HANS W. GOTTINGER
Universität Bielefeld
Fakultät für Soziologie

Information Structures in Dynamic Team Decision Problems

In attacking control problems of various sorts, in particular in economics, the almost universal assumption is made that information being made available to the controlling unit, is centralized. However, in dealing effectively with multi-person decision problems as those arising in decentralized economic systems or organizations this kind of assumption is hard to accept. If, in addition, we have further complications that information may be dynamic (group dynamic) in the sense that information may depend on past performance of other members (decision makers) in a group, new conceptual problems arise which will affect differential game and decentralized control theory. In some form these questions came up when game theorists considered games of extensive form but up to now they have not been treated successfully. Recently, small group researchers in criticizing stochastic stimulus response theories, have pointed out the significance of generating information flows by actions of other members of a group.

This paper attempts to approach these problems in a formal way and generalizes some results obtained by Marschak and Radner for dynamic team decision problems.

1. INTRODUCTION

It has been increasingly recognized over the past years that information should form a part of designing various systems for optimization. In particular, by approaching these problems, we have to mention that there is some need to keep information separate from probability in distinction to conventional information theory, to operationalize the concept of information making it more amenable to some real applications and foremost to study qualitative properties arising in this context.

In conventional control problems it has been a customary assumption that information is centralized. The motivation for this is easy to provide since in all well defined problems control theorists were usually concerned with one decision-maker or policy-maker whose action is geared toward bringing a system under control. So I should point out at the beginning that I emphasize a control-theoretic rather than a decision-theoretic (see Marschak and Radner) or game-theoretic viewpoint (see Kuhn). For technological or military systems such a control task can be established without any difficulty, for social systems this assumption is by far too restrictive unless we are dealing with very simplified hierarchical systems. If we attempt to treat more complex problems such as multi-person optimization problems we are encountered with several decision-makers who all share in controlling a common control variable. If all of these decision-makers are endowed with different "information structures" in the sense of Marschak and Radner [10] we have the situation of a team and the corresponding control problem is a "team decision problem". If decision-makers have different 'preference structures' and conflicting interests the control problem comes close to a game problem.

A second point is based on the additional complication that the information may be *dynamic*, e.g. that information depends on other persons, actions in the past where actions are sequentially chosen given the initial information structure. In this case a series of new conceptual problems arise which have been attacked only recently. Also here, in case of different preference structures among persons, there is a suitable model provided by differential game theory, and, in many respects, this theory merges many interesting aspects of general game theory and optimal control theory.

To some extent, questions relating to the connection of "information sets" and "optimal behavior strategies" have been studied in the early development of games in extensive forms (see Kuhn [8]), however, this has been done in a static framework where the dynamic aspects of information have been neglected.

Let me make a brief digression to talk about the situation in extensive games. Extensive games deal with the case of varying information patterns under successive presentation of alternatives, therefore extending conventional games under perfect information possessing solutions in pure strategies. This is reflected in the branching of a topological tree where the patterns of information act as partitions of the vertices of the tree. The geometric form of game tree representation gives some intuitive insight into the concept of a "nested sequence of information sets" which is analogous to the concept of "inflation" in the sense of N. Dalkey [1]]. By an "inflation" Dalkey means increasing the accuracy of the information. The result of successive application of this process until no further information is obtained is called "complete inflation" of a game*.

In the context of an extensive game information is specified as a function f on X, i.e. it assigns f to any move x of the game so that $f(x)$ is usually an integer. x forms a vertex of a game tree and is said to belong to the information set X.

---

* Incidentally this corresponds to the concatenation of basic computations as found in sequential machine theory [15].

Dalkey then shows that two games which differ only in information have the same *normalized* form, if they have identical complete inflations where "complete inflation of information" generally enables each player to know the opponents' strategies in advance and prior to his own choice.

In the next sections we are dealing with the two aspects of multiperson optimization and control, and, in particular, with dynamic information in a team decision framework.

2. PARTITIONS AND INFORMATION ALGEBRAS

In this section we study the general mathematical background of information structures and give some definitions. The content of information is algebraic or qualitative and the approach resembles that taken in (Gottinger [3]). At the start, let us consider two variables x and y ranging over two finite sets X and Y. Suppose there exists a many-to-one mapping η: $X \to Y$ so that $\eta(x) = y$ and $\eta^{-1}(y)$ exists and constitutes a subset of X. In the context of the team decision structure η would be called an "information function" or "information structure", and X denotes the set of true states of the environment and past history (available to the decision-maker or the team) and Y denotes the set of alternative signals. Such information structures can be generated sequentially so that $(\eta_0, \eta_1, \eta_2,...)$ represents a system of information processing, or processing chain according to Marschak [9]. (In particular, η_0 might denote the information structure generated by observation of nature, and $\eta_1, \eta_2,...$ constitutes a sequence of information generated by intra-team communication.)

Thus the knowledge of x determines uniquely the value of y but this does not hold vice versa.

It is clear that every information structure η induces a partition of X into a class of mutually exclusive subsets each of which is associated with a different y. The knowledge of y enables us to distinguish between those elements of X which belong to different subsets but not between elements within some particular subset.

Accordingly, we could say that y is *less* (or *not more*) *informative* than x depending on whether x does not (or does belong) to some subset $\eta^{-1}(y)$ in X. In other words, the relation of "being not more informative than" (qualitative information) is induced by the partition on X.

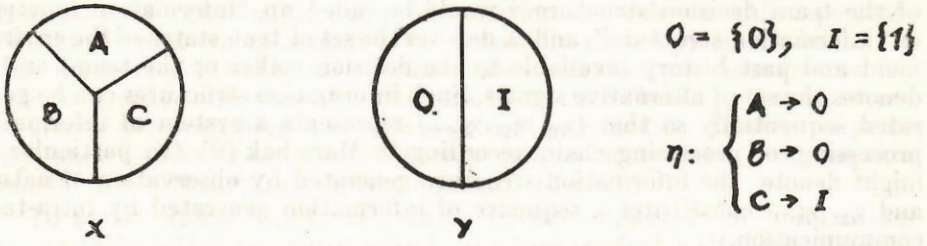
This construction ensures us of the fact that the existence of a state (or states) could only be informative if we get the message about the existence of this state.

Methodologically, we have two ways open to construct an algebra of information sets given the previous considerations. Either we take "qualitative information" as a primitive concept, satisfying certain order properties, and see what kinds of structural properties could be derived. Such an approach has been pursued in [3], it will not be adopted here. Or we could ask the question what kinds of algebraic structures are compatible with qualitative formation. Certain kinds of algebraic structures are quite straightforward, for example,

any partial algebra defined in terms of partial operations can be converted to a partially ordered set and vice versa (see L. Fuchs [2]). We will consider this problem next.

