a few computations and conclusions which have stood out from the tests carried out.

I. ELASTICITY COEFFICIENTS OF EXPENSES FOR THE PURCHASE OF FOODSTUFFS AS A FUNCTION OF INCOME

The consumption demand of the population for foodstuffs is inelastic on the scale of income groups. The mathematical functions used have made this trend stand out, some of them to a high extent, others to a weaker extent.

Here are a few examples:

Elasticity coefficients of expenses for the

	Kind of data used	For-mula K_e (see page 92)
Expenses for the purchase of foodstuffs (total)	Empiric	(1)
	Adjusted	(2)
Expenses for the purchase of:		
Flour products (total)	Empiric	(1)
	Adjusted	(2)
Bred	Empiric	(1)
	Adjusted	(2)
Rice	Empiric	(1)
	Ajusted	(2)
Fats (total)	Empiric	(1)
	Adjusted	(2)
Edible oil	Empiric	(1)
	Ajusted	(2)
Milk and dairy products (total)	Empiric	(1)
	Adjusted	(2)
Fresh milk	Empiric	(1)
	Adjusted	(2)
Sugar	Empiric	(1)
	Adjusted	(2)

* The groups of families are disposed in the increasing order of their mean income, so

The following aspects result from the data shown:

— the elasticity coefficients obtained on basis of the adjusted data present decreasing or increasing trends from one income group to another;

— in the case of foodstuffs some regularities stand out, corresponding approximately to the following three groups:

— products whose consumption is influenced to a lesser degree by income changes and whose demand is estimated as being generally satisfied;

— products which enter frequently in the consumption of the population (bread, rice, flour pastes, sugar, oil, milk, etc.) and for which only small proportions of the income increase are allotted to the increase in expenses. On the scale of income groups the weight of expenses from the total income presents, for these products, a downward trend. The elasticity coefficients are lower than the unit, they decrease as we reach higher income groups, tending towards zero;

— products that are more valuable from the nutrition standpoint and whose consumption is more strongly influenced by changes occurring in the income (poultry, meat, pork, cold meats, cheese, fruit, etc.). The weight of expenses from total income, for these products, increases for the first income groups, remains approximately constant for the middle groups and decreases when reaching the upper income groups. The elasticity coefficients are higher than the unit, with a downward trend when reaching the high income groups.

Among the mathematical functions used in the case of foodstuffs, good results were obtained in the case of an important number of products, by using. L. Törnqvist's formula (I). This formula proves to be adequate in

purchase of foodstuffs as a function of income

Group of mean monthly income per person*								
I	II	III	IV	V	VI	VII	VIII	IX
—	0.722	0.666	0.706	0-644	0.616	0.600	0.577	0.539
0.830	0.777	0.738	0.702	0.667	0.637	0.610	0.578	0.526
—	0.196	0.183	0.184	0.170	0.116	0.117	0.127	0.105
0.330	0.260	0.221	0.191	0.168	0.150	0.136	0.121	0.100
—	0.304	0.215	0.205	0.229	0.144	0.148	0.171	0.142
0.398	0.320	0.276	0.241	0.213	0.192	0.174	0.164	0.130
—	0.330	0.245	0.270	0.230	0.149	0.182	0.199	0.119
0.405	0.326	0.281	0.246	0.219	0.195	0.178	0.160	0.134
—	0.570	0.446	0.587	0.419	0.447	0.418	0.322	0.254
0.647	0.567	0.514	0.469	0.429	0.397	0.370	0.339	0.294
—	0.409	0.392	0.375	0.274	0.253	0.296	0.255	0.164
0.512	0.427	0.377	0.335	0.300	0.273	0.251	0.227	0.192
—	0.704	0.607	0.601	0.543	0.554	0.469	0.456	0.371
0.741	0.671	0.622	0.579	0.539	0.506	0.477	0.444	0.393
—	0.541	0.404	0.398	0.363	0.367	0.253	0.241	0.154
0.510	0.426	0.376	0.334	0.299	0.272	0.250	0.226	0.191
—	0.637	0.618	0.600	0.460	0.480	0.450	0.353	0.302
0.681	0.603	0.552	0.506	0.466	0.433	0.405	0.374	0.326

that group IX represents the families with the highest income.

	Kind of data used	Formula K_e (see page 92)
Vegetables (total)	Empiric	(1)
	Adjusted	(2)
Fruit (total)	Empiric	(1)
	Adjusted	(3)
	Adjusted according to a linear function	(4)
Apples and pears	Empiric	(1)
	Adjusted	(3)
	Adjusted according to a linear function	(4)
Meat (total)	Empiric	(1)
	Adjusted according to a parabolic function	(5)
Poultry meat	Empiric	(1)
	Adjusted according to a parabolic function	(5)
Pork	Empiric	(1)
	Adjusted according to a parabolic function	(5)
Cold meats	Empiric	(1)
	Adjusted according to a linear function	(4)

Formula of the elasticity coefficient

Empirical data: $$K_e = \frac{dy}{y} : \frac{dx}{x} \qquad (1)$$

T. (I) Adjusted data: $$K_e = \frac{x}{y_c} \times \frac{ab}{(x+b)^2} \qquad (2)$$

computing the elasticity coefficients for a wide range of foodstuffs on whose study, moreover, E. Engel formulated his law on the interdependence of expenses and incomes.

The linear function used to study the elasticity of expenses for foodstuffs has led to the conclusion that its appliability is limited. As it also results from the above, for a series of products such as: bread, rice, fats, sugar, milk,etc. the expenses are inelastic along the scale of income groups (the weight of expenses for each of these products along the scale of income groups decreases with respect to the total expenses), however, the results obtained by adjustment about the straight line show a reverse situation (high elasticity).

