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An Updating Model of Direct Consumption Coefficients

DETERMINING THE MODEL AND THE ALGORITHM

The model presented outlines a way of updating the coefficients of direct consumptions, by taking into account some simplifying hypotheses. Let us consider the following problem:

Let us determine a solution of the equation system:

$$\begin{cases} \sum_i a_{ij} \cdot X_j = C_j, & j = 1, 2, ..., n \\ \sum_j a_{ij} \cdot X_j = D_i, & i = 1, 2, ..., n \end{cases} \tag{1}$$

where:

X_j represents the component j of vector X of the global product;

C_j — the component j of vector C of the intermediate consumption;

D_i — the component i of vector D of the intermediate product;

(a_{ij}) — the matrix A of coefficients of direct consumptions.

Vectors X, D and C are considered as given (for the year of planning $t+\Delta t$), and the elements of matrix A are considered to be the unknown values of the problem.

We study the case when system (1) admits one or several solutions. In this case, the selection of a solution out of the set of possible ones is made by taking into consideration certain calculus hypotheses.

Several such hypotheses are known in the literature of speciality, all based on economic considerations. Hypotheses linked with the dependence of the

solution, with the particularities of the technological process and its evolution in time, labour productivity, technical progress, the evolution of energetic resources, etc. are thus enunciated.

We do not intend to enumerate all working hypotheses which are at the basis of the determination of a solution to system(1), however, we want to express our conviction that this method can be an acceptible way for obtaining a solution of the updated technological coefficients which should satisfactorily reflect the evolution of their transformation.

We also consider that the taking into account of ever more simplifying hypotheses, well justified economically, leads to the obtaining of ever more plausible values. The increasing number of such hypotheses taken into consideration, generally, leads to the growing plausibility of the solution obtained from the point of view of economic significance.

THE ECONOMIC-MATHEMATICAL MODEL

The above considerations which have been taken as working criteria have led us to the idea of making efforts for improving methods already existing in literature. In the following, we shall deal with the method of T. I. Matuszewski and others [1], based on linear programming and with an attempt at improving it.

In order to be able to directly refer to the model presented in [1] and to present the form we propose, we give this model in the following. Let us determine a set of values a^*_{ij}, $(i, j = 1, 2, ..., n)$ so that:

$$\sum_i \sum_j \left| \frac{a^*_{ij}}{a_{ij}} - 1 \right| = min, \tag{2}$$

liable to constraints

$$\sum_i a^*_{ij} X^*_j = X^*_{\cdot j} \tag{3}$$

$$\sum_j a^*_{ij} X^*_j = X^*_{i\cdot} \tag{4}$$

$$\frac{1}{2} \leqslant \frac{a^*_{ij}}{a_{ij}} \leqslant 2, \quad i, j = 1, 2, ..., n \tag{5}$$

where: a^*_{ij} represents the technological coefficients which are to be determined for the planned year;

a_{ij} — the given technological coefficients of the basic year;

X^*_j — the global product of branch j in the envisaged year;

$X^*_{\cdot j}$ — the intermediate consumption of branch j in the envisaged year;

$X^*_{i\cdot}$ — the intermediate product of branch i in the envisaged year.

Consequently, the unknown values of the problem are represented by coefficients a^*_i, the other data being known.

As noticed also by the authors of the model presented, constraints (5) have a rather arbitrary character. Anyhow, the considered limits are considered disputable and they might be taken into consideration after thorough

economic analyses. However, the variation of these limits gives the possibility of obtaining different solutions of the model presented which makes us consider that it is a convincing reason for keeping them in the model.

The objective function (2) presented in the model can be justified on the basis of economic hypotheses mainly linked with the general remark that the transformations of the coefficients of direct consumptions are generally sufficiently small in time (for a Δt sufficiently small) and that, generally, there are no jumps in their variation.

Generally, we consider that not only the coefficients of direct consumptions have this property but also the coefficients of the distribution of production h_{ij}. We consider that this new hypothesis is valid because any change in the coefficients of the distribution of the intermediate product of a branch is, generally, accompanied by changes in the coefficients of the distribution of the intermediate product of the other branches due to the interdependences existing between them. As a matter of fact, these interdependences are justified also by the economic significance of total consumptions.

In case that due to economic policies, certain limits to the intermediate consumptions are imposed, constraints (5) can be used for establishing the coresponding limits. In this way and with the aid of the ratios existing between the a^*_{ij}, technological coefficients and the coefficients of the distribution of the intermediate product h^*_{ij} are indirectly influenced by the variation limits of the latter.