We consider all subsets of X for which operations of union ($\cup$), intersection ($\cap$) and complementation ($-$) are defined. This class of subsets of X closed under these operations forms an algebra (field) of sets (which need not be restricted to the finite case and could become a σ-algebra), denoted by **X**. **X** contains the empty set $\emptyset$ and the universal set X. Consider those subsets of X determined by $\eta^{-1}(y)$ for all y which form a particular algebra **A**. **A** could be derived from (say) **X** in such a way that all possible operations of $\cup$, $\cap$ and $-$ are preserved for elements of **A**. Clearly then **X** "covers" **A**, written $\mathbf{X} \supset \mathbf{A}$, and **A** is a subalgebra of **X**. Then **A** containts all "relevant" information for the problem at hand.

Example. Let $\mathbf{X} = \{\emptyset, A, B, C, A \cup B, A \cup C, B \cup C, X\}$. Suppose $\eta^{-1}(y_1 = 0) = A \cup B$, $\eta^{-1}(y_2 = 1) = C$, $\mathbf{A} = \{\emptyset, A \cup B, C, X\}$.



The relevance of information received is hence provided by the choice of the information structure η, more "relevant" information can only be received by an appropriate choice of η' inducing a "finer" partition than the choice of η, hence partitions of X must be η-preserving. Suppose, η' is defined on **X** such that $\eta'^{-1}(y_1 = 0) = A$, $\eta'^{-1}(y_2 = 0) = B$, $\eta'^{-1}(y_3 = 1) = C$, then clearly (and intuitively plausible) η' generates a finer partition on X than does η given the information algebra **X** and for any fixed message set Y. The last qualification is essential, for suppose a message set Y is isomorphic to an action set A, i.e. there exist, in one-to-one assignment, as many different messages as there are actions. Then Y could be split into as many pieces as we wish, and via η^{-1} we could have an appropriately fine partition of X. Hence, by comparing information structures in terms of their information content it is essential to specify the underlying message set. It should be explicitly mentioned here, in relation to Marschak's and Radner's work [10], that the degree of fineness of information structures (i.e. those information structures which induce distinguishably fine partitions of X) implies a comparative relation of being "not more informative than" (i.e. the finer the partition of X the more information is carried by the information structure inducing this partition). A theorem due to Radner and Marschak [10, p.54] then says that for a given best decision function the expected payoff for the information structure η' is at least as large as that for η. This theorem relates qualitative information to payoff relevance.

Summarizing, in this example, we could say that the function η induces a partition of the set X which is coarser than the original partition. The coarseness is reflected by the fact that **X** is a richer collection of subsets than **A** and **X** "covers" **A**. In other words, η generates a subalgebra **A** in **X** and **X** is finer than **A**, or **A** is nested in **X**. Two pieces of information are *equivalent* if their underlying algebras are the same or if they are nested in each other (mutually nested).

The situation becomes more complicated when we are dealing with the situation where more than one agent (e.g. nature) could choose actions and therefore generate information.*

Example. Consider a simple example where $x = (x_1, x_2, x_3)$, not necessarily ordered this way, with subscripts referring to the decision-makers. Each variable x_i can assume values 1 or 0, i.e. only binary information is provided. Suppose it happens that x_1 will appear first and depending on whether $x_1 =$ 0 or 1, x_2 or x_3 will become the second or third state variable. The set of possible x, X, then consists of the set {[000], [001], ..., [110], [111]}, where [...] denotes the state sets over all possible permutations of the variables.

$$\mathbf{X} = \{\emptyset, [000], [001], ..., [111], [000] + [001], ..., [000] + [111], ...,$$
$$[000] + [001] + [010], ..., X\}$$

where '+' here denotes algebraic addition.

Complexity of messages in this algebra could be increased almost indefinitely, by increasing the number of variables, respectively. In this way we could establish a nested sequence of subalgebras differing in terms of coarseness and fineness among each other. The information content in the state sets is preserved under η in Y, where for proper representation X and Y should have the same dimension.

The subalgebras in the previous example are easy to establish. Define

$$A^{(1)} = \{\emptyset, A^1, A^{-1}, X)\}, \qquad A^1 \equiv [000], A^{-1} = X - A^1$$
$$A^{(2)} = \{\emptyset, A^2, A^{-2}, X\}, \qquad A^2 \equiv [00X_3] + [1X_2O], \; A^{-2} = X - A^2$$
$$X_3 = \{x_3 : x_3 \in [0,1]\}, \qquad X_2 = \{x_2 : x_2 \in [0,1]\}.$$

The equivalent function specification is given by

$$y^{(1)} = \eta^{(1)}(x) = \begin{cases} 1, \text{if } x_1 = 0,\ x_2 = 0,\ x_3 = 0 \\ 0, \text{otherwise.} \end{cases}$$

$$y^{(2)} = \eta^{(2)}(x) = \begin{cases} 1, \text{if } x_1 = 0,\ x_2 = 0,\ x_3 \in [0,1] \text{ or} \\ \quad x_1 = 1,\ x_3 = 0,\ x_2 \in [0,1] \\ 0, \text{otherwise.} \end{cases}$$

---

* Incidentally small group researchers, in criticizing stochastic stimulus-response theories, have pointed out that for group dynamic behaviour models it is necessary to distinguish between two kinds of information, that which comes as a stimulus from the physical environment (state of nature), directly observed by the person, and that which is communicated to him by actions of other members within a group.

The information provided by $\mathbf{A}^{(1)}$ tells us whether or not the sequence $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ happened, $\mathbf{A}^{(2)}$ reveals the information whether or not the sequence $x_1 = 0$, $x_2 = 0$, $x_3 \in [0,1]$ or $x_1 = 1$, $x_3 = 0$, $x_2 \in [0,1]$ occurred. We have $\mathbf{A}^{(2)} \not\supset \mathbf{A}^{(1)}$ and $\mathbf{A}^{(1)} \supset \mathbf{A}^{(2)}$. Now, if all messages (signals, propositions) encoded in $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ respectively are correct, $\mathbf{A}^{(1)}$ is considered to be more informative than $\mathbf{A}^{(2)}$ and vice versa when the data are negative.

This is intuitively plausible. The previous examples demonstrate the economy of notation that can be achieved by adopting an algebraic point of view in dealing with "qualitative information" *. Principally, no problems are involved in treating continuous variables, $\mathbf{A}$ and $\mathbf{X}$ simply become σ-algebras which is an immediate extension of finite algebras discussed-so far. Then the function η will be measurable on $\mathbf{X}$. Most results discussed here and later continue to hold in the continuous case with possible technical qualifications. In what follows we limit our discussion to the discrete case. Finally it is interesting to note that we use the same algebraic framework for studying information (independent of probability) as we do for probability.

