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Application of Some Mathematical Methods to the Choice of Publicity Means and to Determine the Efficiency of Commercial Advertising

In the conditions of a general acknowledgement of the necessity for commercial advertising, the author sets the problem of determining its efficiency with the utmost accuracy.

At the IX-th Congress of the Romanian Communist Party, Nicolae Ceauşescu, general secretary of the Central Committee of the R.C.P., stressed the importance of the "harmonious blending of basic and applied research as a condition to increase the efficiency of scientific work".*

This truth is applicable to any research, even if it refers to economic activity of a somewhat auxiliary character, but which must, however, also stand out by undisputed efficiency. Commercial advertising is included in such accessory economic activity.

The usefulness and quite often the necessity of commercial advertising have been generally recognized for the conditions existing in our country. However, the possibility of carrying it out on basis of strictly scientific investigations has been studied only to a small extent. We shall therefore endeavour hereafter to blend basic and applied investigation in order to establish, on basis of the various newer sources standing at our disposal, what the possibilities are, of choosing the optimum means of publicity and also the methods of determining the efficiency of commercial advertising.

If, in view of promoting the sale of product A in the broadest strata of the population — given, for example, the social sanitary function of the

* *IX-th Congress of the Romanian Communist Party*, 19—24 July 1965, p. 61, Editura Politică, Bucharest, 1965.

respective product — the decision is reached to promote an intensification of publicity and certain sums are allotted to this end, e.g. 450,000 lei, the first problem which arises is that of choosing the means, respectively the optimum publicity vehicle in view of attaining the desired result.

The cinema, radio broadcasting, television, national newspapers, local newspapers, weeklies, monthly and quarterly magazines, illuminated signs, shop windows, placards, bill-posting, etc. make up a vast series of such publicity vehicles meant to convey the "publicity message".

To make use of all the existing publicity means would entail in the first place a breaking up of the allotted fund and would therefore be equivalent to an unprofitable waste of the fund conventionnally established by us at 450,000 lei.

Therefore, it now becomes an essential problem to choose the most appropriate publicity means, those having the highest prospects of efficiency; we must now choose one of the hundreds of existing vehicles or else find an optimum combination of vehicles. These combinations may however rise in numbers up to tens of thousands, if not even hundreds of thousands of variants.

Since we are dealing with an entire set of possibilities, of choices as well as with some restrictions, as for instance the limiting of the fund to 450,000 lei, the mathematical technique, known under the name of linear programming, seems to be the proper one in view of obtaining an approximation in this problem of selecting the publicity vehicle.

So far, in 1961, Dr. Learner [1] has already endeavoured to apply linear programming in this sector of activity, with fairly good results.

As it is well known, the main elements of linear programming are the "objective function" and the "restrictions" within whose limits the objective function is to be solved for a maximum. Thus, if, for instance, the aim is to find the maximum level of sale, and this level is the sale per product, P_1 for product *1* and P_2 for product *2*, then the objective function will be: total sale $= P_1 + P_2$.

It is generally known that in questions of publicity both the presentation of the publicity messages and the auditors, respectively the public reached by these messages, are important.

Consequently, the total value of a publicity programme is a function of the number of time or space units used in the available publicity vehicles or means.

If only two publicity vehicles were to be considered and if e_1 and e_2 represent the publicity value of one unit in vehicle *1* and, respectively, in vehicle *2*, then the objective function would be:

Total publicity value $= e_1 + e_2$.

However, restrictions must also be taken into account when solving the problem. These restrictions are expressed by unequalities. Thus, if n_1 and n_2 represent the number of units (pages of advertising or minutes of sound publicity, etc.) then the restrictions would, for example, require that:

$$n_1 \leqslant 12$$
$$n_2 \leqslant 52$$
$$n_1 \geqslant 0$$
$$n_2 \geqslant 0$$

which means that the first restriction requires the number of units (pages, etc.) in vehicle *1* to be smaller or equal to 12, but in no case below zero, hence negative.

Intensive preliminary work is however required in the first place, in view of determining the publicity value of the vehicle with respect to:

— total public reached by the respective vehicle;

— structure of the respective public;

— proportion of the public reached by the respective publicity unit (insertion, minute) in the vehicle;

— qualitative value (coloured advertising, spoken message accompanied by music), of one unit in the vehicle, taking also into account the prestige of that particular publicity vehicle.

It might thus be found, for example, that one page of advertising for the respective product in four colours, in magazine "*C*", reaches a public representing 30 per cent of the total readers of that magazine (for example 939,600 out of the total 3,132,000 readers), and that regarding the qualitative value (prestige, etc.) that vehicule stands at a level of 80 per cent of what would represent 100 per cent as the ideal, hence maximum prestige.

