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An Open Dynamic Model of Inter-Branch Connections

The author presents the variant of a dynamic model of inter-branch connections which he proposes for the Romanian economy. The presented model is a simulation model.

The main aim of the presented model is to allow the simulation of the various trajectories of development of the branches of the national economy with respect to various hypotheses made on the various components of the net final consumption.

By net final consumption we mean the final consumption from which the volume of productive investments is subtracted.

It results that we shall introduce into the model for each year of the planning or forecasting horizon, as exogenous elements calculated on the basis of other models, the following elements:

— the volume and structure of consumption

— the volume and structure of export;

— the volume and structure of other elements of the net final consumption.

In the case of the optimization model, a number of the above elements will not be given exogenously but will result from the solving of the model.

WORKING HYPOTHESES

In the following, we shall present the main working hypotheses which are at the basis of the model. Some of the hypotheses will be presented in several variants which will generate several variants of the model.

Hypothesis 1: the distribution equation

Considering that the model is an open model which takes into account the foreign trade activity, the following variants can be taken into consideration for the distribution equation:

1.1. Classic equation

$$X_t = A_t X_t + \hat{Y}_t - \hat{M}_t + \hat{E}_t \tag{1}$$

where we noted

$A_t = \| a_{ij}^t \|_{n \times n}$ — the matrix of specific consumptions from home production and from import in year t;

$X_t = \| X_j^t \|_{n \times 1}$ — the vector of production in year t;

$\hat{Y}_t = \| Y_j^t \|_{n \times 1}$ — the vector of final consumption (without export) in year t (from home production and import);

$\hat{M}_t = \| M_j^t \|_{n \times 1}$ — the vector of imports in year t;

$\hat{E}_t = \| E_j^t \|_{n \times 1}$ — the vector of exports in year t;

1.2. The distribution equation of home production:

$$X_t = A_t^{int} \cdot X_t + \hat{Y}_t^{int} + \hat{E}_t \tag{2}$$

where the notations are the same as above, and

$A_t^{int} = \| a_{ij}^{t,\,int} \|_{n \times n}$ — is the matrix of specific consumptions from home production in year t;

$\hat{Y}_t^{int}$ — the vector of final consumption from home production in year t;

Variant 1.1. has the advantage that it applies to the matrix of total specific consumptions (both from home production and from imports) which, as is known, has a greater stability that the matrix of partial specific consumptions (from home production or from imports).

Another advantage of the variant 1.1. is that it is closer to the way of planning practised.

In our opinion, these advantages are real only in case that the vector of imports for each year of the plan period is given exogenously as part of the model.

But, in this case the model is not longer an open model, but a closed one, because the outfit from imports for production, for investment and for consumption is calculated somewhere outside the model.

If we want the model to be an open model indeed, we must introduce into it the following relations as well (in variant 1.1.):

$$M_i^t \geqslant \sum_{j=1}^{u} a_{ij}^{t,\,imp} \cdot X_j^t + \sum_{j=1}^{n} \eta_{ij}^t \cdot \bar{I}_i^j + \eta_j^{t,\,c} \cdot C_t \tag{3}$$

$$i = 1,\ 2,\ \ldots,\ n$$

where we noted with:

$A_t^{imp} = \| a_{ij}^{t,\,imp} \|_{n \times n}$ — the matrix of specific consumptions from imports in year t;

$\bar{I}_t^j$ — the volume of investments from home production put into operation in year t in branch j;

$\hat{\eta}_t = \| \eta_{ij}^t \|_{n \times n}$ — the matrix of "specific investments" from imports in year t.

The coefficient η_{ij}^t shows the share of investments from imports necessary in year t in branch j from the profile of branch i for each unit of fixed assets put into operation from the home production.

$\hat{\eta}_t^c = \| \eta_j^{t,c} \|_{n \times 1}$ — the vector of consumption from imports in year t:

$\eta_j^{t,c}$ — shows the share of individual and social consumption which is necessary to be imported into branch j for each consumption unit given from the home production;

C_t — the volume of consumption in year t(scaler).

The right side of relation (3) shows the non-competitive imported outfit in year t and relation (3) shows the fact that the imports to be made in branch i secure the necessary outfit from imports for productive consumption, for investments and for consumption in this branch.