3. INFORMATION STRUCTURES AND DECISIONS

Suppose first, we have n payoff-relevant and information-generating action variables x_k, $k \in K = \{1, ..., n\}$, each controlled by a separate decision-maker 'agent'. *A sequential action* of a person acting more than once at different times could be considered as that of several persons each acting once and only once at a particular point in time.** Each x_k takes values on a finite set X_k to which there is associated an algebra $\mathbf{X}_k$ so that $x = (x_1, ..., x_n) \in X = \prod_{k \in K} X_k$ and $\mathbf{X} = \prod_{k \in K} \mathbf{X}_k$. The information received by each agent must be some function of x, denoted by $y_k = \eta_k(x)$ taking values over a finite set Y_k. The collection of information functions $\eta = (\eta_1, ..., \eta_n)$ constitutes the *information structure* of the problem***. According to the previous section, η could be characterized by $(X, \mathbf{X}, \mathbf{A}_k, k \in K)$ where $\mathbf{A}_k$ is the algebra induced on $\mathbf{X}$ by η_k. Now we would like to show the interplay between information and control of action in this simple kind of situation.

After having received the message y_k the k-th agent wants to control his action x_k via a pure strategy (decision function or control law δ_k which is a mapping from Y_k to the set X_k of feasible actions chosen from a set of admissible pure strategies ν_k). This is in agreement with Marschak's and Radner's [10] description of a decision function, and the following example, taken from their book, illustrates the idea.

---

* The applicability of this point of view seems to be fairly general and certainly covers information analysis as performed in sequential machines (see Hartmanis and Stearns [15], and in connection with organization theory, Gottinger [49].

** This means, basically, that one person dynamic control problems could be converted to many person static control problems and vice versa.

*** The definition of the information structure is formally equivalent, but in substance different from that given in the previous section. Here it is assumed that the conglomerate of the individual action variables (including nature) constitutes the 'environment' from which every decision-taker receives his information, rather than the physical environment in terms of states of nature only. This assumption is more in the spirit of group behaviour models.

Example. A profit-maximizing firm in a perfectly competitive environment (consisting of a conglomerate of appropriate actions of all competitive firms in the market) with increasing marginal costs is given a signal of price changes for various products in the coming year. The firm transforms the signal into an appropriate action, resulting in the decision rule: 'choose that level of production that will make marginal cost equal to price'. By this the action variable has to be controlled, otherwise the adopted decision rule would be violated.

The action of the k-th agent, after all, can be represented by the equation

$$(*) \qquad x_k = \delta_k(\eta_k(x)), \quad k \in K.$$

Clearly, for arbitrary η and $\delta = (\delta_1, ..., \delta_n) \in \nu$, there may not exist a unique solution x_k, provided a solution exists resulting in a possibly meaningless problem.

The clarification of the relationship between information and decision or control is an important first step, however, some additional problems arise if we attempt to formulate a more general problem. In the context of statistical decision theory we would conceive a prior probability distribution over the states of nature as nature's mixed (randomized) strategies. Also in certain game-theoretic situations decision makers would adopt mixed strategies which are probability distributions over the class of pure strategies.

These cases can be handled by the introduction of a sample space Ω, the points of which represent n-vectors ω. On Ω we define a σ-algebra of subsets, S, and assign a probability measure P on S. Then (Ω, S, P) is called a *probability space.* Since both X and Y are finite, ν must be finite too, and hence it is natural to assume Ω to be finite.

A component of ω, ω_k, ranges over a finite set having the cardinality of ν_k. The probability distribution of all $\omega \in \Omega$ such that $\omega_1 = r$, $\omega_2, = s$, ..., $\omega_n = t$ is given by $P(\{r, s, ..., t\})$. By construction, every set of admissible pure strategies is convexified by randomization and properly should be indexed by an appropriate $\omega \in \Omega$ which is distributed according to P. Adoption of pure strategies then simply corresponds to degenerate probability distributions over Ω. Then, most naturally, a randomized decision rule is given by δ_k: $\Omega \times Y_k \to X_k$ by convexifying and enlarging the set ν_k, so that the action is represented by

$$(**) \qquad x_k = \delta_k(\eta_k(x), \omega), \quad k \in K.$$

This will correspond to a situation where the decision-maker realizes some noise or measurement uncertainty in the signal and adopts a mixed strategy. The payoff is defined by

$$e(x_k) = e[\delta_k(\eta_k(x), \omega), x,],$$

where e is the expectation operator.

Let us give some examples pertaining to pure or mixed decision rules given the information structure. We will deal with the most general cases as those of multi-person dynamic control problems which arise in various forms in sequential decision theory, game theory, and team decision theory.

Example 1 (Deterministic multi-stage dynamic control problem).

The order of decision-makers in terms of actions is completely fixed in advance. Let the first decision-maker be nature choosing the initial state

of the dynamic system, given the information structure $y_1 = \eta(x_1)$ and decision rule $\delta(\eta(x_1)) = x_1$. [More explicitly, when talking about a dynamic problem, we may let depend the information structure on t, $y_1 = \eta(x, t)$ so that $\delta(\eta(x, t)) = x_1$].

Then decision-makers 2 through n choose the values of the control to be applied at the subsequent stages. The states of the dynamic system at all stages are functions of x. The usual information structure of the problem is $y_k = \eta(x_1, ..., x_{k-1})$, $k \in K$ which is equivalent to saying that person k knows the present state. There is no probabilistic consideration and specification of the probability space $(\Omega, \mathcal{S}, P)$ is irrelevant.

Example 2 (Noise and random effects).

Suppose we want to model n random variables. Let nature be the first k decision-makers. They receive no information (i.e. $\mathbf{A}_k$ is the trivial algebra $\{\emptyset, X\}$ for $k \leqslant n$ and η_k are constant functions). The k decision-makers adopt mixed strategies. Hence δ_k: $\Omega \to X_k$ can be identified as random variables distributed according to P. These random variables can then be used to represent random initial conditions, disturbances, measurement noises and all other random effects in the system. No assumption of indepedence of these random variables is required, nor is the time order of action of these decision-makers relevant in the actual system. The point is that so long as nature does not receive any information about x, and its decision only depends on Ω, we might as well assume that they act all at once in the very beginning. But their actions may be revealed to other (human) decision-makers at different times and with different accuracy through the specification of the η_k for the other decision-makers.

Example 3 (Multi-stage stochastic control problems)

In multi-stage stochastic control problems an important part of the problem is the pattern of information available to the controller (or decision-maker) at various instances of time. The possible patterns may include the following: he may know the state of the system at all past times, or only at the present; he may know only some functions of the past; he may or may not remember what he did in the past, etc A strategy for a stochastic decision problem must be a recipe which specifies at each time and for each condition of the controller's knowledge a mode of action.

This example is a combination of examples 1 and 2. We wish to model the usual dynamic stochastic system with noisy observations. It has to be pointed out that η_k is now a many-to-one function of x_j, $j < k$. The algebras induced by η_k, $k \in K$, $\mathbf{A}_k$, furthermore, have the *sequential nesting property*, i.e. $\mathbf{A}_k \subset \mathbf{A}_{k+1} \subset \mathbf{A}_{k+2} \subset ...$, in other words, in the sequence each information structure is finer than its predecessor (adaptation of increasing information). If we consider all different decision-makers as a single, sequential decision-maker acting at different times, it is said to have perfect memory (or perfect recall) if sequential nesting holds.

