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Fuzzy Algorithms in Economic Systems

The efficiency of the conventional quantitative techniques (we mean here operations research, the theory of probabilities, simulation, etc.) in analysing the physical systems has stimulated the extension of these techniques also in the analysis of economic, social and biological systems.

So far, the results were not those expected because:

— the economic social systems... are systems of a behavioural type, that is systems in which the behaviour of the subsystems is playing a decisive part;

— the hypothesis that these systems can be described by differential equations, with finite differences or integral equations is simplifying the complexity of the processes in these systems surpassing by far the possibilities of such equations;

— the principle of incompatibility [6] acts within these systems; thus, as the complexity of a system increases, the possibility of making precise suppositions on its behaviour decreases;

— the conventional techniques and methods have a deterministic or stochastic character, therefore, in the best case they are built for measuring *the incidence* and not *the imprecision*, etc.

Considering only those shown above, it becomes obvious that the effort to adapt some inadequate methods should be replaced by the seeking of adequate methods of analysis and modelling.

In 1965 ZADEH proposed a method based on the premise that human thinking operates not with numbers but with fuzzy sets[1] defined to be classes of objects in which the passing from membership to nonmembership is made gradually and not abruptly as in the classical theory of sets.

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[1]) An excellent monograph on the applications of fuzzy sets is [5]

1. FUZZY SYSTEMS AND OPTIMALITY

Consider X and S two sets and T a set of finite or infinite indices. We call a discrete system the set:

$$\mathcal{S} = \{(x, s, t)/x \in X, s \in S, t \in T] \tag{1}$$

X will be the set of admissible inputs and S the set of admissible states. If we note with $\Omega_{\mathcal{S}} = S \times X$, the field of admissibility of $\mathcal{S}$, we have, for $s_0, x_0, x_1, \ldots, x_T$ given,

$$S_{t+1} = f(x_t, s_t) \tag{2}$$

$$y_t = g(x_t, s_t) \tag{3}$$

where $f\colon \Omega_{\mathcal{S}} \to S$, $g\colon \Omega_{\mathcal{S}} \to Y$, and y_t is the output at moment t and Y is the set of outputs (we use the notation $(x, s, t) = (x_t, s_t)$.

If $\forall (x_t, s_t) \exists ! \, S_{t+1}$ which satisfies 2) and $\exists ! \, y_t$ which satisfies 3) $\mathcal{S}$ is called deterministic ($\mathcal{S}_D$). Otherwise, we speak of a non-deterministic system ($\mathcal{S}_N$). In a $\mathcal{S}_N$ the probabilities:

$$p(s_{t+1}/(x_t, s_t)) \tag{4}$$

$$p(y_t/(x_t, s_t)) \tag{5}$$

can be attached to a pair (x_t, s_t).

The theory of fuzzy systems sets out from the remark that probabilities 4) and 5) measure incidence, generated as a result of a series of experiments according to the law of large numbers. But, there are cases when the testing possibilities are reduced, if not inexistent (for example the economic or social systems).

Such systems have a complex structure. Hence the lack of precision (fuzzyness) of our knowledge about the respective systems. The use of the concept of fuzzy sets to the systems [1], [3] allows the measuring of this imprecision of the behaviour of the states and outputs in circumstances of imprecision.

Thus, if the grades of membership are built*:

$$\mu(s_{t+1}/_1(x_t, s_t)) = \varkappa(s_{t+1}) \tag{6}$$

$$\mu(y_t/\,(x_t, s_t)) = \varkappa(y_t) \tag{7}$$

where $\varkappa(\cdot)$ are the membership functions generating the respective grades of membership, and these grades are attached to $\mathcal{S}_N$, we say that we have obtained a fuzzy system ($\mathcal{S}_F$).

For example: "It is raining now, so the paint will *not* dry *very well* and the delivery term will be *a little* delayed".

We thus have a system with a deterministic input (it is raining). Its state is described by the degree of humidity of the paint and the output is the delivery term. The underscored terms determine the fuzzyness of the state and of the output.

If U is the set of admissible decision in the system $\mathcal{S}_F$, we can construct:

$$\mu(s_{t+1}/(s_t, u_t)) = \varkappa\,(u_t) \tag{8}$$

where $u_t \in U$ is the decision at moment t.

If $\exists v \subseteq U$ so that $\forall v_t \in V$, $\exists u_t \in U$ so that $\varkappa(u_t) \leqslant \varkappa(v_t)$ we can say that the decision u_t is finer than the decision v_t.