Therefore: $3{,}132{,}000 \times 0.3 \times 0.8 = 751{,}680$ which is the synthetic number of the total publicity value of vehicle "*C*", to which the conventional symbol X_5 is given.

Consequently to these preliminary studies and the advertising rates being known, the following table may be drawn up (conventional examples). (A similar model was elaborated by Batten, Barton, Durstine and Osborn in [*2*].)

Vehicle and unit		*Cost* (lei)	*Publicity value* (thousand readers)
Magazines (in 4 colours)			
A-page	(X_1)	9,650	193.0
1/2 page	(X_2)	5,650	91.5
B-page	(X_3)	50,140	882.5
1/2 page	(X_4)	29,750	470.4
C-page	(X_5)	43,170	751.7
1/2 page	(X_6)	31,850	477.8
D—page	(X_7)	4,950	81.6
E—page	(X_8)	13,055	155.3
F—page	(X_9)	11,300	209.1
1/2 page	(X_{10})	6,550	98.9
G—page	(X_{11})	23,050	246.6
H — page	(X_{12})	13,450	150.6
1/2 page	(X_{13})	8,010	79.4
Newspapers			
1000 line insert in 5 national newspapers	(X_{14})	15,500	261,9
15 special newspapers	(X_{15})	30,200	546.7
25 regional newspapers	(X_{16})	40,000	662.7

			(thousand persons)
Television			
4 minutes	(X_{17})	148,000	1,296.5
Cinema			
60 seconds bi-weekly in:			
10 cinemas in the capital	(X_{18})	11,800	132.2
30 cinemas in capitals of regions	(X_{19})	18,600	249.2
50 cinemas in other centres	(X_{20})	23,000	259.7

Before being able to solve the problem of selecting the publicity vehicles by means of the method of linear programming, as already mentioned, it is necessary for the restrictions to be specified.

Let us assume that they are the following:

1. All the vehicle units should be larger than or at least equal to 0.

2. The number of inserts should not extend over a period longer than 26 weeks:

$$X_1 \leqslant 6 \qquad X_{11} \leqslant 26$$
$$X_2 \leqslant 6 \qquad X_{12} \leqslant 6$$
$$X_3 \leqslant 6 \qquad X_{13} \leqslant 6$$
$$X_4 \leqslant 6 \qquad X_{14} \leqslant 26$$
$$X_5 \leqslant 26 \qquad X_{15} \leqslant 26$$
$$X_6 \leqslant 26 \qquad X_{16} \leqslant 26$$
$$X_7 \leqslant 26 \qquad X_{17} \leqslant 26$$
$$X_8 \leqslant 26 \qquad X_{18} \leqslant 26$$
$$X_9 \leqslant 6 \qquad X_{19} \leqslant 26$$
$$X_{10} \leqslant 6 \qquad X_{20} \leqslant 26$$

3. It is considered essential for magazines B and C to each have at least 1 insert, hence:

$$X_1 \geqslant 1 \qquad X_3 \geqslant 1$$

We also consider is absolutely necessary for least one insert to appear in each of the 15 special newspapers, hence

$$X_{15} \geqslant 1$$

4. Likewise for $X_{17} \geqslant 1$.

5. Based on past experience the management of the enterprise considers that at least 1/2 of the 450,000 lei budget, therefore 225,000 lei, should be alloted to the magazines:

$$C_1X_1 + C_2X_2 + C_3X_3 \ldots + C_{13}X_{13} \geqslant 225{,}000 \text{ lei}$$

(C_1, C_2, C_3, etc. representing the unit cost per magazine).

6. Total expenses may not exceed the amount of 450,000 lei.

$$C_1X_1 + C_2X_2 + C_3X_3 \ldots + C_{20}X_{20} \geqslant 450{,}000 \text{ lei}$$

The procedure followed by the electronic computer in the case of the example mentioned in [2] was therefore the following:

1. To satisfy the restrictions, including in the solution the minimum number of units of every vehicle for which the minimum was pre-determined; hence, one unit (one insert) each in X_1, X_3, X_{15} and X_{17} ($X_1 = 1$, $X_3 = 1$, $X_{15} = 1$, $X_{17} = 1$).

2. To take into account the maximum publicity value per invested leu, per vehicle unit, and in case the allotted amount (450,000 lei) has not thus been used up, to include the vehicles having the next highest publicity value. There results consequently from the table indicating the publicity value for the 20 vehicles, that the maximum publicity value per leu is afforded by the vehicle X_1 which, being a monthly magazine and taking into account the restrictive limitations of the period (26 weeks, respectively 6 months) therefore affords X_1 a frequency of 6, respectively the publication of the insert in numbers of that magazine.