Example 4 (Dynamic Team Decision Problems).

Under a dynamic team decision problem (which is different from the team problem in a dynamic environment according to Marschak and Radner [10, Chap. 7]) we understand that information is not only generated by nature (observation), even not by nature changing in time, as in the Marschak-Radner case, but also and foremost by a list of sequential actions taken by

various members of the team the information of which is acquired by every successive agent (communication), hence the dynamics involved is essentially group dynamics resulting in an "inflation of information structures" as time goes on. The latter aspect is important and has been emphasized by small group researchers.

This is a restriction of Example 3, if the sequential nesting property does not necessarily hold, there need not exist perfect memory, and every team member acts once by explicit consideration of time, hence differs only from some other member in terms of the information structure but not in terms of the payoff functions. Also the case of noisy information can be adequately treated in this framework (see Marschak and Radner [10]).

Now suppose the nesting property does not hold due to limited memory or communication difficulties preventing decision-makers from transmitting information to each other perfectly. Then new complications will arise which brings us to a class of problems being considerably more difficult than those of Example 3.

Example 5 (General Extensive Games)

There is a further generalization of examples 1—4 to cases where the payoff for each decision-maker may be different and more specifically to cases where

(a) the order of actions of the decision-makers may be random (e.g. who plays next depends on the toss of a coin)

(b) the occurrence of random events may depend on actions of other decision-makers (e.g. the winner of a game will be decided by a toss if the score is tied, etc.).

These cases can be modelled along the lines of discussion in Example 2 taking into account equation (**).

Additional specification of the probability space (Ω, S, P)will also be necessary since different controllers (players) or coalition of controllers who may consist of several decision-makers are assumed to use independent (extraneous) random devices for mixing their strategies. This would require, for example, for any two controllers k and $k+1$, to satisfy the independence assumption of probability theory:

$$P(\{\omega\colon \omega_k = r,\ \omega_{k+1} = s\}) = P(\{\omega\colon \omega_k = r\}) \cdot P(\{\omega\colon \omega_{k+1} = s\}).$$

In Example 3 and 4 the *Sequential Nesting Property* can be given in the following illustration:
By assumption,

$$\delta_j(w) = x_j,\ j < k,$$

$$\delta_k(\eta_k,\ (x_j,t)) = x_k,$$

$$\delta_{k+1}(\eta_{k+1}(x_j,\ t),\ x_k) = x_{k+1},\ldots$$

In Example 3 $k, k+1, k+2,\ldots$ refer to a single decision-maker acting sequentially at different times, in Example 4, $k, k+1, k+2,\ldots$ could be identified as different members of a team acting sequentially, and always acquiring information from those members acting immediately before.

If sequential nesting does not hold, then we have a different situation as illustrated below.

In this case we have a team where each member receives information from nature only but does not care or does not recall what the other members

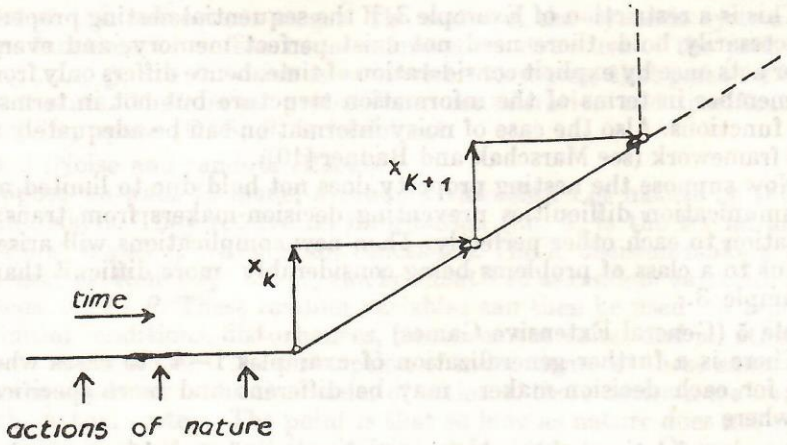


Fig. 1

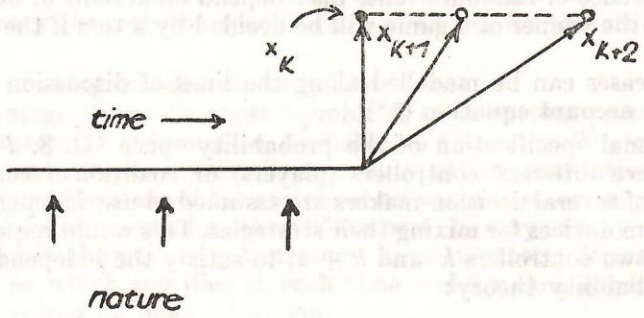


Fig. 2

did acting before him. An analogous interpretation applies to a single decision-maker acting sequentially in time: he may not recall what he did previously. His information structure essentially assumes a constant pattern.

4. ASSUMPTION OF CAUSALITY

A crucial question arises in the control theoretic formulation of dynamic decision problems, e.g. under which conditions does the closed loop equation (**) exist and possess a unique solution?

For very simple dynamic, nonstochastic systems one could make a point that (1) the time order of the various decision variables selected are fixed in advance, (2) that each decision maker acts as if he takes only full responsibility

for his own action (independence assumption), (3) that each agent has perfect memory, e.g. knows about all actions of the previous players.

For complex systems each of these assumptions is questionable. For these systems an organizational point of view on the interdependencies of decisions, delay, imperfect memory and ignorance plays a dominant role. This is why team theory, although denying conflicting interests, is a considerable step forward to a unified treatment of complex designs. Let us turn now to the discussion of the important result of Witsenhausen [13], [14], in which he showed that the requirement of causality is sufficient for answering the crucial question. Intuitively, the causality condition means that, for any play of the game or dynamic decision problem, the actions of the agents can be ordered and that the information available to an agent may depend on decisions of agents acting earlier but cannot depend on the decisions of agents acting at concurrent or later times. The difficulty here is to translate the intuitive idea of causality into meaningful mathematical conditions on η. For further considerations we decompose the total number of agents into K blocks so that for characteristic agents k in K we let $k_1,...,k_\alpha$ be all agents with characteristic k. Consider first the case where the order of agents is fixed in advance, and the first α-1 agents $k_1,...,k_{\alpha-1}$ have already taken their decisions. Let $\mathbf{X}(k_1,..., k_{\alpha-1})$ denote the algebra carrying the information on all the $\alpha-1$ previous decisions of $k_1,...,k_{\alpha-1}$ (in that order) which is a subset of X. *Then causality requires that the information algebra of the last decision-maker α who acts afterwards must satisfy*

$$(C_1) \qquad \mathbf{A}_{k_\alpha} \subset \mathbf{X}(k_1, ..., k_{\alpha-1}).$$

Note that if the $(\alpha + 1)$-th decision-maker acts concurrently with the α-th decision-maker, then his information algebra should also satisfy (C_1) and not be contained in $\mathbf{X}(k_1, ..., k_{\alpha-1}, k_\alpha)$, so k_α is the generic notation denoting all decision-makers who act concurrently after the first α-1 decision-makers.