There also resulted from the computations made, that the semi-logarithmic function is of restricted applicability. The results obtained by using this function could be accepted for: bread, rice, edible oil. For the remaining products the elasticity coefficients present values that are high in comparison with those resulting from other methods. Since parameter b of the equation

Group of mean monthly income per person								
I	II	III	IV	V	VI	VII	VIII	IX
—	0.621	0.544	0.515	0.459	0.391	0.388	0.358	0.270
0.646	0.565	0.512	0.467	0.427	0.395	0.368	0.338	0.292
—	1.689	1.596	1.571	1.471	1.338	1.351	1.278	1.243
0.898	0.922	0.932	0.939	0.943	0.946	0.948	0.949	0.949
0.990	0.994	0.994	0.996	0.996	0.996	0.997	0.998	0.998
—	1.708	1.604	1.563	1.392	1.326	1.271	1.120	1.051
1.581	1.119	0.946	0.830	0.746	0.682	0.631	0.578	0.501
0.755	0.812	0.841	0.865	0.881	0.896	0.906	0.916	0.931
—	1.445	1.304	1.354	1.247	1.164	1.030	1.013	0.822
1.528	1.208	1.056	0.934	0.821	0.717	0.611	0.467	0.135
—	2.442	2.070	2.505	2.628	2.177	1.945	1.729	1.426
4.227	1.990	1.498	1.201	0.874	0.781	0.596	0.347	0.263
—	2.282	1.947	2.112	1.989	1.838	1.507	1.431	1.210
1.908	1.379	1.169	1.014	0.879	0.784	0.641	0.483	0.121
—	1.345	1.196	1.236	1.345	1.467	1.227	1.393	1.266
1.297	1.196	1.152	1.123	1.103	1.089	1.079	1.068	1.054

T. (II) Adjusted data $$K_e = \frac{x}{y_c} \times \frac{a(b + c)}{(x + b)^2} \qquad (3)$$

Data adjusted according to a linear function $$K_e = \frac{x}{y_c} \times b \qquad (4)$$

Data adjusted according to a parabolic function $$K_e = \frac{x}{y_c} (b + 2\,cx) \qquad (5)$$

usually acquires large values there results an exaggerately high elasticity coefficient for the lower groups. As it is also shown in other studies, the elasticity decrease for foodstuffs is weaker than evidenced by the semi-logarithmic adjustment.*

As it results from the diagram hereafter, L. Törnqvist's formula (II) could only be accepted for apples and pears. The same conclusion also results from the examination of the trend of elasticity according to the kinds of data.

The curve of the empiric data and of the adjusted data for apples and pears is shown in Fig. 1.

* The logarithmic function cannot be applied to foodstuffs since the elasticity coefficients would, in this case, be constant along the scale of income groups, which is not normal ($K_e = b$). The exponential function has not been used since, for foodstuffs, the variation of expenses from one income group to another is not great, while this function is applied precisely in the cases where the expenses present larger variations along the scale of income groups.

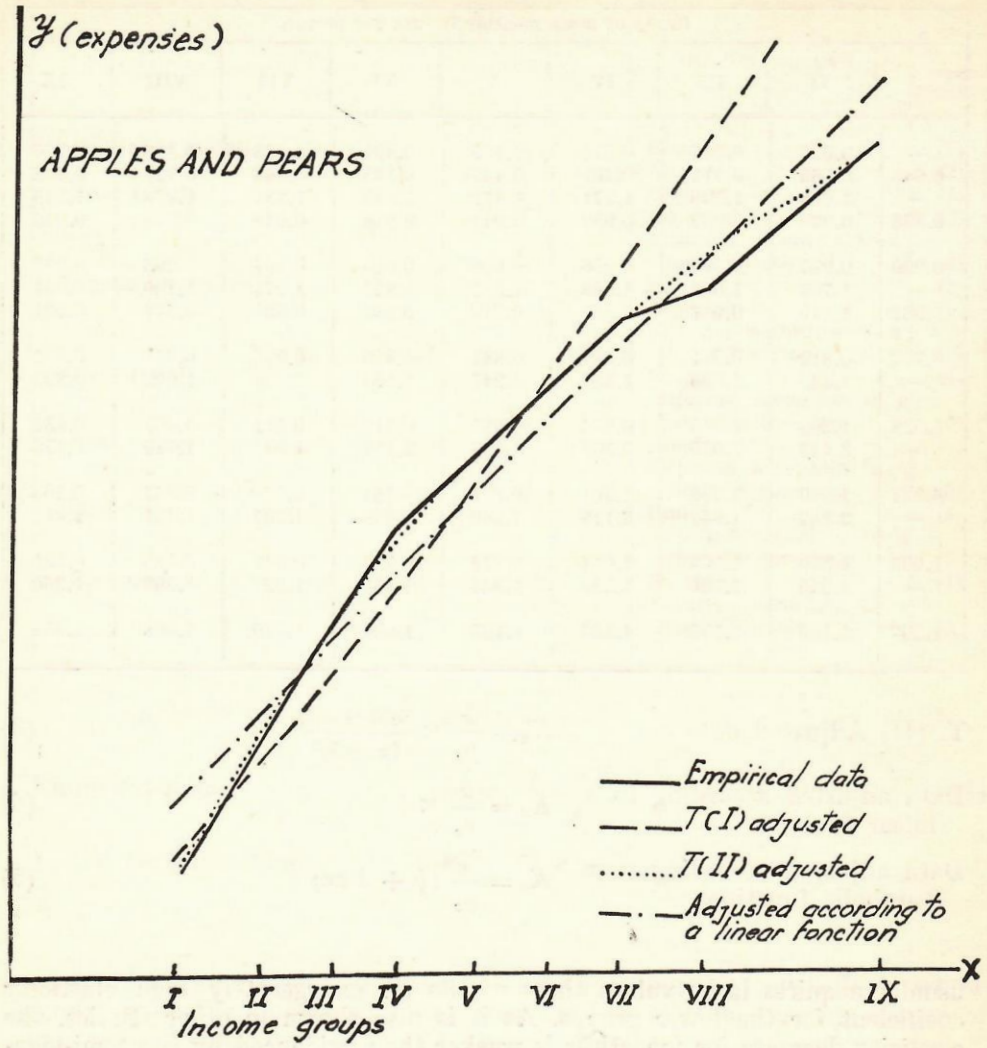


Fig. 1

The elasticity coefficients for fruit, determined on basis of L. Tornqvist's formula, are very close to the unit just like the ones actually resulting by adjustment about a straight line, with the only difference that the latter are closer to the unit and constant for some income groups.