We call policy P a subset after t of U. For example:

$$P_u = \{u_1,...,u_T\} \tag{9}$$

or

$$P_v = \{v_1,...,v_T\} \tag{10}$$

are policies.

We can attach to any policy P a vector of membership functions of the decisions. For example, for P_u we have:

$$\varkappa_{P_u} = \{\varkappa(u_1),...,\varkappa(u_T)\} \tag{11}$$

An optimal policy P^* is

$$P^* = \underset{P}{opt}\{\varkappa_p\} \tag{12}$$

The decisions within a policy P can be deterministic, stochastic or fuzzy. If in a policy there is at least one fuzzy decision we say that the respective policy is fuzzy.

The decisions of a policy are made through algorithms. In the large sense of the word, an algorithm is considered to be an ordered set of instructions [5]. Within the fuzzy algorithms there is at least a fuzzy instruction. These instructions may be:

a) assignation instructions

$$x = large$$

b) conditional instructions

$$\text{IF}x = small\ \text{THEN}y = very\ small$$

c) instructions of unconditional action

Slightly decreases on x.

A classification of the fuzzy algorithms is given in [6].

Examples of fuzzy algorithms in production scheduling **Example no. 1.** In an enterprise section, a number of p products: $P_1, P_2,...,P_p$ must be turned out on a plan horizon T. The plan horizon T is divided into n periods (decades, weeks or even days). For each period $i(i = 1,2,...,m)$, we know the available assets accumulated for resources R_{ij} (semifabs production capacity, etc.), where $j = 1,2,...,q$. For each product $P_k(k = 1,2,...,p)$ a quantity q_i^k must be delivered at the requested term T_i $(i = 1,2,...,m)$. A series of data are known in connection with the situation of the contracts, the rhythmicity of work, past experience etc.

*) For this, see [6] and [7].

The planning of the turning out of products must greatly observe the terms imposed on T_i and not exceed, if possible, the available resources.

In order to solve the problem, it is necessary to establish first the priority of the product. For this the following criteria are taken into account:

— the type of the contract (firm contracts, contracts being finalized, preliminary orders, the estimation of future orders);

— the rhytmicity of work (on batches at fixed terms, on batches at non-fixed terms, the delivery of the entire quantity);

— the experience of the past (products with a great delay probability, products with a small delay probability).

On the basis of these criteria, a scheduling fuzzy subroutine (called the PREORD subroutine) has been elaborated. A whiffletree is built according to the scheme in fig. 1. The root of the whiffletree contains the group G_1 of products with a maximum urgency. The grades of membership each criterium (represented by the knot) are written on the arcs in relation to the criterium on the previous level. The grade of membership to the respective criterion is written on the knot for each product P_k. For each branch of the whiffletree, we calculate:

$$\mathcal{S}_{\alpha k} = Min(\mu_i^o, \varkappa_{k1}^i, \mu_{ji}, \varkappa_{k2}^j, \mu_{ji}^h, \varkappa_{k3}^{h'}, \mu_{ji}^{he}, \varkappa_{k4}^{e'}).$$

Then, we calculate:

$$Max[\rho_{\alpha k}] = \rho_k^*$$

The subroutine then schedules the products, the following rules being observed:

— a product P_k has priority to another product P_s if:

$$0.9\rho_k^* \geqslant \rho_s^*$$

— a product P_k is indiferent to P_s, if:

$$\mathcal{S}_k^* \geqslant \rho_s^*$$

but

$$0.9\rho_k^* \leqslant \rho_s^*.$$

In continuation, the steps of the fuzzy algorithm of planning the execution of products are:

Step 1. We apply to PREORD subroutine which schedules the products. If two or several products are indifferent a random number is genetated on the basis of which the priority order is established.

Step 2. The number of periods t is initialized during which products:

$$t_i = T_i.$$

will be turned out considering that the established term is expressed in number of plan periods (decades, weeks, days) as well as the available quantity left

$$\delta_{ij} = R_{ij}$$

Step 3. Quantity q_i^k is allocated for the period t_i, the average rate $\frac{q_i^k}{t_i}$ ($i = 1,2,...,m$ $K = 1,2,...,p$) being obtained. The quantity achieved accumulated for h periods is established, that is $\frac{q_i^k}{t_i}$ ($h = 1,2,...,t$).

Step 4. The left quantity δ_{ij} is calculated for each resource, at the end of each period:

$$\delta_{ij} = \delta_{ij} - h\frac{q_i^k}{t_i} C_{ij}, \quad h = 1,2,...,t_i$$
$$j = 1,2,...,q$$

Step 5. The available quantity left δ is compared to zero. If it is much smaller than zero:

$$\delta_{ij} \ll 0$$

the number of planned periods is reduced by one unit:

$$t_i = t_i + 1$$

and the algorithm is repeated from step 3.