Indeed, we have

$$h^*_{ij} = a^*_{ij} \frac{X^*_j}{X^*_i}, \quad i, j = 1, 2, \ldots, n. \tag{6}$$

As, from (5) we generally have

$$u_1 a_{ij} \leqslant a^*_{ij} \leqslant u_2 a_{ij}, \quad i, j = 1,2, \ldots, n \tag{7}$$

(u_1, u_2 represent the two limits of ratio (5), $u_1, u_2 \geqslant 0$) resulting from ratios (6) and (7) is

$$u_1 a_{ij} \frac{X^*_j}{X^*_i} \leqslant h^*_{ij} \leqslant u_2 a_{ij} \frac{X^*_j}{X^*_i}, \quad i, j = 1,2, \ldots, n \tag{8}$$

where u_1, u_2, X_i, X_j, a_{ij} are given and have the above shown significance.

Similarly, variation limits can be imposed for h^*_{ij} and indirectly determine the variation limits for a^*_{ij}. Indeed, if we suppose that

$$\bar{u}_1 \leqslant h^*_{ij} \leqslant \bar{u}_2 \tag{9}$$

from (8) we obtain

$$\begin{cases} \bar{u}_1 = u_1 a_{ij} \dfrac{X^*_j}{X^*_i} \\ \bar{u}_2 = u_2 a_{ij} \dfrac{X^*_j}{X^*_i} \end{cases} \tag{10}$$

or

$$\begin{cases} u_1 = \bar{u}_1 \dfrac{X_j^*}{a_{ij} X_j^*} \\ u_2 = \bar{u}_2 \dfrac{X_i^*}{a_{ij} X_j^*} \end{cases} \tag{11}$$

For this reason, we shall replace the objective function (2) by function

$$F(a_{ij}^*,\ h_{ij}^*) = \sum_i \sum_j \left| \frac{a_{ij}^*}{a_{ij}} - 1 \right| + \sum_i \sum_j \left| \frac{h_{ij}^*}{h_{ij}} - 1 \right| = min \tag{12}$$

where:

h_{ij}^* represents the coefficients of the distribution of the intermediate production for the envisaged year, being unknown;

h_{ij} — the coefficients of the distribution of the intermediate production given for the basic year.

The constraints remain the same in this stage of the presentation of the model.

To eliminate the nonlinearity of function (12), we shall use the same method used in [1]; for this reason and for an easier expression, we must take over certain ratios and notations.

Consequently, equation (12) can also be written:

$$F(a_{ij}^*) = \sum_i \sum_j \left| a_{ij}^* X_j^* - a_{ij} X_j^* \right| \frac{1}{a_{ij} X_j^*} + \sum_i \sum_j \left| a_{ij}^* X_j^* X_i - a_{ij} X_i^* X_j \right| \frac{1}{a_{ij} X_i^* X_j} \tag{13}$$

by using ratio $h_{ij} = \dfrac{a_{ij} X_j}{X_i}$. The following notations are made:

$$\begin{cases} X_{ij}^* = a_{ij}^* X_j^*; \\ X_{ij} = a_{ij} X_j^*; \end{cases} \tag{14}$$

$$\begin{cases} \bar{X}_{ij}^* = a_{ij}^* X_j^* X_i \\ \bar{X}_{ij} = a_{ij} X_i^* X_j \end{cases}$$

$$i,\ j = 1,\ 2,\ \ldots,\ n.$$

By using notations (14), function (13) can be written

$$F(X_{ij}^*, \bar{X}_{ij}^*) = \sum_i \sum_j \left| X_{ij}^* - X_{ij} \right| \frac{1}{X_{ij}} + \sum_i \sum_j \left| \bar{X}_{ij}^* - \bar{X}_{ij} \right| \frac{1}{\bar{X}_{ij}} \tag{15}$$

The above constraints can also be written

$$\sum_i X_{ij}^* = X_{.j}^*,\ j = 1,\ 2\ \ \ldots,\ n \tag{16}$$

$$\sum_j X_{ij}^* = X_{i.}^*.\quad i = 1,\ 2,\ \ldots,\ n \tag{17}$$

$$u_1 \leqslant \frac{X_{ij}^*}{X_{ij}} \leqslant u_2,\quad i,j = 1,\ 2,\ \ldots,\ n. \tag{18}$$