According to previous considerations, the situation becomes even more complicated if we allow the possibility that the order of actions may be random. Agent α need not know who has acted before him or that he is in fact the last in line. Let us reconsider a previous example of Section 2. Suppose that we know x_1 has chosen 0, followed by x_2 resulting in a value 0 or 1 (which we know). The algebra characterizing this knowledge is certainly contained in the algebra describing the actions of x_1 and x_2, in general:

$$\mathbf{X}(x_1, x_2) = \{\emptyset, [00X_3], [01X_3], [10X_3], [11X_3], [1X_2X_3], [0X_2X_3], ..., X]\}.$$

In view of randomization x_3 could be chosen as the second agent, and then $\mathbf{X}(x_1, x_3)$ could be appropriately defined.

Now let there be a fourth decision-maker choosing x_4 which can take only one value. The information algebra $\mathbf{A}_{x_4}$ could easily be $\mathbf{A}^{(2)}$ specified in section 2 without violating our intuitive notion of causality. Yet $\mathbf{A}_{x_4} \equiv \equiv \mathbf{A}^{(2)} \not\subset \mathbf{X}(x_1, x_2)$.

This indicates that equation (**) of Section 2 has to be modified. Therefore, let

(i) r be a particular ranking of α decision-makers in a given order $(k_1, ...,k_\alpha)$.

(ii) $F^\alpha_{\alpha-1}(r) \equiv (k_1, ..., k_{\alpha-1})$ the projection (a truncated map) of r onto its

first α-1 components, being equivalent to the statement $r \in (F^{\alpha}_{\alpha-1})^{-1}(k_1, ..., k_{\alpha-1})$, where $(F^n_{\alpha})^{-1}(r)$ for $\alpha < n$ denotes all arrangements of n decision-makers acting in accordance with the ranking r.

Then $\mathbf{A}(k_1, ..., k_{\alpha-1})$ denotes the information algebra conveying the information as to what the k-group of α-1 decision-makers has done.

As a special rule, say a protocol, $\varnothing$ has to be introduced which for a given x will generate a particular arrangement of the n decision-makers. Hence $\varnothing$: $X \to S_n$ where $S_\alpha = \{(k_1, ..., k_\alpha)\}$ for $1 \leqslant \alpha \leqslant n$ is the set of possible arrangements of α players in a group k.

Then, according to Witsenhausen [13], the causality condition can be put in the following form:

(C_2) There exists a $\varnothing$ such that for $1 \leqslant \alpha \leqslant n$ and

$$r = (k_1, ..., k_\alpha) \in S_\alpha \text{ one has}$$

$$\mathbf{A}_{k_\alpha} \cap (F^{n}_{\alpha} \circ \varphi)^{-1}(r) \subset \mathbf{X}(k_1, ..., k_{\alpha-1}).$$

Property (C_2) is distinguishable from (C_1) to the extent of the additional term $(F^{n}_{\alpha} \circ \varphi)^{-1}(r)$. This term could be interpreted as the set of all x being consistent with the fact that the first α decision makers can be ordered according to r. Since the information algebra $\mathbf{A}_{k_\alpha}$ does not contain this ordering property, it is the intersection of both terms that must be contained in $\mathbf{X}(k_1, ..., k_\alpha)$.

By definition, the part of $\mathbf{A}_{k_\alpha}$ which is consistent with the actual ordering must depend only on the actions that have already taken place. If the order is fixed in advance, then all x are mopped into one given order and $(\varphi)^{-1}(F^n_\alpha)^{-1}(s) = X$; ($C_2$) reduces to ($C_1$) in this case. Witsenhausen showed that under condition (C_2) for every decision rule $\delta \in \nu$ the closed loop equations (**) can be solved in a unique way and the solution process is recursive. The problem can actually be organized as a dynamic multi-person system (extensive game) and played for every $\omega \in \Omega$.

In what follows we shall only be interested in systems where the order of decision-makers is fixed in advance. Thus, to the extent that the maximal information a decision-maker is permitted to have will only consist of actions of other decision-makers (having acted before but not concurrently with him) the causality condition of (C_1) will be satisfied.

5. STATIC AND DYNAMIC INFORMATION STRUCTURES

To develop a specific problem, let us partition the set of all agents, A, into mutually exclusive sets $A_0, A_1, ..., A_k$, i.e. $K + 1$ classes of different decision-makers. Let A_0 denote the set of nature's agents and let σ constitute their actions. The symbol x will be reserved for the actions of all agents in $A_1, ..., A_K$. We further assume β_α, $\alpha \in A_0$, the decision rule to be independent of x. Consequently, σ (equivalent to β_{A_0})* are simply random variables defined on a probability space (Ω, S, P) and they all act at once in the very beginning.

*$\beta_{A_0} : \Omega \to X_0$

Finally, to simplify matters, we consider only pure strategies for x. These assumptions constitute considerable restrictions from the very general model discussed in previous sections. However, it covers most of the interesting problems as exposed under examples 2—4 in Section 3.

Let there be α decisions possible in each A_k, $k = 1, ..., K$ and $(K+1) \cdot \alpha = N$. Define $A_1 \equiv \{\alpha \mid y_\alpha = \eta_\alpha(\sigma)\}$, i.e. those agents in A_1 whose information depends on σ only. By recursion, we could define

$$A_2 \equiv \{\alpha \mid y_\alpha = \eta_\alpha(\sigma, x_i) \ i, \in A_1\},$$

$$A_K = \{\alpha \mid y_\alpha = \eta_\alpha(\sigma, x_i), i \in \bigcup_{j=1}^{K-1} A_j, \alpha \notin \bigcup_{j=1}^{K-1} A_j\}.$$

Hence, A_k are sets of agents who for our purposes can be considered to be acting concurrently at one time instant. By causality, their information can only depend on the actions of those who act before them. Since we fixed the order in advance, this partition is not random. The information algebra of agents α in A_1, for example, can be defined as a subalgebra $\mathbf{A}_{k_\alpha}$ of $\mathbf{S}$ by virtue of η_{k_α} for $k = 1$.

