INPUT-OUTPUT-ANALYSIS AND OPERATIONS RESEARCH TECHNIQUES

B. Korte, M. Grotschel, Institut für Ökonometrie und Operations Research, Universität Bonn, 53 Bonn, Nassestr. 2 (F.R.G.)

Consider the matrix $X = (x_{ij})$, $i, j = 1, ..., n$, of intermediate inputs and the vector $x = (x_1, ..., x_n)^T$ of gross products. The matrix of input coefficients $\bar{X} = (\bar{x}_{ij})$ is given by

$$\bar{x}_{ij} = \frac{x_{ij}}{x_j} \quad i, j = 1, ..., n \tag{1}$$

Let $y = (y_1, ..., y_n)^T$ be the vector of final demand. The identity

$$(I - \bar{X})\, x - y = 0 \tag{2}$$

is called the Leontieff-system.

An often suggested model in the literature is to maximize a linear or concave objective function subject to a set of restrictions containing the Leontieff system and nonnegativity constraints $x \geqq 0$, $y \geqq 0$. The further restrictions are normally taken to consist of inequalities derived from the Leontieff-Walras-production function or some other production functions, and/or upper and lower bounds on the variables x, y.

Thus we obtain an optimization model

$$\begin{aligned} &max\ f(x, y, z) \\ &\text{s.t. } (I - \bar{X})\, x - y = 0 \\ &f_j(x, y) + z_j = b_j \qquad j = 1, ..., m \qquad (3) \\ &x, y, z \geqq 0 \qquad (4) \end{aligned}$$

It is shown that the optimal solution of this problem does not depend on (2), i.e. the Leontieff system is redundant, if f is concave and f_j, $j, ..., m$ is convex. This implies that the optimal solution is only dependent on the added restrictions, which means that this frequently used model does not reflect the input output structure of the economy: it only reflects the preconception of the model builder.