In case that the hypothesis is adopted that only uncompetitive import is made, inequality (3) becomes equality and if in the distribution equation (1) we replace the expression of imports given by relation (3), we obtain:

$$X_t = A_t \cdot X_t + \hat{Y}_t - (A_t^{imp} X_t + \hat{\eta}_t \cdot \bar{I}_t + \hat{\eta}_t^c C_t) + \hat{E}_t. \tag{4}$$

Considering that:

$$A_t^{int} = A_t - A_t^{imp}$$

relation (4) is also written:

$$X_t = A_t^{int} X_t + \hat{Y}_t - \hat{\eta}_t \hat{I}_t - \hat{\eta}_t^c C_t + \hat{E}_t. \tag{4'}$$

As can be seen, by setting out from the "classical" distribution equation (1) and making the hypothesis that only uncompetitive import is made (only those products are being imported which cannot be turned out in this country), we get to relation (4') which is quite similar to the distribution equation (2). The difference between the distribution equation (4') and (2) consists in the fact that the former outlines the import for investments and for consumption in each branch.

But, this is made at the cost of additionally calculating the matrices of "specific investments" from imports $\hat{\eta}_t$ and of the vectors of consumption from imports η_t^c.

Considering that investments and consumption from imports go directly to the beneficiaries without generally, passing through the productive system, we consider that the outlining of these sizes in the specific branches is not strictly necessary.

Hypothesis 2: *balance of foreing trade*:

$$V_t = E_t - \sum_{j=1}^{n} m_j^t \cdot X_j^t - \sum_{j=1}^{n} \eta_j^t \cdot \bar{I}_t^j - \eta_t^c \cdot C_t \tag{5}$$

where E_t is the total volume of export in year t, m_j^t is the outfit of raw and related materials from imports per production unit of branch j in year t,

η_j^t is the outfit of investments from imports per unit of fixed assets put into operation in branch j in year t, and η_t^c is the outfit from imports in year t per consumption unit from home production.

In case that we calculate matrices A_t^{imp}, $\hat{\eta}_t$ and vector $\hat{\eta}_t^c$, we have:

$$m_j^t = \sum_{i=1}^{n} a_{ij}^{t\,imp}; \quad \eta_j^t = \sum_{i=1}^{n} \eta_{ij}^t; \quad \eta_t^c = \sum_{i=1}^{n} \eta_{it}^{t\,c}.$$

We mention that in case that the distribution equation (1) is not adopted and thus, matrices A_t^{imp}, $\hat{\eta}_t$ and vector $\hat{\eta}_t^c$ are not calculated, the aggregate sizes m_j^t, η_j^t and η_t^c can also be calculated directly.

The direct estimation of these sizes has the advantage that on the one hand, it requires a smaller volume of work and, on the other hand, it leads to more exact results. This assertion is based on the fact that, in our opinion, the direct estimation of the outfit of investments from imports per unit of fixed assets put into operation in year t in branch j may lead to more exact results than the estimation of a similiar coefficient for each specific branch (coefficients η_{ij}^t).

Taking into consideration those shown above, the further presented model refers to ratio (2).

Hypothesis 3: the evolution equation of fixed assets

$$K_{t+1}^j = K_t^j - \mu_t^j K_t^j + \bar{I}_t^j + \eta_t^j \cdot \bar{I}_t^j \tag{6}$$

where we noted with:

K_t^j — the volume of fixed assets at the beginning of year t in branch j;
μ_t^j — the coefficient of disuse following physical and moral wear.

As can be seen, equation (6) shows that the volume of fixed assets in branch j at the beginning of year $t + 1$ is equal to that existing at the beginning of the previous year from which the volume of annulments is subtracted and the volume of investments both from home production and from imports is added.

Relation (6) is also written:

$$K_{t+1}^j = (1 - \mu_t^j) \cdot K_t^j + (1 + \eta_t^j) \cdot \bar{I}_t^j. \tag{6'}$$

Hypothesis 4: equation of investments

The volume of investments from home production put into operation in branch j in year t is:

$$\bar{I}_t^j = \sum_{i=1}^{n} \sum_{\tau=t-\theta_j}^{t} \bar{I}_{\tau,t}^{i,j} \tag{7}$$

where we noted with:

$I_{\tau,t}^{i,j}$ — the volume of investments made from the production of year τ of branch i for capacities to be put into operation in branch j in year $t (t \geqslant \tau)$
θ_j — the maximum "lag" period of branch j.