The graphic representation of expenses according to adjusted data and empirical data for (total) fruit and cold meats, shows that the adjustment according to a linear function is acceptable. Likewise, for (total) fruit L. Törnqvist's formula (II) also adjusts the data satisfactorily starting from the II-nd income group and higher.

The curves of empirical and adjusted data for these products are as follows (Fig. 2 and 3):

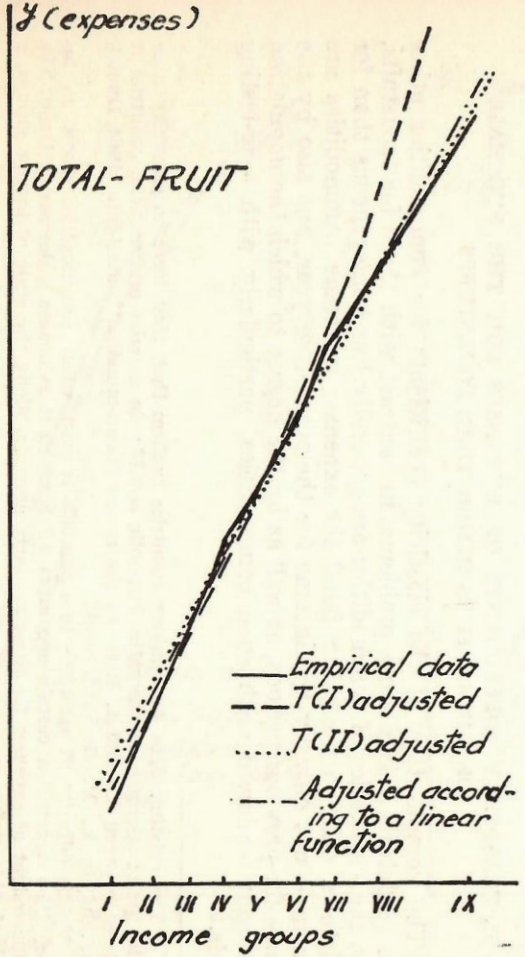

y (expenses)
TOTAL- FRUIT
Empirical data
T(I) adjusted
T(II) adjusted
Adjusted according to a linear function
I II III IV V VI VII VIII IX
Income groups

Fig. 2

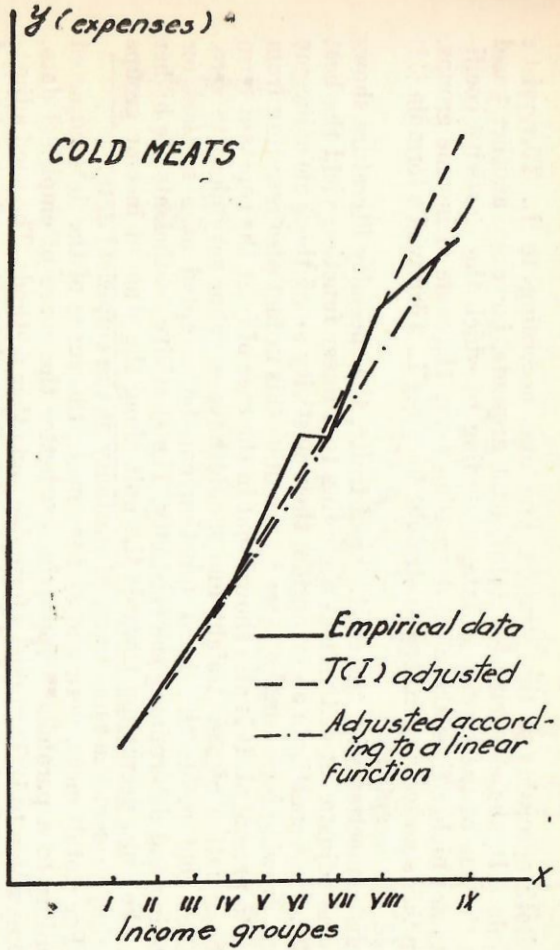

y (expenses)
COLD MEATS
Empirical data
T(I) adjusted
Adjusted according to a linear function
X
I II III IV V VI VII VIII IX
Income groupes

Fig. 3

Both according to the straight line and according to L. Törnqvist's formula (II), the elasticity for (total) fruit presents, however, another trend than the one of the empirical data, according to which the elasticity coefficients are higher than the unit, decreasing along the scale of income groups. From this standpoint neither the straight line, nor L. Törnqvist's formula (II) should be accepted.

In connection with the elasticity of fruit*, the specialty literature shows that the adjustment of the data according to a linear function yield the best results. Fruit constitute an exception, their elasticity along the scale of income groups drawing increasingly closer to the unit, this in fact also resulting from the data presented. It is also shown that in the case of fruit the constant term in the equation of the straight line should have a value very close to zero.

For cold meats the linear function can be accepted as a method for adjusting and determining the elasticity. The elasticity coefficients are higher than the unit, decreasing towards the unit along the scale of income groups and they present the same trend of elasticity as the empirical data.

For (total) meat, pork and poultry meat, the curve of the data adjusted according to a parabola** approximates better the curve of empirical data, in comparison to L. Törnqvist's formulae and other methods. The good adjustment of the data is confirmed both graphically and by the value and trend of the elasticity coefficients along the scale of income groups, in comparison with the elasticity according to empirical data.

Schematically, the curve of empirical data and of adjusted data for (total) meat, pork and poultry meat, shows as in Fig. 4, 5, 6.