3. Proceeding further in this manner and also taking into account restrictions 1—6, the solution resulting from the computer is the following:

$X_1 = 6$ $\quad$ $X_{11} = 0$
$X_2 = 0$ $\quad$ $X_{12} = 0$
$X_3 = 2$ $\quad$ $X_{13} = 0$
$X_4 = 0$ $\quad$ $X_{14} = 0$
$X_5 = 0$ $\quad$ $X_{15} = 2$
$X_6 = 0$ $\quad$ $X_{16} = 0$
$X_7 = 3$ $\quad$ $X_{17} = 1$
$X_8 = 0$ $\quad$ $X_{18} = 0$
$X_9 = 6$ $\quad$ $X_{19} = 0$
$X_{10} = 0$ $\quad$ $X_{20} = 0$

Total expenses = 449,230 lei. Total publicity value = 6,812.3 thousand readers.

Therefore, for the planned publicity compaign, as a function of the specific features of the object for which this campaign is contemplated, as a function of the publicity quality of the available vehicles (magazines, newspapers, etc.), as a function of the total available funds and of a few other considerations (restrictions), applying the mathematical method of linear programming, the advertising campaign should unfold as follows:

1) Magazine A ..... 6 one-page insert in 4 colours
2) Magazine B ..... 2 one-page insert in 4 colours
3) Magazine D ..... 3 one-page insert in 4 colours
4) Magazine F ..... 6 one-page insert in 4 colours
5) The 15 specialty newspapers ..... 2 one-page insert in 4 colours
6) Television ..... 1 four-minute insert

Hence, the above 6 publicity vehicles or means have been selected out of the twenty under consideration, as being the most adequate.

It should be pointed out that the "restrictions" have, for the most, been established by the management of the enterprise on basis of considerations proceeding to a great extent from experience or subjective, discretionary estimations. The solution obtained by the application of linear programming

can therefore only be assigned a guiding value, nevertheless, elaborated scientifically.

The optimum turning to account of investments in general, as well as of publicity investments, is not only possible, but also necessary.

The objectives of our 5-year plan for 1966—1970 also comprise considerably increased tasks both for home and foreign trade. The expenses destined to publicity will also increase. It should, however, be remembered that "mathematical programming enjoys particular conditions in socialist economy owing to the planned character of the latter" [*1*]. In the conditions of the development of our country's economy according to plan, there is no doubt that the application of linear programming is fully achievable also in the domain of publicity.

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The mode and means by which one can verify if an advertising campaign or action has reached its aim, how the effect of the advertising campaign on the activity of a commercial enterprise can be appreciated and, finally, the size of the publicity budget required, are questions regarding which the solutions so far reached — notwithstanding the vast literature dealing with these problems and all the experiments carried out and tests elaborated to this end — are still a long way from supplying wholly acceptable formulae. This situation can be accounted for by the fact that so many factors, difficult to measure, are involved that they make it difficult to determine the most suitable method of mathematical analysis.

Generally, empirism still predominates. A percentage in value of the sales volume is allotted to publicity and the past estimates become criteria for the present. There are even authors who, considering that advertising is to the greatest extent, an art, reject the idea of measuring efficiency by the stringency of a mathematical analysis. We also find, on a close level, certain authors who place the main emphasis on "economic psychology" of the behaviourist type, such as Bouquerel, Daniel Adam, Tagliacarne, Pierre-Louis Raynaud and others.

The huge variety of publicity vehicles and advertising forms, the wide range of broadcasting systems and the continuous increase in number and differentiation of the multitude of goods and their assortments clearly constitute a very serious hindrance in elaborating any adequate computation formula of the efficiency of commercial advertising.

The problem becomes still more complex as a function of the policy of variable or constant prices as well as in relation with a fixed or variable budget for publicity.

The correlation between publicity and demand (of goods or services) can be estimated from two different positions, not contradictory, but complementary: from the position of the publicity action having the effect of decreasing the elasticity of demand, as well as from the position of demand with effects on the elasticity of publicity. In the former case only the curve of demand is displaced, in the latter the correlation is complete, in a reciprocal and continuous adjustment within the limits of the inelasticities reached [*3*, p. *63—64*].

Thus, at the price level OP, the new demand $A'R'$ is clearly larger than the old demand AR, but concomitantly the demand $A'R'$ has become less elastic, particularly if the advertising offensive has also been used as an occasion to raise the product price, along the ordinate Op.

The displacement towards the right of the straight line AR towards $A'R'$ therefore exhausts the elasticity due to the increase in selling price loaded by advertising expenses and raised towards p.