We shall introduce the following matrices of the specific investments:

$$b_{ij}^{\tau,t} = \frac{I_{\tau,t}^{i,j}}{\bar{I}_t^j} \tag{8}$$

$$B_{\tau,t} = \| b_{ij}^{\tau,t} \|_{n \times n}. \tag{9}$$

Size $b_{ij}^{\tau,t}$ shows the share of investments to be made from the production of year τ of branch i for capacities to be put into operation in branch j in year $t (t \geqslant \tau)$ in the total volume of the capacities to be put into operation in year t in branch j.

Obviously:

$$\sum_{i=1}^{n} \sum_{\tau=t-\theta_j}^{t} b_{ij}^{\tau,t} = 1. \tag{10}$$

The balance equation of investments made from the production of year t of branch i is:

$$I_t^i = \sum_{j=1}^{n} \sum_{\tau=t}^{t+\theta_j} I_{t,\tau}^{i,j} = \sum_{j=1}^{n} \sum_{\tau=t}^{t+\theta_j} b_{ij}^{t,\tau} \bar{I}_\tau^j. \tag{11}$$

It results from equation (6')

$$\bar{I}_t^j = \frac{1}{1+\eta_t^j} [K_{t+1}^j - (1-\mu_t^j) K_t^j]. \tag{12}$$

Considering (12), the balance equation of investments is written:

$$I_t^i = \sum_{j=1}^{n} \sum_{\tau=t}^{t+\theta_j} \frac{b_{i,j}^{t,\tau}}{1+\eta_\tau^j} [K_{\tau+1}^j - (1-\mu_\tau^j) K_\tau^j]. \tag{11'}$$

Noting with:

$$\gamma_{ij}^{t,\tau} = \frac{b_{ij}^{t,\tau}}{1+\eta_\tau^j} \quad \text{and} \quad g_t^j = 1 - \mu_t^j$$

we can write:

$$I_t^i = \sum_{j=1}^{n} \sum_{\tau=\cdot}^{t+\theta_j} \gamma_{ij}^{t,\tau} [K_{\tau+1}^j - g_\cdot^j K_\cdot^j]. \tag{12'}$$

Remarks:

1. In case that we do not consider the "lags", for each year we shall have a single matrix of specific investments

$$B_t = \| b_{ij}^{t-1,t} \|_{n \times n} \quad \text{and} \quad b_{ij}^{\tau,t} = 0 \quad \text{for } \tau < t-1$$

$$\text{or } B_t = \| b_{ij}^{t,t} \|_{n \times n} \quad \text{and} \quad b_{ij}^{\tau,t} = 0 \quad \text{for } \tau < t.$$

2. In case that we suppose that the "lags" depend only on the beneficiary investment branch, acting uniformly on all components in various branches, we have:

$$b_{ij}^{\tau,t} = \lambda_j^{\tau,t} \cdot b_{ij}^t \tag{13}$$

where $\lambda_j^{\tau,t}$ are the lag coefficients.

Thus, in this case, every year we need a single matrix of specific investments whose elements are calculated as follows:

$$b_{ij}^t = \frac{\sum_{\tau=t-\theta_j}^{t} I_{\tau,t}^{i,j}}{\bar{I}_t^j}$$

and they represent the share of investments from the production of branch i of the total investments put into operation in year t in branch j, and $\lambda_j^{\tau,t}$ is calculated as follows:

$$\lambda_j^{\tau,t} = \frac{\bar{I}_{\tau,j}^j}{\bar{I}_t^j}$$

where we noted with $\bar{I}_{\tau,t}^j$ the volume of investments made from the home production of year τ for capacities to be put into operation in branch j in year $t(t \geqslant \tau)$.