Although the trend of the elasticity coefficients according to the adjusted data follows the trend of the empirical data along the scale of income groups, the parabolic function strongly influence the elasticity for the first few income groups, and it leads to a weak elasticity for the upper income groups. The elasticity increase or decrease for the highest and lowest income groups is actually weaker than is shown by the parabolic function.

II. ELASTICITY COEFFICIENTS OF EXPENSES FOR THE PURCHASE OF COMMODITIES OTHER THAN FOODSTUFFS

The computation of the elasticity coefficients for commodities other than foodstuffs raises more problems, in contrast with those for foodstuffs, since the categories of commodities are generally less homogeneous than for foodstuffs, and on the other hand the expenses for some commodities are influenced not only by the income for the respective year, but also by the income for previous periods, as well as by the degree to which the population is already endowed with such commodities, particularly with long-lasting goods.

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* The existing data for various countries confirm that fruit have an elasticity close to the unit: German Democratic Republic = 0.73; in a series of European countries it oscillates between 0.6 and 1. Refer to the review "Consommation", avril-juin 1964, Dunod, Paris, and to L. M. Goreux's work.

** The adjustment according to a parabola is used when consumption shows an increase, then reaches a certain maximum, after which it evidences a decreasing trend. The specific weight of expenses for these products presents, along the scale of income groups, a trend of this nature.

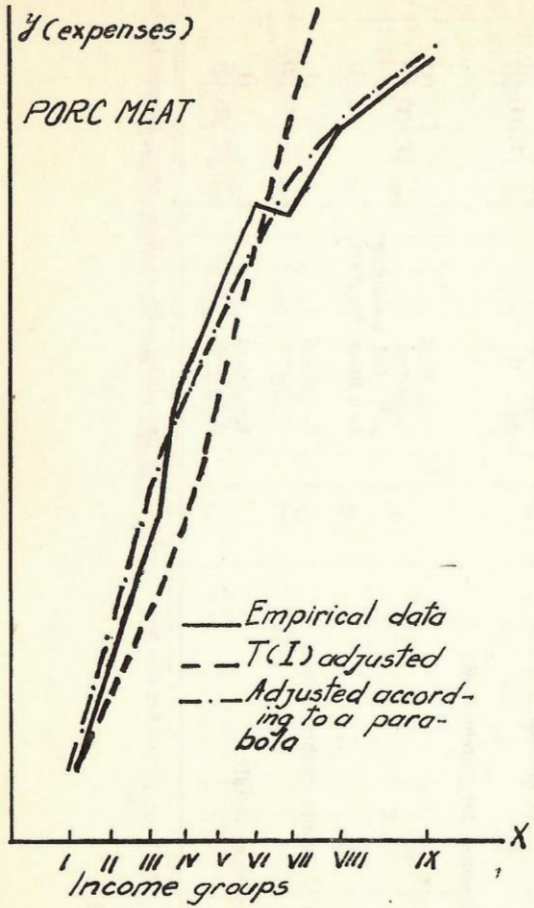


Fig. 4

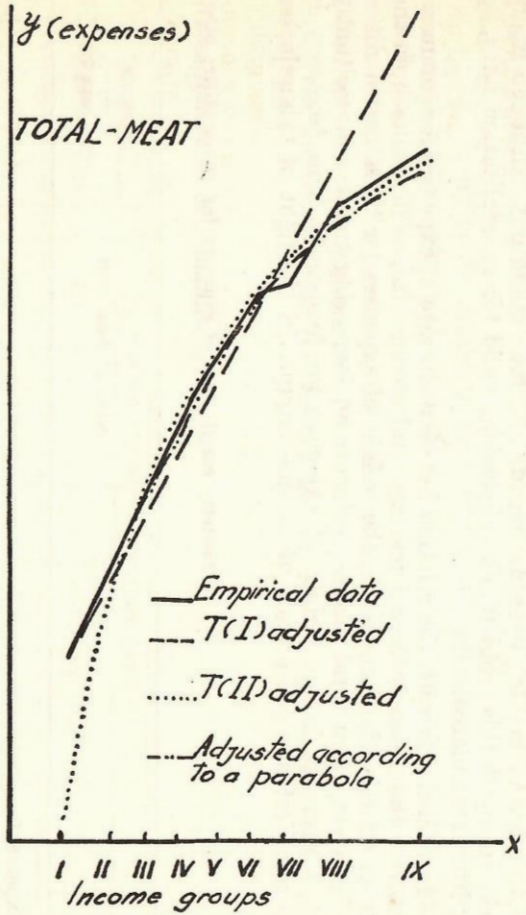


Fig. 5

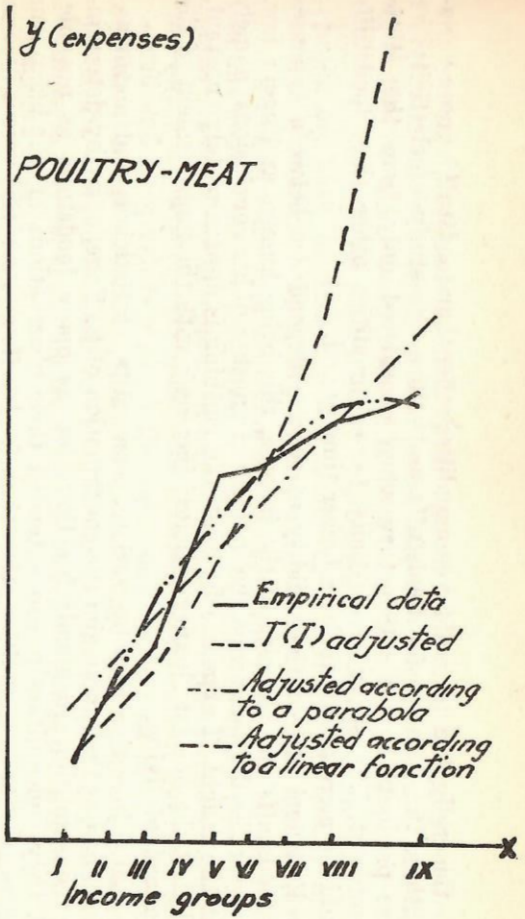


Fig. 6

Generally, the demand for commodities other than foodstuff is more variable than the demand for foodstuffs and, thus, the study of elasticity for these products is not satisfactory when considered only from the static standpoint. The study of elasticity for commodities other than foodstuffs would be more adequate for a longer time period.