We make the following changes in variables

$$X^*_{ij} = X_{ij} + X^+_{ij} - X^-_{ij}; \tag{19}$$

$$\bar{X}^*_{ij} = \bar{X}_{ij} + \bar{X}^+_{ij} - \bar{X}^-_{ij}; \tag{20}$$

$$i, j = 1, 2, \ldots, n$$

where variables X^+_{ij}, X^-_{ij}, $\bar{X}^+_{ij}$, $\bar{X}^-_{ij}$, are positive. By using notations (19) and (20), constraints (16) and (17) become

$$\sum_i (X^+_{ij} - X^-_{ij}) = X^*_{\cdot j} - \sum_i X_{ij}, \tag{21}$$

$$j = 1, 2, \ldots, n$$

$$\sum_j (X^+_{ij} - X^-_{ij}) = X^*_{i\cdot} - \sum_j X_{ij}; \tag{22}$$

$$i = 1, 2, \ldots, n.$$

By using notations (14) we can outline also constraints:

$$\bar{X}^*_{ij} = X^*_{ij} X_i \tag{23}$$

$$i, j = 1, 2, \ldots, n$$

or

$$(X^+_{ij} - X^-_{ij}) X_i - (\bar{X}^+_{ij} - \bar{X}^-_{ij}) = \bar{X}_{ij} - X_i X_{ij} \tag{24}$$

$$i, j = 1, 2, \ldots, n$$

which reflects the dependence ratio between the two types of variables corresponding to variables a^*_{ij} and h^*_{ij}.

It is also noticed that ratio (24) observes the condition of proportionality between the elements of vectors X^+_{ij}, X^-_{ij} and respectively between $\bar{X}^+_{ij}$ and $\bar{X}^-_{ij}$. Consequently, vectors X^+_{ij} and X^-_{ij} cannot belong to the same basic solution, thus both $\bar{X}^+_{ij}$ and $\bar{X}^-_{ij}$ have the same property.

It results that for the same pair of indices (i, j) X^+_{ij} and X^-_{ij} respectively $\bar{X}^+_{ij}$ and $\bar{X}^-_{ij}$ cannot be different from zero in the same time. Under these conditions, constraints (18) can be written:

$$(u_1 - 1)\, X_{ij} \leqslant X^+_{ij} \leqslant (u_1 - 1) X_{ij} \tag{25}$$

$$(1 - u_2)\, X_{ij} \leqslant X^-_{ij} \leqslant (1 - u_1) X_{ij} \tag{26}$$

where

$$u_1 < u_2$$

$$i, j = 1, 2, \ldots, n$$

and the objective function (15) becomes:

$$F(X_{ij}^{+},\ X_{ij}^{-},\ \bar{X}_{ij}^{+},\ \bar{X}_{ij}^{-}) = \sum_i \sum_j (X_{ij}^{+} + X_{ij}^{-})\bar{C}_{ij} +$$

$$+ \sum_i \sum_j (\bar{X}_{ij}^{+} + \bar{X}_{ij}^{-})\bar{C}_{ij} \qquad (27)$$

where

$$\bar{C}_{ij} = \frac{1}{\bar{X}_{ij}},\ C_{ij} = \frac{1}{X_{ij}}.$$

The experiments of the model were made for the Hungarian balancesheets by using the statistical data published in [3].

Annexes 1—3 present part of the data obtained for which constraints (25), (26) were used only for the technological coefficients $a_{ij} = 0$ causing them to remain unchanged.

It can be noticed that for the technological cefficients with higher values, ratio a_{ij}^{*}/a_{ij} is close to 1 and for those with lower values this ratio has, generally, a greater deviation from 1.