If

$$\delta_{ij} < 0 \text{ or } \delta_{ij} > 0$$
$$n$$

it is considered that δ_{ij} is the new quantity of accumulated resources.

Step 6. k is compared to the number r of products which do not surpass the plan horizon T.

If

$$k < r$$

the algorithm is repeated from step 3.

If $k = r$ go to step 7.

Step 7. We consider:

$$R_{ij} \geqslant 0, (\forall)_j.$$

Step 8. We seek if there are products for which

$$T_i > I$$

We consider the set $\mathcal{M}$ of products satisfying this inequality. We calculate the cardinal of this set:

CARD $\mathcal{M} = p'$

Step 9. p' is compared to zero. If $p' = o$ the results are printed. If $p \neq 0$ go to step lo.

Step 10. We choose: $min\ T_i$

Step 11. We calculate:

$$min \frac{\delta_{ij}}{C_{ij}}$$

Step 12. We calculate:

$$p = r + p'$$

and the algorithm is repeated from step 4.

The logical scheme of this algorithm is presented in Fig. 2.

2. A FUZZY ALGORITHM IN PRODUCTION SCHEDULING

In a workshop with m machines, n jobs are processed, each job undergoing a single processing. A job can be processed on each of the machines, but the quality of processing is different, that is on some of the machines it is *very good*, on others *good*, on others *less good*, etc. Each job is ordered by a certain beneficiary. The urgency of delivery of the job *(extremely* urgent, *less* urgent, etc.) results in keeping with the importance of the beneficiary and with the delivery term.

The processing time is known for each job and the available durations for each machine (these sizes are supposed to have a deterministic or stochastic character, resulting after some measurements).

From past experience and from various data with a technological character, it is known that some of the machines have a *very large* probability* of strangulation, others *large*, others *small*, etc.

The processing order of the jobs as well as their allocation on machines are required so as to observe the delivery terms in keeping with the importance of the beneficiaries and to ensure an *ever better* quality of the technological processing.

The steps of the algorithm:

1° CALL ORD (U, X, Y)
2° CALL ORD (R, X, Y).
3° CALL ALOC (U, R).
4° If the number of jobs left is nearly zero THEN go to 8°.
5° IF *nearly* all machines are loaded
THEN go to 9°.
6° CALL REORD (U, X, Y).
7° CALL REORD (R, X, Y). GO to 3°.
8° STOP.
9° print "solution impossible". STOP.

Subroutine ORD (z, X, Y) orders (lexicographically for example) a set according to two fuzzy criteria. If $z = U$, the machines are set in order, a case in which X = strangulation probability (fuzzy understood, that is considering values of the form: *very very large*, *very large*, *large*, *neither large nor small*, etc.) and Y = the available duration of the machine *(very small*, etc.*)*. For example, if a machine has a very very small strangulation probability and a very very long time available, it will be nearly first on the list. If $z = R$ jobs are set in order according to criteria X = importance of the beneficiaries and Y = the reserve left of the job.

*) In this place, probability has a fuzzy meaning. This is developed in [5] and [6].