For $a \in A_2$ let us consider a fixed set of decision functions β. Then we may write

$$y_\alpha = \eta_\alpha(\sigma, x_{A_1}) = \eta_\alpha(\sigma, \beta_{A_1}(\eta_{A_1}(\sigma) \equiv f_\alpha(\sigma) \text{ for all } \alpha \in A_2.$$

Thus again we may define $\mathbf{A}_{k_\alpha} \subset \mathbf{S}$ (for $k = 2$) and recursively, $\mathbf{A}_{k_\alpha} \subset \mathbf{S}$ for $k = 1, ..., K$. The sequence $\mathbf{A}_{1_\alpha}, \mathbf{A}_{2_\alpha}, ...$ forms itself a nested sequence of information sets.

Note that the information algebra as defined is dependent on the choice of β.

The causality assumption could be pictured as shown in Fig. 3 — for the case of only three agents in every group acting concurrently.

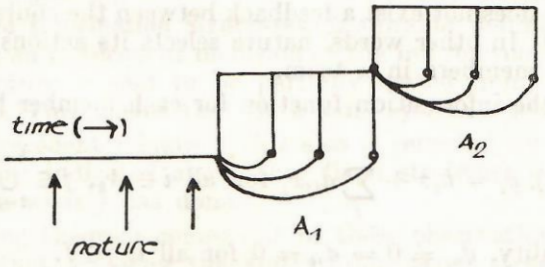


Fig. 3

Here the property of sequential nesting holds, and we have a dynamic information structure. A static information structure would be a degenerate case of this. One can easily see that Fig. 3 represents a more general situation then that in Fig. 1.

At this point it is appropriate to classify information structures into two types. An information structure is *static* if $\mathbf{A}_k$ is empty for $k > 1$; otherwise it is dynamic. In static problems it is irrelevant in what order x_a appears after receiving the signal y_a. Agents then can be regarded as acting all at once or at different points in time without changing the problem. This is in the spirit of conventional statistical decision theory where σ is regarded as the state of the world (pure strategy of nature) and where information is used to incorporate the knowledge of σ independent' of the actual actions that follow. In fact, most of team theory fits into this description and since every single decision-maker acting sequentially in time could be considered as a team where all its members act all at once given the environment it also applies to the theory of individual decision-making in a dynamic context.

The dynamic case is more representative in situations of multiperson stochastic control where a piece of information (e.g. a noisy measurement of the 'state of the dynamic system') usually depends on control decisions at earlier times.

Let us now turn to a particular dynamic team decision problem, linear in the information structures of its members but quadratic in its payoffs. Let there be $i = 1, \ldots, n$ members in the team, the probability distribution of nature's pure strategies, a random vector σ is known to all members, and, to simplify matters, is Gaussian. Information could be received in such a dynamic team in two steps. First, for every member $i = 1, \ldots, n$, the information y_i is assumed to be a linear function of σ, and second additional information is received from control actions of other members acting before but not concurrently with its own action. Such a model assumes that you cannot escape from the past to shape the future, which is well reflected in the causality condition. This model is still rather restrictive since it still assumes that members acting concurrently at a particular point in time make their decisions completely independent of each other whereas, for example, it might be better to assume that they base their actions on expectations on what other members might do (adaptation).

However, now we will leave these considerations aside. Furthermore, it should be noted that the environment will not change in the course of the actions, so there does not exist a feedback between the course of actions and the environment. In other words, nature selects its actions completely independent of the members in a team.

Thus, let the information function for each member be

$$(*)\quad y_i = h_i\sigma + \sum_j d_{ij}x_j \text{ for all } i \in A_k,\ j \in \bigcup_{l=0}^{k-1} A_l$$

where, by causality, $d_{ij} \neq 0 \Rightarrow d_{ji} = 0$ for all $i, j \in I$.

Thus, information to member i depends linearly on σ and on those x_j which act preceding to i. In particular, j is a precedent of i, written $j < i$ if $d_{ij} \neq 0$ and $<$ is transitive.

If the last term of (*) does not exist, i.e. if $d_{ij} = 0$ for all i and j we have the situation of a static team and most of the results of Marschak and Radner [10] apply.

Corresponding to (*) we would establish the payoff function to be optimized:

$$(**) \qquad \Phi(\beta_1, ..., \beta_N) = E\left[\frac{1}{2} x'Qx + x'S\sigma + x'c\right]$$

where Q is a symmetric positive definite matrix, S is a matrix and c a vector of appropriate dimension, $'$ denotes vector transposition.

We will discuss this situation in view of results obtained by Ho and Chu [7].

Let us only mention here that the above assumptions are made to derive explicit results and to clarify the role played by information structures in dynamic optimization problems. Given (*) and (**) there exists a considerable flexibility in our specification of the information structure so that all decentralized stochastic control problems of the linear-quadratic-Gaussian type can be approached in that manner. We consider three main types of η: static, partially nested, decomposed tree. The latter two are all dynamic information structures. The illustration of η can be considerably improved by the introduction of a precedence diagram which is a graphical representation of (*). In this diagram we represent each x_k, $k = 1, ..., K$ by a node so that the ordering is according to the precedence relation $<$ which is a partial ordering. It shows whose information will be affected by whose actions and so typically will have a hierarchical communication line indicating the message generating process, i.e. a lattice-type structure.

a) **Static Information Structures.** In this case we have $A_k = \Phi$ for $k \geq 2$, the precedence diagram is simply a horizontal array of unconnected nodes and the information is generated only by the states of nature, i.e. $y_i = h_i\sigma$ for all i. It then has been shown by Radner [11] (see also Radner and Marschak [10]) that the β which minimizes Φ is unique and linear in their information, i.e.

$$\beta_i = c_i y_i + d_i \text{ for all } i \in A_1,$$

where c_i and d_i can be explicitly obtained by solving a system of linear equations in a linear team.

b) **Partially Nested Information Structures.** A suitable definition of 'partially nested' can be given in terms of inclusion of the information subalgebras. An information structure is said to be partially nested (p.n.) if $j < i$ implies $\mathbf{A}_j^\beta \subset \mathbf{A}_i^\beta$ for all β. This means that in a p.n. system agent i will always know what his precedent j knows. This also is reflected by the hierarchical communication line indicated above. For fixed strategies β agent i will be able to determine what j has done.

The following theorem comes out of these observations. The theorem essentially says that knowing the static information structure and given the causality condition in terms of a precedence diagram it is possible to compute from it the information resulting from a dynamic team.

Theorem. In a dynamic team with p.n. information structure

$$y_i = h_i\sigma + \sum_j d_{ij}x_j \text{ all } j \text{ precede } i \ (j < i)$$

is equivalent to a static information structure of the form

$$\hat{y}_i = h_i\sigma \text{ for any admissible } \beta.$$

Sketch of Proof. The proof could be based on the property of recursive computability in a dynamic team. For $i \in A_1$ we actually have $y_i = \hat{y}_i$. By induction, or $i \in A_2$, $j \in A_1$;

$$(*) \qquad \hat{y}_i = y_i - \sum d_{ij}\beta(\hat{y}_j).$$

Now, by the p.n. property, knowing y_i implies knowing y_i so that one particular y_j may be indentified with $\hat{y}_i$, so $\hat{y}_i$ is computable from y_i and viceversa, we have

$$(**) \qquad y_i = \hat{y}_i + \sum d_{ij}\ \beta(\hat{y}_j) \text{ for } i \in A_2,\ j \in A_1.$$

This shows, in this particular case, that y_i and $\hat{y}_i$ are equivalent. This process can be extended immediately to include the sets $A_3 ..., A_K$.