E. Engel proved the validity of reverse dependence between expenses for foodstuffs and the size of the income, this being known as Engel's law. However, he did not prove the validity of another of his conclusions, namely that the weight of expenses for rent and clothing is approximately constant, and that the one for "luxury articles" increases with the improvement of the standard of living.

Later investigations by various economists, statisticians and mathematicians had as their main aim the confirmation of E. Engel's law by determining consumption (demand) functions for as many foodstuffs as possible, but they concomitantly also extended their observations on the behaviour of expenses for commodities other than foodstuffs and services versus income size, in view of making eventual trends and laws stand out. Although many works approach this problem, no conclusions valid for generalization have, so far, been formulated.

In connection with the relation between the size of expenses for commodities other than foodstuffs and services, and income size, as it results from the data of selective investigations, the weight of expenses for these commodities and services, from total income, presents an increasing trend, concomitantly with a decrease in the weight of expenses for foodstuffs. This tendency is also manifest along the scale of income groups. In the light of this relation

Elasticity coefficients of expenses for commodities other

	Kind of data used	Formula K_e (see page 92)
Expenses for purchase of commodities other foodstuffs (total)	Empirical	(1)
	Ajusted	T. (I) (2)
Expenses for purchase of:		
Cloth (total)	Empirical	(1)
	Adjusted	T. (I) (2)
	Adjusted according to a linear function	(4)
Ready-made clothing (total)	Empirical	(1)
	Adjusted	T. (I) (2)
Footwear (total)	Empirical	(1)
	Adjusted	T. (I) (2)

* The groups of families are disposed in the increasing order of their mean income, so that

an attempt was made to find, in the case of some more important categories of commodities other than foodstuffs, also the corresponding consumption function on whose basis it would be possible to study elasticity.

In the table on page 102—103 the elasticity coefficients are shown, computed on basis of data adjusted according to various methods, for some expenses connected with the purchase of commodities other than foodstuffs.

The schematic representation of the curve shows that for cloth, both the adjustment according to a straight line and according to L. Törnqvist's formula (I) could be accepted. However, the latter adjusts the empirical data better, and on the other hand the elasticity coefficients present the same trend and have values close to the ones computed on basis of the empirical data. Although the linear function also adjusts well, it cannot be accepted since the computed elasticity coefficients do not present, along the scale of income groups, the same trend as the elasticity determined on basis of the empirical data.

The curve of empirical data and that of adjusted data for cloth are shown in Fig. 7.

For ready-made clothing and footwear, adjustment accordding to L. Tornqvist's formula (I) leads to better resuts than formula (II), particularly for the first three income groups. For the remaining income groups, according to both formulae, the value of the coefficients is sufficiently close and presents a common trend.

On the whole, however, we can accept adjustment according to formula (I), since:

than foodstuffs, as a function of income

Group of mean monthly income per person *								
I	II	III	IV	V	VI	VII	VIII	IX
—	1.050	1.270	1.147	1.211	1.277	1.232	1.296	1.308
1.069	1.100	1.127	1.182	1.188	1.221	1.255	1.302	1.402
—	2.074	1.925	1.739	1.879	2.113	2.256	2.325	2.061
1.196	1.298	1.396	1.515	1.664	1.837	2.049	2.403	3.602
2.558	1.767	1.541	1.414	1.333	1.280	1.241	1.160	1.160
—	0.748	0.884	0.587	0.594	0.923	0.794	0.710	0.695
0.900	0.866	0.839	0.813	0.787	0.764	0.743	0.717	0.671
—	1.057	0.933	0.924	0.848	0.923	0.856	0.886	0.759
0.947	0.926	0.911	0.895	0.879	0.864	0.850	0.832	0.800

group IX represents the families with the highest income.