Hypothesis 5: structure of the final consumption

$$\hat{Y}_t^{int} = \hat{I}_t + \hat{C}_t + \hat{R}_t \tag{14}$$

where:

$\hat{I}_t = \| I_t^i \|_{n\times 1}$; I_t^i representing the volume of investments obtained from the production of year t of branch i;

$\hat{C}_t = \| C_t^i \|_{n\times 1}$; C_t^i representing the volume of consumption obtained from the production of year t of branch i;

$\hat{R}_t = \| R_t^i \|_{n\times 1}$; R_t^i — the contribution of branch i to other components of the final consumption.

We shall suppose that the structure vectors of consumption and of the rest of the final consumption are given for each year. In this case, noting with C_t (scaler) the volume of consumption in year t and with R_t (scaler) the volume of the rest of the final consumption, we have:

$$C_t^i = \alpha_t^i C_t; \qquad R_t^i = \beta_t^i R_t. \tag{15}$$

Obviously:

$$\sum_{i=1}^{n} \alpha_t^i = 1 \qquad \text{and} \qquad \sum_{i=1}^{n'} \beta_t^i = 1.$$

We have:

$$Y_t^{i,int} = I_t^i + \alpha_t^i C_t + \beta_t^i R_t, \qquad i = 1, 2, \ldots n. \tag{14'}$$

Hypothesis 6: export

We shall suppose that the structure vector and the volume of export is given exogenously for each year. Thus:

$$E_t^i = \delta_t^i \Sigma_t \tag{16}$$

where E_t is the volume of export given exogenously. We have:

$$\sum_{i=1}^{n} \delta_t^i = 1.$$

Hypothesis 7: constraints of the model

Within the model we shall introduce the following three types of constraints:

a. constraints on the production capacities;

b. constraints on labour force;

c. constraints on the trade balance of foreign payments.

To write the constraints on the production capacities we shall note with f_j^t the necessary fixed assets per product unit in branch j in year t. We have:

$$f_j^t \cdot X_j^t \leqslant \bar{K}_j^t, \qquad j = 1, 2 \ldots n \tag{17}$$

where $\bar{K}_j^t$ is the average (annual) level of fixed assets in branch j in year t.

We shall consider that

$$\bar{K}_j^t = K_j^t + \lambda_t^j [K_{t+1}^j - K_t^j] \tag{18}$$

where coefficient λ_t^j shows the rate of the putting into operation throughout the year of the new capacities in branch j (in case of a uniform rate $\lambda_t^j = 0.5$).

Noting with l_j^t the necessary labour force per product unit in branch j in year t, the labour force constraints is:

$$\sum_{j=1}^{n} l_j^t \cdot X_j^t \leqslant L_t \tag{19}$$

where we noted with L_t the total volume of labour force available in year t.

To write the constraint on the trade balance of foreign payments we shall replace in relation (5) $\bar{I}_t^j$ by the expression given by relation (12). We have:

$$E_t - \sum_{j=1}^{n} m_j^t \cdot X_j^t - \sum_{j=1}^{n} \frac{\eta_j^t}{1 + \eta_j^t} [K_{t+1}^j - g_t^j K_t^j] - \eta_t^c \cdot C_t \geqslant S_t \tag{20}$$

where we noted with S_t the planned balance of the trade balance of foreign payments for year t.

Remark

In certain variants for solving the model, we may give up constraint (19) or/and (20).

Considering equations (2), (14') and (16) we can write:

$$X_i^t = \sum_{j=1}^{n} Q_{ij}^{t,int} \cdot X_j^t + I_t^i + \alpha_t^i \cdot C_t + \beta_t^i \cdot R_t + \delta_t^i \cdot E_t.$$

Further, we shall replace I_t^i by its expression given by relation (12) and will take into consideration constraints (17), (19) and (20). Under these conditions, the model is:

$$\begin{cases} X_i^t = \sum_{j=1}^{n} Q_{ij}^{t,int} \cdot X_j^t + \sum_{j=1}^{n} \sum_{\tau=t}^{t,\theta_j} \gamma_{ij}^{t,\tau}[K_{\tau+1}^j - g_\tau^j K_\tau^j] + \alpha_t^i \cdot C_t + \beta_t^i R_t + \delta_t^i \cdot E_t^i \\ \qquad i = 1, 2, ..., n; \quad t = 0, 1, 2, ..., T_i \\ f_j^t \cdot X_j^t \leqslant K_j^t + \lambda_t^j [K_{t+1}^j - K_t^j] \\ j = 1, 2, ..., n; \quad t = 0, 1, 2, ..., T; \\ \sum_{j=1}^{n} l_j^t \cdot X_j^t \leqslant L_t, \quad t = 0, 1, ..., T; \\ E_t - \sum_{j=1}^{n} n_j^t X_j^t = \sum_{j=1}^{n} \frac{\eta_j^t}{1+\eta_j^t} [K_{t+1}^j - g_t^j \cdot K_t^j] - \eta_t^c \cdot C_t \geqslant S_t, \; t = 0, 1, 2, ..., T; \\ X_j^t \geqslant 0; \quad K_t^j \geqslant 0. \end{cases}$$

To write the model in a matrix form, we shall use the already introduced notations and in addition we note with:

$$\Gamma_{t,\tau} = \| \gamma_{ij}^{t,\tau} \|_{n \times n}; \qquad K_t = \| K_t^j \|_{n \times 1};$$
$$\alpha_t = \| \alpha_t^i \|_{n \times 1}; \qquad \beta_t = \| \beta_t^i \|_{n \times 1};$$
$$\delta_t = \| \delta_t^i \|_{n \times 1}; \qquad m_t = \| m_t^j \|_{1 \times n}$$
$$\eta_t = \left\| \frac{\eta_t^j}{1+\eta_t^j} \right\|_{1 \times n}; \quad l_t = \| l_j^t \|_{1 \times n}.$$

We also consider the following diagonal matrices:

$$\| g_t^j \|_{n \times n} = G_t; \qquad \| f_j^t \|_{n \times n} = F_t; \qquad \| \lambda_t^j \|_{n \times n} = \Lambda_t.$$

With this, the dynamic model of inter-branch connections is written:

$$\begin{cases} X_t = A_t^{int} \cdot X_t + \sum_{\tau=t}^{t+\theta} \Gamma_{t,\tau}[K_{\tau+1} - G_\tau K_\tau] + \alpha_t C_t + \beta_t R_t + \delta_t E_t \\ F_t \cdot X_t \leqslant K_t + \Lambda_t [K_{t+1} - K_t] \\ l_t \cdot X_t \leqslant L_t \\ E_t - m_t X_t - \eta_t [K_{t+1} - G_t k_t] - \eta_t^c \cdot C_t \geqslant S_t \\ X_t \geqslant 0; \quad K_t \geqslant 0 \\ t = 0, 1, 2, ..., T. \end{cases}$$

DATA NECESSARY TO THE SIMULATION MODEL

For simulating the various plan or prognosis variants, on the basis of the above model, the following data are necessary for each year of the considered time period:

1. Volume and structure of:
— consumption (C_t and α_t);
— export (E_t and δ_t);
— of other elements of the final consumption (R_t and β_t).
2. The matrix of coefficients of the specific consumptions from home production: A_t^{int}.
3. The matrix of specific investments: $B_{\tau,\,t}$.
4. The diagonal-matrix of the necessary fixed assets per product unit: F_t.
5. The line vector of the necessary labour force per product unit: l_t.
6. The volume of existing labour force (available): L_t.
7. The diagonal matrix of the rate of putting into operation the new capacities: Λ_t.
8. Coefficients of annulment on branches: μ_j^t.
9. Specific coefficients of import (on branches):
— for productive consumption: m_j^t
— for investments: η_t^j
— for consumption: η_t^c
10. The planned balance of the trade balance of foreign payments: S_t
11. The original and final data for vector K_t of fixed assets.

MODÈLE DYNAMIQUE DES CONNECTIONS INTERBRANCHES

L'auteur présente une variante de modèle dynamique des relations interbranches, qu'il propose pour l'économie roumaine.
Le modèle présenté est un modèle de simulation.

ОТКРЫТАЯ ДИНАМИЧЕСКАЯ МОДЕЛЬ МЕЖОТРАСЛЕВОГО БАЛАНСА

Автор настоящей статьи представляет вариант динамической модели межотраслевых связей, которую он предлагает для румынской экономики.
Представляемая модель является моделью симулирования.