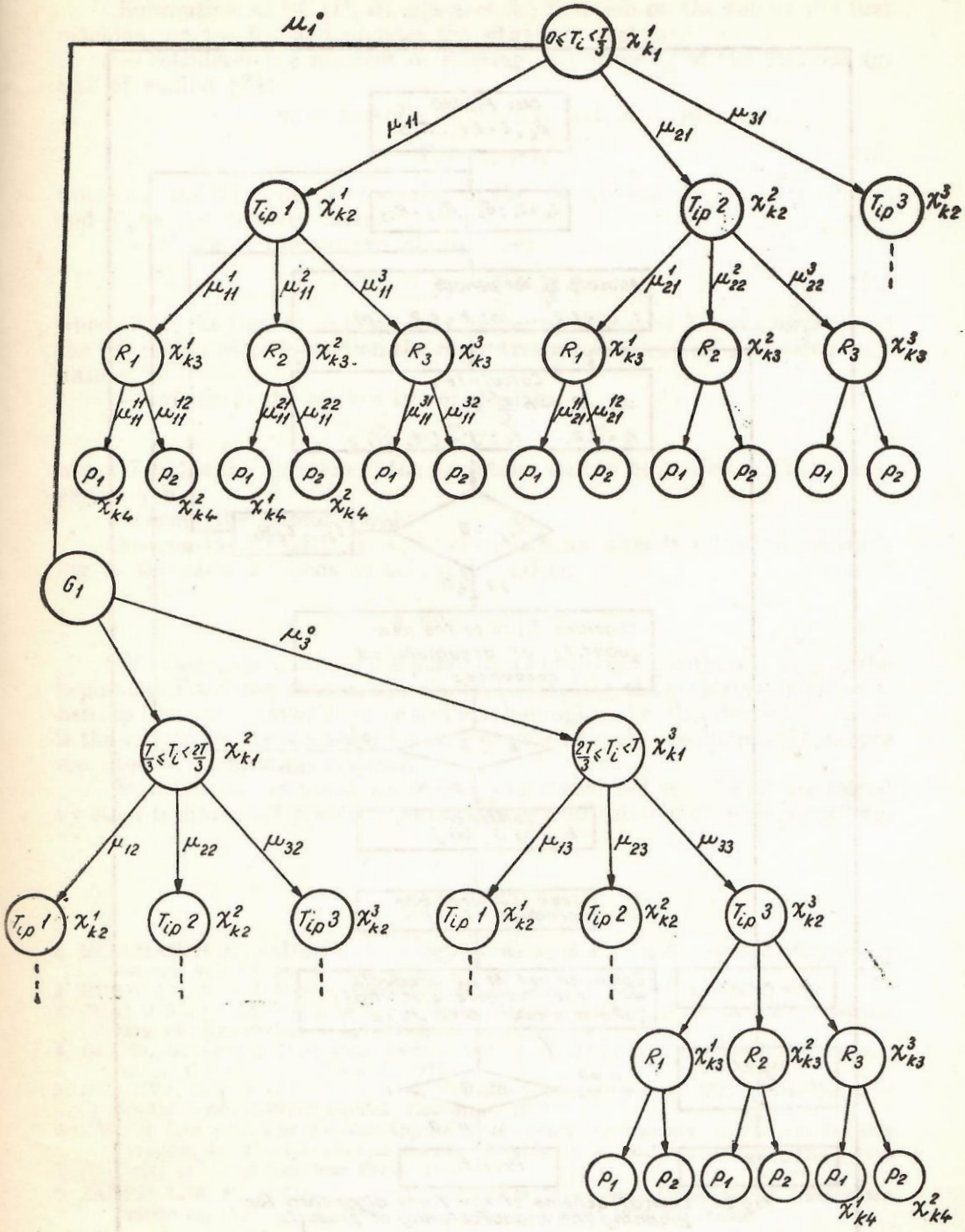

μ_1^0
$0 \le T_i < \frac{T}{3}$
χ^1_{k1}
μ_{11}
μ_{21}
μ_{31}
$T_i p\ 1$
χ^1_{k2}
$T_i p\ 2$
χ^2_{k2}
$T_i p\ 3$
χ^3_{k2}
μ^1_{11}
μ^2_{11}
μ^3_{11}
μ^1_{21}
μ^2_{22}
μ^3_{22}
R_1 χ^1_{k3}
R_2 χ^2_{k3}
R_3 χ^3_{k3}
μ^{11}_{11} μ^{12}_{11}
μ^{21}_{11} μ^{22}_{11}
μ^{31}_{11} μ^{32}_{11}
μ^{11}_{21} μ^{12}_{21}
p_1 p_2
χ^1_{k4} χ^2_{k4}
G_1
μ_3^0
$\frac{T}{3} \le T_i < \frac{2T}{3}$
χ^2_{k1}
$\frac{2T}{3} \le T_i < T$
χ^3_{k1}
μ_{12} μ_{22} μ_{32}
μ_{13} μ_{23} μ_{33}