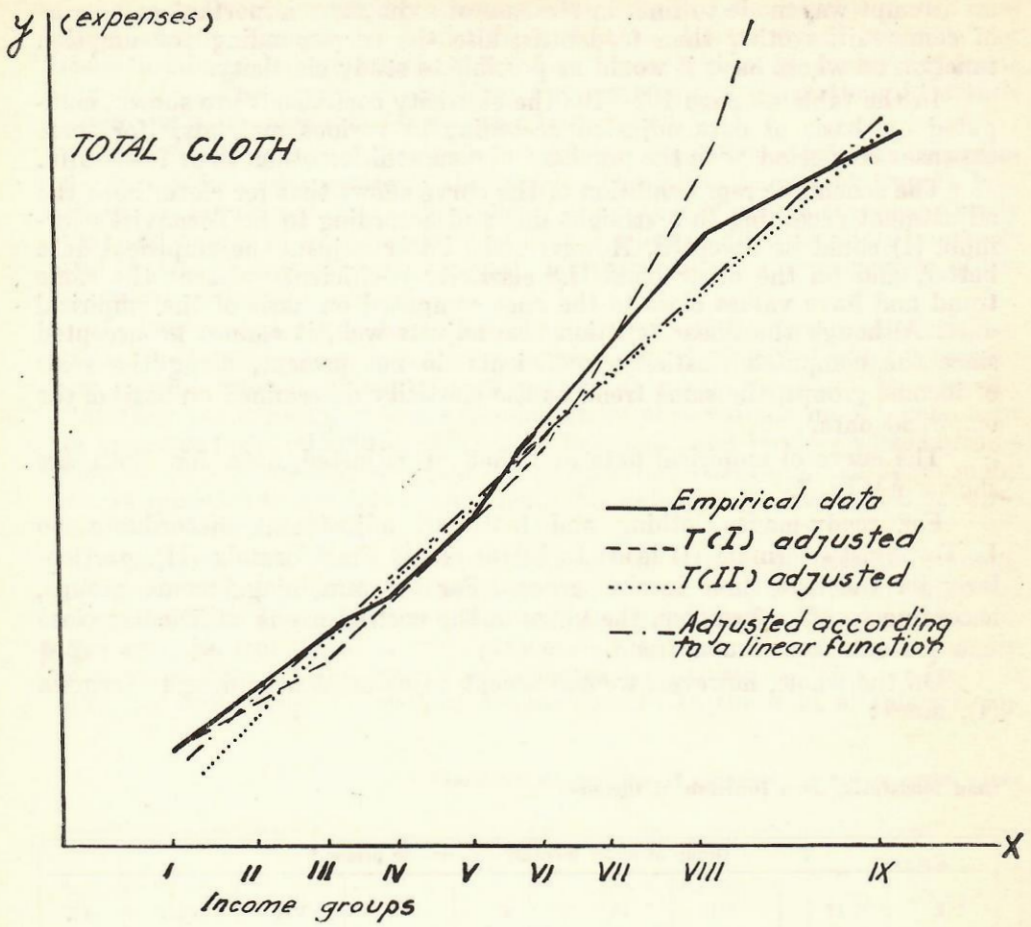


Fig. 7

— formula (I) is applied in the conditions set by L. Törnqvist, respectively, the specific weight of expenses for these products decreases along the scale of income groups:

— the trend of expense elasticity according to the adjusted data corresponds to the one according to the empirical data;

— the value of the elasticity coefficients computed according to formula (I) is closer to their value according to the empirical data.

Schematically, the curve of expenses for these commodities is shown in Fig. 8 and 9.

The exponential function, as an adjustment method, has not yielded good results for commodities other than foodstuffs, although it is recommended, taking into account the larger variation of the expenses. It was proved that it overestimates the expenses along the scale of income groups and, particularly, the higher and lower groups.

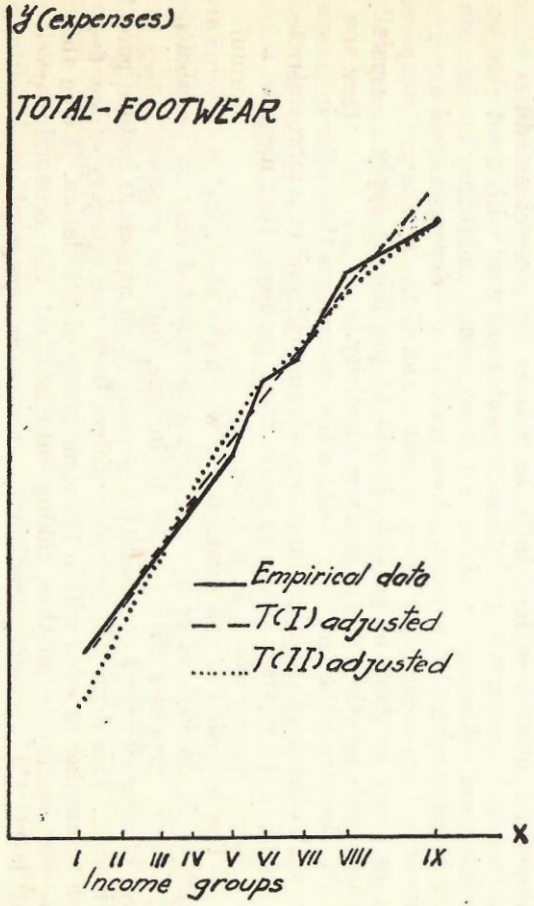

y (expenses)
TOTAL - FOOTWEAR
Empirical data
T(I) adjusted
T(II) adjusted
X
I II III IV V VI VII VIII IX
Income groups

Fig. 8

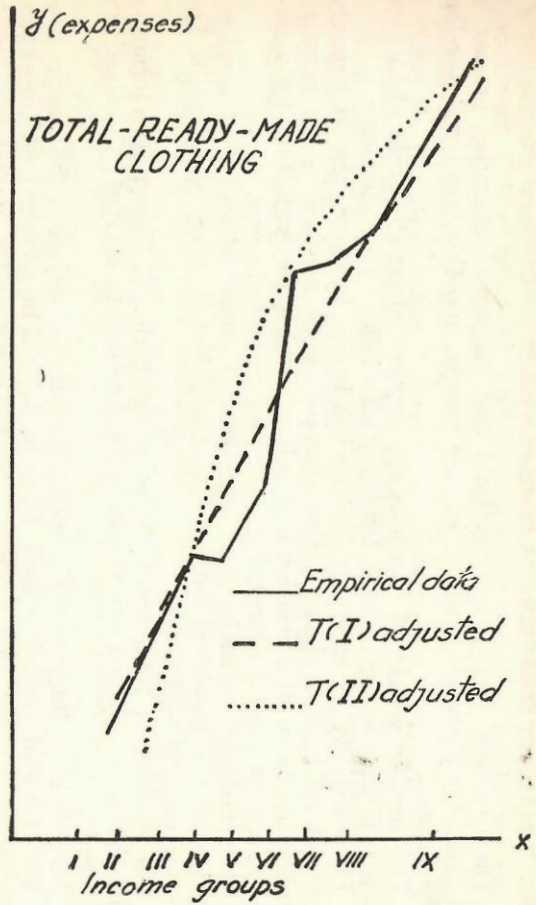

y (expenses)
TOTAL - READY - MADE CLOTHING
Empirical data
T(I) adjusted
T(II) adjusted
X
I II III IV V VI VII VIII IX
Income groups

Fig. 9

III. ELASTICITY COEFFICIENTS OF EXPENSES FOR PAYMENT OF SERVICE

The expenses for services, in total and for some categories, present a high elasticity along the scale of income groups, and this trend will develop in the future.