REFERENCES

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TABLE 1

MATRIX OF DIRECT EXPENDITURE COEFFICIENTS-1970 ESTIMATE WITH THE AID OF THE 1968 STATISTICAL MATRIX

No.	Name Branch	1	2	3	4	5	6	7	8	6	10	11	12	13	14	15	16
1	Mining Industry	.0547	.1729	.0837	.0095	.0863	.1653	.0092	.0070	.0041	.0009	.0018	.0127	.0320	.0070	.0037	.0000
2	Electric Energy	.0389	.0994	.0281	.0135	.0265	.0338	.0146	.0058	.0148	.0036	.0016	.0365	.0162	.0127	.0038	.0000
3	Metallurgy	.0438	.0171	.3552	.1817	.0560	.0229	.0100	.0076	.0530	.0003	.0000	.0063	.0150	.0013	.0066	.0000
4	Machine-building industry	.0740	.0675	.0539	.2288	.0558	.0356	.0346	.0104	.1078	.0377	.0339	.1042	.1109	.0191	.0227	.0000
5	Construction materials	.0025	.0109	.0098	.0100	0.578	.0084	.0035	.0076	.1165	.0080	.0031	.0148	.0021	.0065	.0045	.0000
6	Chemistry	.0332	.0631	.0889	.0390	.0470	.1830	.0731	.0222	.0195	.0627	.0286	.0407	.0664	.0141	.0201	.0000
7	Light industry	.0207	.0076	.0145	.0291	.0356	.0381	.2977	.0219	.0544	.0062	.0133	.0211	.0164	.0512	.0669	.0000
8	Food industry	.0011	.0005	.0012	.0011	.0019	.0211	.0251	.1472	.0000	.0664	.0012	.0000	.0018	.0040	.0044	.0000
9	Engineering	.0055	.0138	.0131	.0035	.0163	.0150	.0035	.0048	.0025	.0036	.0025	0.938	.0094	.0222	.0319	.0000
10	Agriculture	.0036	.0000	.0000	.0001	.0003	.0192	.0449	.5142	.0007	.2510	.0101	.0055	.0000	.0053	.0055	.0000
11	Sylviculture	.0218	.0000	.0002	.0002	.0025	.0003	.0327	.0003	.0015	.0011	.1801	.0009	.0000	.0010	.0000	.0000
12	Water Administration	.0039	.0037	.0014	.0021	.0089	.0042	0021	.0028	.0027	.0034	.0016	.0000	.0014	.0017	.0000	.0000
13	Transport-telecommunication	.0658	.0291	.0229	.0217	.0645	.0214	.0134	.0282	.0873	.0063	.0490	.0055	.0064	.1084	.1236	.0000
14	Home trade	.0117	.0073	.0192	.0288	.0143	.0360	.0285	.0115	.0434	.0005	.0051	.0062	.0246	.0074	.0260	.0000
15	Foreign Trade	.0044	.0007	.0163	.0187	.0072	.0258	.0202	.0119	.0036	.0049	.0008	.0016	.0039	.0003	.1255	.0000
16	Taxes-Import	.0055	.0010	.0207	.0142	.0065	.0230	.0155	.0078	.0030	.0045	.0023	.0005	.0030	.0001	.0000	.0000

TABLE 2

MATRIX OF DIRECT EXPENDITURE COEFFICIENTS—1969 STATISTICS

No.	Name Branch	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	Mining Industry	.0523	.1746	.0857	.0090	.0939	.1619	.0097	.0073	.0041	.0009	.0022	.0129	.0299	.0069	.0039	.0000
2	Electric Energy	.0365	.1061	.0291	.0134	.0287	.0316	.0151	.0058	.0146	.0029	.0022	.0339	.0168	.0124	.0041	.0000
3	Metallurgy	.0423	.0168	.3498	.1827	.0551	.0221	.0099	.0073	.0556	.0005	.0000	.0077	.0151	.0012	.0004	.0000
4	Machine-building industry	.0853	.0657	.0517	.2372	.0535	.0343	.0339	.0103	.1037	.0366	.0426	.0862	.1051	.0175	.0254	.0000
5	Construction materials	.0023	.0103	.0102	.0096	.0555	.0085	.0038	.0073	.1182	.0074	.0040	.0150	.0022	.0063	.0048	.0000
6	Chemistry	.0321	.0603	.0930	.0379	.0454	.1917	.0725	.0227	.0211	.0595	.0353	.0416	.0685	.0137	.0211	.0000
7	Light industry	.0179	.0074	.0148	.0316	.0336	.0388	.2993	.0221	.0547	.0058	.0131	.0210	.0154	.0458	.0668	.0000
8	Food industry	.0012	.0005	.0015	.0011	.0019	.0204	.0249	.1534	.0000	.0629	.0015	.0000	.0019	.0027	.0048	.0000
9	Engineering	.0047	.0124	.0124	.0036	.0146	.0144	.0032	.0055	.0036	.0033	.0029	.1072	.0122	.0183	.0334	.0000
10	Agriculture	.0035	.0000	.0000	.0000	.0005	.0189	.0434	.5022	.0007	.2602	.0127	.0069	.0000	.0051	.0061	.0000
11	Sylviculture	.0211	.0000	.0000	.0000	.0025	.0007	.0358	.0000	.0013	.0009	.1405	.0009	.0000	.0010	.0000	.0000
12	Water Administration	.0031	.0030	.0015	.0018	.0093	.0054	.0026	.0031	.0034	.0024	.0022	.0000	.0012	.0016	.0000	.0000
13	Transport-tele-comuunication	.0634	.0276	.0234	.0203	.0624	.0231	.0140	.0310	.0836	.0060	.0622	.0090	.0065	.1037	.1246	.0000
14	Home trade	.0113	.0074	.0190	.0217	.0146	.0391	.0305	.0149	.0460	.0002	.0065	.0077	.0244	.0071	.0299	.0000
15	Foreign Trade	.0065	.0008	.0164	.0182	.0073	.0191	.0147	.0103	.0013	.0080	.0040	.0000	.0063	.0009	.1137	.0000
16	Taxes-Import	.0076	.0013	.0207	.0137	.0085	.0232	.0158	.0080	.0029	.0036	.0029	.0004	.0040	.0002	.0000	.0000