Fig. 1

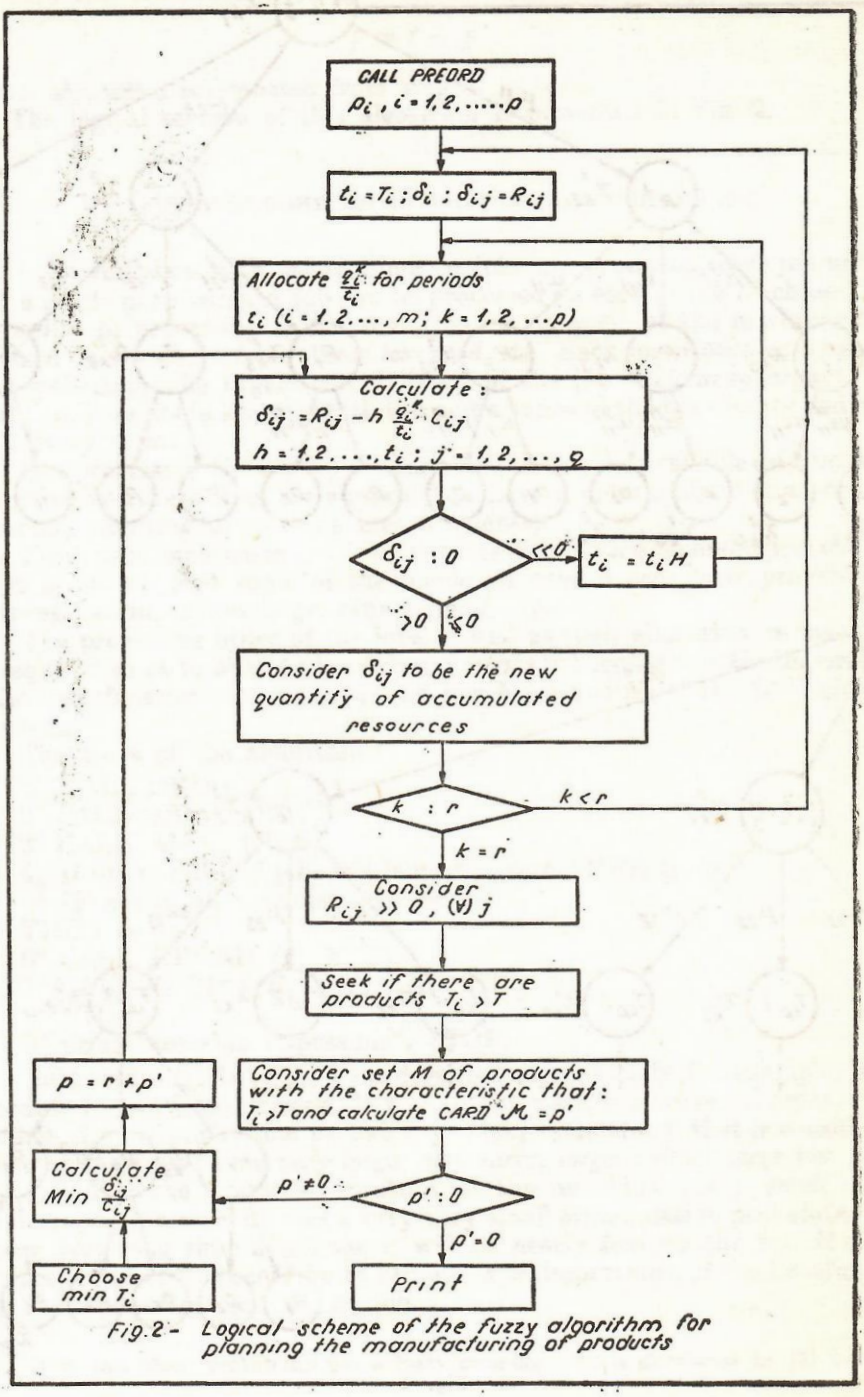


Fig.2 - Logical scheme of the fuzzy algorithm for planning the manufacturing of products

Subroutine ALOC (U, R) allocates the first job on the list on the first machine on the list and updates the situation, that is:

— calculates the moment of starting of processing of the machine (τ) and of ending (T):

$$\tau_i = max(T_d, T_{i-1}); \quad (i = 1, 2, ..., n)$$

$$T_i = \tau_i + t_i \tag{13}$$

where t is the duration of processing of the job on the respective machine. and T_d is the delivery term,

— calculates the reserve of machine:

$$D_r = T_f - T - \Sigma t_r^u \tag{14}$$

where T_f is the final term of activity of the machine and Σt_r^u the duration of the job left to be processed on the respective machine in best processing conditions.

— calculates the reserve left of the jobs:

$$Res = T_i - T_d - \Sigma t_r^n \tag{15}$$

where $T_p \Sigma t_r^n$ is the duration of the operations still to be carried out in the respective job.

— erase the allocated job.

Subroutine REORD (z, X, Y) changes a list already set in order according to the same methods as subroutine ORD.

3. CONCLUSIONS

Of course, the study of the possibilities of fuzzy algorithms is only at the beginning. Problems, such as optimality, scheduling, the construction of membership functions have still to be studied thoroughly. Another difficult problem is the solving on the computer because at present, only deterministic concepts are used in computing systems.

But, the way in which some problems considered as difficult are solved by other techniques forecasts the ever stronger affirmation of fuzzy modelling.

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