On the basis of this Radner's theorem can be extended to a dynamic team. Let us summarize the result: In a p.n. linear-quadratic-Gaussian dynamic team, the optimal β_i is unique and linear in y_i.

Some special cases have been discussed in [7]. Note that there is no assumption on the independence of any of the random variables.

The equivalence of dynamic η with static η for p.n. systems is not merely restricted to the case of the recurrence relation $y_i = h_i\sigma + \sum_{j<i} d_{ij}x_j$. Slightly more general results are possible [7].

Whenever such equivalence can be established (in case of mutually nested information structures), known results on existence and uniqueness of stochastic control in a static team (Marschak and Radner [10]). can be extended to the dynamic case without difficulty.

c) **Decomposable Tree Structures.** The requirement of p.n. information structure can be further relaxed if additional assumptions on the payoff functions and on the distribution of σ are permitted.

Assume the precedence diagram gives rise to a system of directed trees, an exposed in Fig. 1, i.e. there are no loops and all links branch downward. Furthermore, suppose the payoff function is decomposable into a sum of quadratic payoffs, e.g.

$$\Phi = \sum_{k=1}^{K} \Phi^k = \sum_{k=1}^{K} E\left\{\frac{1}{2}\, x'^k Q^k x^k + x'^k S^k \sigma^k + x^k c^k\right\}$$

where $x^k = [x_1^k, ..., x_a^k]$ lists all members (out of different static teams) on a given branch of the directed team and K is the total number of branches.

Actually here we have a divisionalized structure as in the case of utility trees [5] and the payoff function is additively separable.

Note that some agent may belong to two branches.

Finally third, suppose the random effects of each branch are independent, i.e.

$$p(\sigma^1, \sigma^2, ..., \sigma^k) = p(\sigma^1), p(\sigma^2), ..., p\ (\sigma^k).$$

This means that the probability space $(\Omega, \mathbf{S}, P)$ can be decomposed into

$$\Omega = \prod_{k=1}^{K} \Omega_k, \; \mathbf{S} = \prod_{k=1}^{K} \mathbf{S}_k, \; P = \prod_{i=1}^{K} P_k.$$

Under these somewhat more restrictive asumptions we can model the case where a person has to control a series of independent tasks with eventually overlapping areas of control. The last control action of one task may give rise to several different control actions of different tasks as reflected in the precedence of the tree structure. Furthermore, because of separability each time sequence of tasks is measured according to a separate performance index (payoff function).

Another example is that of a divisionalized firm as that described by T. Groves [6], as an organization consisting of several partially autonomous units linked only through a central administration. The administration's head only initiates policies in each division which then carries them out according to the local environment which is only known to the particular division.

Principally, such problems can be solved as a series of K classical linear-quadratic-Gaussian problems. Let us now state a condition to verify this: Let $\mathbf{A}_i^\beta$ be the information (sub) algebra of agent i induced on $\mathbf{S}$ for fixed β. Define then $\mathbf{B}_i^\beta$ as the algebra induced on $\mathbf{S}$ by $x_i = \beta_i(y_i)$. It is evident that $\mathbf{B}_i^\beta \subset \mathbf{A}_i^\beta$ since x_i is a contraction of y_i. Then we state the fundamental property:
(*) $i(j) < i(l) \Rightarrow (\mathbf{A}_{i(j)}^\beta \cap \mathbf{S}_i) \cup \mathbf{B}_{i(j)}^\beta \subset \mathbf{A}_{i(l)}^\beta$ for all β states that along any branch of the tree agent $i(l)$ always knows 1) what his predecessors in the same branch have done (e.g. $\mathbf{B}_{i(j)}^\beta$, $i(j) < i(l)$ but not necessarily what they know ($\mathbf{A}_{i(j)}^\beta$, $i(j) < i(l)$). 2) what his predecessors know about the random variables σ_i which affect the payoff Φ^i (e.g. $\mathbf{A}_{i(j)}^\beta \cap \mathbf{S}_i$).

Now let us consider a specific case for illustration purposes. Consider two branches on the precedence tree in the following figure.

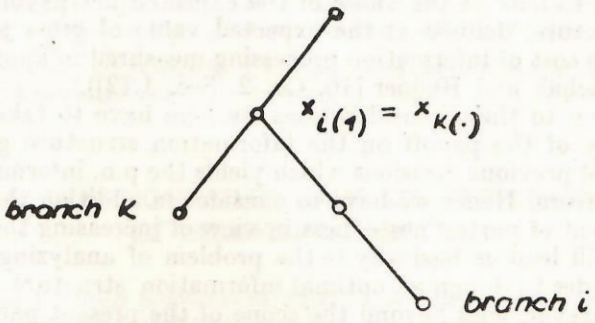


Fig. 4

All decision-makers on branch i, say, have access to two kinds of information. First it is information concerning other random variables σ^j, $j \neq i$, not affecting i. These informations are not *payoff-relevant* for optimizing Φ^i, and there-

fore will not be considered. Second, the other kind of information is given by 1) and 2) above. By the separability property of the payoff function $\emptyset$, optimizing $\emptyset$ is equivalent to optimizing $\emptyset^i$, $i \in K$ for all decision-makers in a particular branch except for the starting agent $i(1)$ (see Fig.) with $x_{i(1)} = x_k(\cdot)$ since this is a point of overlap with branch k. Now the problem of optimizing Φ^i for $x_i(2), x_i(3), \ldots, x_{i(m)}$ with 1) and 2) represents a straightforward linear-quadratic-Gaussian stochastic control problem or a team problem with p.n. information structure [in which all agents are totally ordered].

According to the results obtained by Striebel [12] it is known that knowledge of $\mathbf{B}^{\beta}_{i(j)}$ and not $A^{\beta}_{i(j)}$ is *sufficient* for the solution via backward induction or by the solution method leading immediately to the previous theorem. We can show that all optimal strategies are linear, and in particular linear in $x_i(1) = x_{k(.)}$. This means, by separability, that we could establish for the next step a new payoff function $\Phi^i + \Phi^k = \Phi^{(i,k)}$ which is now a quadratic function of the starting point $x_{k(1)}$. Going backwards on successive branches in the same fashion we could reduce the precedence tree to an array of nodes as in a static team and the payoff $\emptyset$ to a quadratic function of the action variables associated with these nodes only. The reduced problem then is solved by Radner's theorem resulting in linear strategies once again for these starting decision-makers. Thus, this leads us immediately to the statement of the following theorem.