TABLE 3

RATIO BETWEEN THE ESTIMATED COEFFICIENTS AND THE STATISTICAL ONES 1969 (1969 E/1969)

No. Name Branch	1	2	3	4	5	6	7	9	9	10	11	12	13	14	15	16
1 Mining Industry	1.0447	.9901	.9765	1.0533	.9184	1.0212	.9449	.9525	1.0055	1.0075	.8475	.9860	1.0705	1.0233	.9472	1.0000
2 Electric Energy	1.0661	.9372	.9633	1.0076	.9240	1.0680	.9716	1.0075	1.0179	1.2295	.7156	1.0767	.9661	1.0237	.9318	1.0000
3 Metallurgy	.10371	1.0178	1.0155	.9941	1.0176	1.0364	1.0040	1.0449	.9526	.6851	1.0000	.8210	.9908	1.0650	1.4211	1.0000
4 Machine-building industry	.8677	1.0265	1.0416	.9644	1.0422	1.0390	1.0198	1.0105	1.0395	1.0277	.7957	1.2082	1.0553	1.0951	.8911	1.0000
5 Construction materials	1.0901	1.0518	.9588	1.0415	1.0400	.9924	.9337	1.0410	.9856	1.8069	.7800	.9860	.9616	1.0236	.9251	1.0000
6 Chemistry	1.0337	1.0456	.9564	1.0277	1.0370	.9546	1.0026	.9784	.9223	1.0544	.8112	.9786	.9697	1.0336	.9507	1.0000
7 Light industry	1.1562	1.0158	.9847	.9194	1.0594	.9805	.9947	.9933	.9939	1.0656	1.0107	1.0062	1.0628	1.1182	1.0007	1.0000
8 Food industry	.9244	.8937	.8483	.9818	.9756	1.0359	1.0117	.9593******		1.0566	.8014	1.0000	.9535	1.4721	.9169	1.0000
9 Engineering	1.1751	1.1101	1.0566	.9581	1.1159	1.0431	1.1126	.8834	.6972	1.0992	.8520	.8749	.7722	1.2091	.9560	1.0000
10 Agriculture	1.0283	1.0000	1.0000******		.7174	1.0118	1.0334	1.0238	1.1031	.9649	.7965	.8050	1.0000	1.0292	.8896	1.0000
11 Sylviculture	1.0327	1.0000**********			1.0000	.5121	.9292******		1.1497	1.1719	1.2817	1.0700	1.0000	1.9796	1.0000	1.0000
12 Water Administration	1.2313	1.2330	.9904	1.1710	.9500	.7892	.8313	.8776	.7993	1.3808	.7156	1.0000	1.1085	1.0524	1.0000	1.0000
13 Transport-telecommunication	1.0385	1.0536	.9805	1.0712	1.0328	.9269	.9614	.9096	1.04421	.0592	.7866	.6153	.9816	1.0447	.9922	1.0000
14 Home trade	1.0345	.9806	1.0111	1.3263	.9800	.9206	.9364	.7727	.9433	2.7000	.7725	.7997	1.0090	1.0508	.8714	1.0000
15 Foreign Trade	.6841	.8588	.9884	1.0280	.9904	1.3520	1.3727	1.1519	2.6767	.6082	.1950******		.6137	.3793	1.1041	1.0000
16 Taxes. Import	.7192	.7891	.9995	1.0404	.7609	.9893	.9808	.9795	1.0221	1.2353	.8041	1.2326	.7600	.8125	1.0000	1.0000

The elements of the matrix marked with asterisks have values close to 1, the asterisks appearing because of the ratio denominator which is close to O.