The specific weight of expenses for services along the scale of income groups manifests an increase, a tendency also followed by elasticity.

The elasticity of expenses for services is not, however, always strongly influenced by income changes. For example, some services in the network of public health or of education and culture, which are gratuitous for wage-earners, are, to a small extent, influenced by income changes.

The elasticity coefficients of expenses for some categories of services, according to the empirical data and to the adjusted ones, show as in the table on page 108—109.

For services, in total and by expense categories, L. Törnqvist's formulae adjust the data well and can therefore be accepted to compute the elasticity coefficients.

According to the manner in which the curve of adjusted data approximates the curve of empirical data, formula (II) can be accepted for the expenses for (total) services, and formula (I) for the expenses for making and mending clothing and footwear.

For these categories of expenses elasticity follows the trend of the elasticity of the empirical data.

In the expenses for cinema, theatre and other cultural-sporting expenses, L. Törnqvist's formula (II) gives better results than formula (I). The curve of data adjusted according to formula (II) approximates better the curve of empirical data and the trend shown by the elasticity of these expenses is the same as that of the empirical data.

As it results from the computations presented by groups of expenses, L. Törnqvist's formulae and particularly the first one, can be used with results considered as being sufficiently good in characterizing elasticity for an important number of foodstuffs, as well as for some commodities other than foodstuffs and services. If these formulae are used with great care and discrimination, taking the conditions set into account, and if they are harmoniously blended with the economical analysis of the dependence of expenses on income, from commodity to commodity, and if the results are compared with as many methods as possible, it will be possible to apply successfully L. Törnqvist's functions to the study of elasticity.

We also stress the fact that, besides the advantages they offer in comparison with other mathematical functions whose applicability is more restricted, the use of L. Törnqvist's formulae raises some problems that must be taken into account, among which we mention:

— The inclusion of the products for which the elasticity coefficients are computed, in the various groups set cannot be effected only on basis of the behaviour of expenses with respect to income, but the appreciation of the degree of necessity for a commodity is dependent on numerous factors besides the income. This degree of necessity differs from one category of the population to another or even within the same group of population. It also differs from one country to another, taking into account the economic development level, the customs, traditions, preferences, geographical conditions,

etc. Excepting the commodities of great consumption frequency which are obviously of first necessity within certain physiological limits, for any category of the population, for products more valuable from the nutrition standpoint, for clothing, footwear, long-lasting commodities of a relatively high value, the inclusion in one of the aforesaid groups is generally difficult. A series of economic (besides income) and demographic factors intervene, which differentiate these necessities considerably (degree of endowment with commodities, living conditions, family size, structure of family by sex and age groups, etc.). Such a grouping could only be carried out at the level of homogeneous collectivities of population;

— On the other hand, the practical application of formulae (II) and (III) is difficult both from the standpoint of the volume of computations required and due to the fact that formula (II) is not applicable to income groups of a lower level (as it also resulted from the computations presented for ready-made clothing and footwear in the case of the first three income groups). Under these conditions we should use, for some expense groups, two procedures for adjusting the data and namely for groups with a lower income level one method, and for groups with a higher income level another method. These divisions of the income groups are not the most fitting in the analysis for the trends of elasticity;

— In the case of formula (II) some difficulties are met due to the fact that it is not possible to determine the value of the parameters a b, c by the least squares method, but by the approximation method through chosen points. According to this method the choice of the best variant requires several computation tests with a higher number of combinations of the chosen points, but since the number of income groups is small (9—10 income groups are usually established), a larger number of points cannot be included in the computation and it is not possible to effect the remaining combinations. On the other hand, the great volume of computations required by these numerous variants renders the use of this method less effective.

In the work previously mentioned, H. Wold shows that in case of greater variations in expenses from one income group to another, L. Törnqvist's formulae are often used with good results; however, these functions cannot offer a good adjustment for more than two phases of a curve, which, due to irregularities, present several phases.

The following have stood out in connection with the use of adjustment methods of the empirical data and the computation of the elasticity coefficients:

— in the first stage — that of the actual adjustment of the data — most of the mathematical functions can be used with good or almost good results;

— in the stage where the elasticity coefficients are computed on basis of the adjusted data, there are cases when a certain mathematical function, even though it adjusts the data well, does not lead to good results of the coefficients, the latter presenting, along the scale of income groups, trends which cannot actually occur (e.g. adjustment according to a straight line for some foodstuffs, etc.).

The methods used in adjusting the data and, respectively, in determining the elasticity coefficients cannot be valid generally. Whether it is for food-

stuffs, commodities other than foodstuffs, or services, that mathematical function must be used which, from one category of expenses to another, adjusts better the data on whose basis the computed elasticity coefficients present a trend close to the elasticity of the empirical data, or which is normal from the economic point of view. The choice of the type of function that is most adequate to adjust the data requires an accurate knowledge of the various types of distributions, both empirical and theoretical.

In practice, it is difficult to associate the empirical distribution with the corresponding algebraic expression since the curve of the adjusted data does not pass through all the points of the observed data. It is therefore necessary to test several kinds of mathematical functions and choose the one which approximates the empirical data better.

It is also possible for the deviation between the empirical and the adjusted data to be rather small but, nevertheless, the function does not correspond to the actual trend of the phenomenon. Therefore, when adopting one method of adjustment of another, it is not sufficient to take into account the trend of the adjusted and empirical curves; one should also take into account the values and trends presented by the elasticity coefficients which may sometimes be opposite to the actual ones.

In practice there may also be situations where no adjustment method can be applied due to some structural changes having occurred in the development of a phenomenon, and it is essential to know this fact thoroughly in order not to effect mechanical adjustments.