Theorem. In a dynamic team with information structure as given by (*) the optimal strategies for all decision-makers are linear.

FINAL REMARKS

In the context of team decision problems, the evaluation of information structures is linked up to its payoff relevance. However, as noted by Marschak and Radner [10], the information structure that results in the highest expected payoff may involve such high costs of information processing that it makes some other information structure with lower costs more preferable. In other words, one has to look at the value of the expected net payoff given the information structure, defined as the expected value of gross payoff minus a term, called the cost of information processing measured in appropriate utility units (see Marschak and Radner [10, Ch. 2, Sec. 1.12]).

In addition to these considerations we here have to take into account the dependency of the payoff on the information structure generated by a successive list of previous decisions which yields the p.n. information structure for a dynamic team. Hence we have to consider in addition the length of the tree or the extent of partial nestedness in view of increasing the expected net payoff. This will lead us basically to the problem of analyzing optimal stopping rules in order to design an optimal information structure for a dynamic team. This, however, goes beyond the scope of the present paper.

Some interesting conclusions could be drawn about the economies of information in planning and forecasting. The view adopted here to represent economic organizations in a group dynamic framework emphasizes the need of a rational sequential allocation of information in the system so that information about previous decisions are made available to decision-makers acting

in the future (indicative planning), for example to anticipate shortages, bottlenecks or, in general, crisis situations.

Sequential reshuffling of information by controlling agencies takes account of the observation that decision-makers tend to memorize only partially 'past history' (which would affect their possible decisions). Increased delays in decision-making (due to partial ignorance) or overshot reactions (due to non-optimal stopping), all of this would result in destabilizing conditions in the system.

REFERENCES

1. DALKEY N.: *Equivalence of Information Patterns and Essentially Determinate Games.* Contr. Theory of Games II, 193—216.
2. FUCHS L.: *Partially Ordered Algebras I.* Coll. Math. 14, 1966, 115—130.
3. GOTTINGER H. W.: *Qualitative Information and Comparative Informativeness.* Kybernetik 13, 1973, 81—94.
4. GOTTINGER H. W.: *Computable Organizations: Representation by Sequential Machine Theory.* Electronics Research Lab., University of California, Berkeley, Memo. 426, March 1974, to appear in Annals of Systems Research, 1974.
5. GOTTINGER H. W.: *Beiträge zur funktionalen Separabilität bei Nutzenfunktionen I.* Zeitschr. ges. Staatswissenschaft 125, 1969, 406—436.
6. GROVES T.: *Market Information and the Allocation of Resources in a Dynamic Team Model.* Social Syst. Research Institute, University of Wisconsin (Madison), Report 7210, 1972.
7. HO Y. C. and CHU K. C.: *Team Decision Theory and Information Structures in Optimal Control Problems, I and II.* IEEE Trans. Automatic Control, AC-17, 15—22 and 22—28.
8. KUHN H. W.: *Extensive Games and the Problem of Information,* Contr. Theory of Games II, 193—216.
9. MARSCHAK J.: *Economics of Information Systems.* Journal American Statistics Association, 66, 1971, 192—219.
10. MARSCHAK J. and RADNER R.: *Economic Theory of Teams.* Yale University Press New Haven, 1972.
11. RADNER R.: *Team Decision Problems.* Ann. Math. Statist. 33, 1962, 857—881.
12. STRIEBEL C.: *Sufficient Statistics in the Optimum Control of Stochastic Systems.* Journal Math. Analysis and Applications 12, 1965, 576—592.
13. WITSENHAUSEN H.: *Information Structures, Feedback and Causality.* SIAM Journal Control 9, 1971, 149—160.
14. WITSENHAUSEN H.: *On Information Structures of Games with Continuous Variables and a Finite Number of Decisions.* Proc. First International Conf. Theory and Applic. Diff. Games, Amherst, 1969, I—I—I—7.
15. HARTMANIS J. and STEARNS R. E.: *Algebraic Structure Theory of Sequential Machines.* Prentice-Hall, Englewood Cliffs, N. J., 1966.

STRUCTURES INFORMATIONNELLES DANS LES PROBLEMES DYNAMIQUES DE DECISION COLLECTIVE

Pour aborder les divers problèmes de direction, et spécialement en économie, on prend presque toujours comme point de départ l'hypothèse que l'information mise à la disposition du noyau de direction est centralisée. Pourtant, lorsqu'on travaille réellement avec les problèmes de décision, qui impliquent plusieurs personnes, problèmes qui existent dans les systèmes ou les organisations économiques décentralisés, cette hypothèse s'avère difficilement acceptabe. Si par surcroît, il existe des complications supplémentaires, et notamment si l'information est dynamique (dynamique de groupe), c'est-à-dire qu'elle peut être dépendante des actions passées des autres membres (décidents) appartenant au groupe, de nouveaux problèmes de concepion

apparaissent, qui affecteront de façon différente la théorie de la direction décentralisée et la théorie des jeux. Ces problèmes sont apparus sous une certaine forme lorsque les théoriciens des jeux ont abordé les jeux du type extensif, mais jusqu'à présent ils ont été traités sans succès. Récemment, les chercheurs des groupes restreints, faisant la critique de la théorie de la réaction aux stimulents stochastiques, ont mis en évidence l'importance des flux informationnels engendrés par les actions des autres membres d'un groupe.

Le présent ouvrage tend à aborder ces problèmes dans un mode formel et à généraliser certains résultats obtenus par Marschak et Radner pour les problèmes dynamiques de décision collective.

ИНФОРМАЦИОННЫЕ СТРУКТУРЫ В ДИНАМИЧЕСКИХ ЗАДАЧАХ ПРИНЯТИЯ КОЛЛЕКТИВНОГО РЕШЕНИЯ

При рассмотрении различных вопросов руководства, в особенности в экономике почти всегда исходят из предположения, что информация, предоставленная в распоряжение руководства, является централизованной. Всеже, когда фактически работают по вопросам принятия решений в отношении многих людей, как, например, по вопросам, возникающим в децентрализованных экономических системах или организациях, подобное предположение трудно принять. Помимо этого, если имеются и дополнительные осложнения, а именно информация является динамической (в групповом отношении динамической), то-есть могла бы зависить от прошлых действий других (принимающих решение) членов группы, возникают новые проблемы концепции, отражающиеся различным образом на теории децентрализованного руководства и теории игр. В определенной форме эти проблемы возникли, когда теоретики игр приступили к рассмотрению игр экстенсивного типа, но до сих пор их рассмотрение не было успешным. Недавно исследователи небольших групп, критикуя теорию реакции на стохастические стимулы, выявили важность информационных потоков, порождаемых действиями других членов группы.

В настоящей статье делается попытка приступить к рассмотреноию этих проблем формальным образом и обобщить определенные результаты, полученные Маршаком и Раднером по динамическим задачам принятия коллективного решения.