The computations and considerations presented in this work, in connection with the establishment of the mathematical functions which characterize the dependence between expenses and income by kinds of commodities and groups of expenses, constitute the results of investigations undertaken in the last few years in the Central Department of Statistics, and of collaboration with a series of central institutions interested in the subject, such as: the Ministry of Home Trade, Centre of Mathematical Statistics of the Academy

Elasticity coefficients of expenses

	Kind of data used	Formula K_e (see page 92)
Expenses for services (total)	Empirical Ajusted	(1) .T. (II) (3)
Expenses for making and mending clothing and footwear	Empirical Adjusted	(1) .T. (I) (2)
Expenses for cinema, theatre and other cultural-sporting expenses	Empirical Adjusted Adjusted	(1) .T. (II) (2) .T. (II) (3)

* The groups of families are disposed in the increasing order of their mean income, so that

of the Socialist Republic of Romania, State Committee of Planning and Committee for Prices.

To enable reaching more substantiated conclusions in connection with these methods as well as other aspects regarding the study of demand, it is necessary to intensify the investigations in view of further studying thoroughly L. Törnqvist's formulae, in view of applying them as accurately as possible as well as in view of using all the mathematical functions which express most accurately the correlation between expense and income or that of other factors.

For the future it is necessary for studies regarding elasticity to be made not only statically, but also dynamically, particularly for commodities other than foodstuffs whose elasticity is influenced by several factors and whose tendency of development can only stand out over a longer time period.

The study of consumption demand must not stop at the average elasticity per person but, in the future, it will need to broaden its possibilities in view of also studying the elasticity of expenses for commodities and services as a function of demographic factors such as : family size (family of 2 persons, 3 persons, 4 persons, adults and children, etc.), number of children in a family, sex, age, etc., which bear a direct influence on the size and structure of demand. It is also necessary, in the case of some foodstuffs such as fruit and vegetables, for the studies regarding elasticity to be made, taking into account the seasonal factor in their production and consumption. The necessity also stands out, for all groups of expenses, that investigations should also be undertaken according to regional characteristics.

On the other hand, besides the elasticity of expenses as a function of income and other factors, investigations must be extended with regard to the elasticity of food consumption per total and by categories of the population.

Considering the new tasks set in connection with the study of consumption demand, a co-ordination in the efforts of the institutions interested in

for services as a function of income

Group of mean monthly income per person *								
I	II	III	IV	V	VI	VII	VIII	IX
—	1.665	1.591	1.633	1.741	1.677	1.775	1.781	1.694
1.150	1.225	1.293	1.374	1.469	1.573	1.693	1.875	2.368
—	2.057	2.303	2.228	2.189	2.791	2.722	2.978	2.743
1.250	1.389	1.526	1.705	1.942	2.243	2.640	3.447	8.219
—	1.325	1.962	1.847	1.933	1.875	2.048	2.041	2.330
1.192	1.296	1.393	1.512	1.661	1.831	2.039	2.389	3.564
0.903	1.003	1.078	1.156	1.236	1.318	1.405	1.527	1.820

group IX represents the families with the hig est income.

the subject is imperative in view of further developing and improving these methods and in view of testing and generalizing new methods of obtaining, processing and analyzing the data and of effectively being informed about the changes occurring in the demands of the population.

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CALCUL DES COEFFICIENTS D'ELASTICITE EN SE BASANT SUR LES DONNEES AJUSTEES A L'AIDE DE DIVERSES FONCTIONS MATHEMATIQUES

Cette étude présente les résultats obtenus dans le calcul des coefficients d'élasticité des dépenses effectuées pour marchandises et services en fonction du niveau des revenus, les données ajustées des investigations sélectives des budjets de famille étant utilisées dans ce but.

On indique les avantages résultant de l'utilisation des ajustements analytiques en les comparant aux données empiriques observées et on souligne les fonctions mathématiques d'une grande applicabilité.

On insiste tout spécialement sur les formules mathématiques de L. Tornqvist et sur la possibilité de les employer pour l'ajustement analytique des données ainsi que pour le calcul des coefficients d'élasticité.

On met ensuite en relief certains des résultats obtenus par l'emploi de ces fonctions dans l'étude de l'élasticité des dépenses pour marchandises alimentaires.

L'étude comprend également certaines considérations sur le calcul des coefficients d'élasticité des dépenses pour marchandises non-alimentaires et services, par rapport au revenu, en se basant sur les mêmes fonctions mathématiques.

On montre, finalement, la nécessité d'intensifier les recherches en vue de continuer à développer et perfectionner l'emploi de diverses fonctions mathématiques en vue d'étudier également l'élasticité en conjonction avec d'autres facteurs économiques et démographiques qui permettraient d'aborder la relation demande-revenu d'une façon plus complexe.

РАСЧЕТ КОЭФИЦИЕНТА ЭЛАСТИЧНОСТИ НА ОСНОВЕ ДАННЫХ, ПОДОБРАННЫХ ПРИ ПОМОЩИ РАЗЛИЧНЫХ МАТЕМАТИЧЕСКИХ ФУНКЦИЙ

Автор стати представляет результаты, полученные при расчете коэфициентов эластичности затрат на товары и обслуживание, в зависимости от уровня расходов, используя данные, подобранные при выборочном исследовании семейных бюджетов.

В работе показаны преимущества использования аналитических подборок по сравнению с эмпирическими данными и выявлены широко применяемые математические функции.

Особое внимание уделено математическим формулам Л. Тернгвиста и возможности их применения при аналитической подборке данных и при расчете коэффициентов эластичности.

В продолжение, подчеркиваются некоторые результаты, полученные путем применения этих функций при изучении эластичности затрат на продовольственные товары.

Статья содержит, также, отдельные положения, связанные с расчетом коэффициентов эластичности затрат на непродовольственные товары и услуги в зависимости от дохода, на основе этих же математических функций.

В конце работы показана необходимость интенсификации исследований для дальнейшего развития и усовершенствования применения различных математических функций к изучению эластичности и в соотношении к другим экономическим и демографическим факторам, которые позволили бы более полное исследование отношения потребностей и дохода.