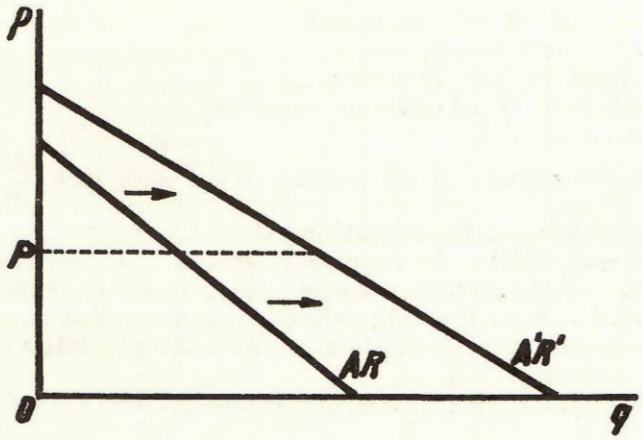

Fig. 1

It is obvious that a price decrease along the ordinate Op concomitantly accompanied by an advertising offensive would have further displaced the straight line $A'R'$ along the abscissa Oq.

From the numerous experiments and tests methodically carried out and whose data were scrupulously recorded, a series of authors have reached the conclusion that three parameters should be considered in view of determining the connection of the correlation or, rather, of the inter-action between sale and advertising: the constant of sale decrease, the level of saturation (ceiling) and the constant of sale increase. This, obviously, in the hypothesis that, effectively, when advertising ceases, the sales will decrease in a constant ratio which will, however, vary from one product to another. If, for example, this constant of sale decrease were 10 per cent, from an initial sale of 100,000 pieces yearly, the sale in the following year, without any advertising, will only be 90,000 and in the subsequent respective years, 81,000, 72,900, 65,610, etc. The problem is reversed for the constant of sale increase. On basis of experience, one thing is nevertheless certain, viz. that advertising has no continuous and unlimited effects in promoting sales, a level of stagnation being therefore reached at a certain moment [*4, p. 111—114*].

It should be mentioned that not all these parameters are applicable to any product for sale. The constant of sale decrease can only be identified in the case of certain goods that may become obsolete or may be replaced by new assortments, thus continuously losing part of their customary pur-

chasers and this would obviously occur without the influence of any publicity action.

The following mathematical model has been elaborated on basis of the above parameters: [*5, p.370—381*]

$$\frac{dS}{dt} = rA\,\frac{M-S}{M} - LS$$

where:

L = coefficient of sale decrease;
M = level of saturation;
r = constant of sale increase;
A = coefficient of advertising expenses;
S = volume of sales.

This equation shows that the increase in the sales rate $\left(\frac{dS}{dt}\right)$ is proportional to the intensity of the advertising effort (A) influencing the potential customers ($M-S$), minus the reductions arising from customer loss (LS).

The values of L, M and r are obtained, based on experience in the preceding period and on the data derived from controlled advertising experiments. The procedure of pre-tests also yields the absolute values required to compute efficiency.

After discontinuing the advertising campaign, the sales decrease exponentially:

$$S_{(t)} = S_{(T)}\, e^{-L(t-T)} \qquad (t > T)$$

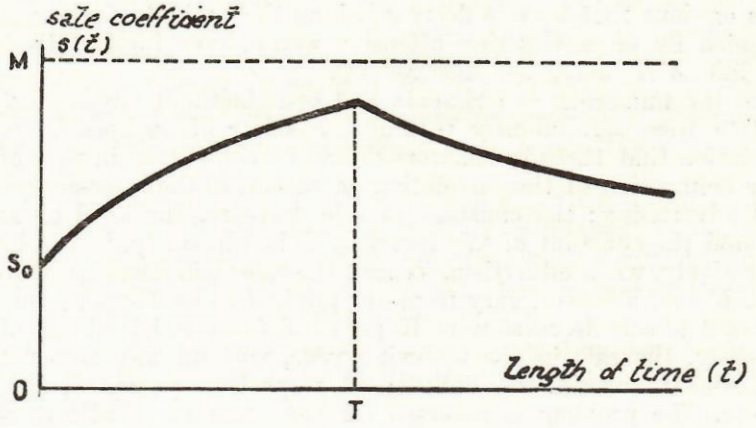

Fig.2

Indeed, under the impulse of the advertising campaign, the sales develop with a marked increase coefficient which then enters a phase of decrease after reaching the level of saturation during the campaign $O—T$.

Publicity actions for the same product, carried out in three or four districts or quarters that are not too distinct, can be of particular documentary value. In the first district, for instance, the advertising campaing lasts 6 months, in the second, with the same initial date, it extends over a period of 12 months; in the third district the publicity action begins at the end of the one in the first district and lasts also 6 months, and the fourth district serves as a "witness" and no publicity action is undertaken in it.

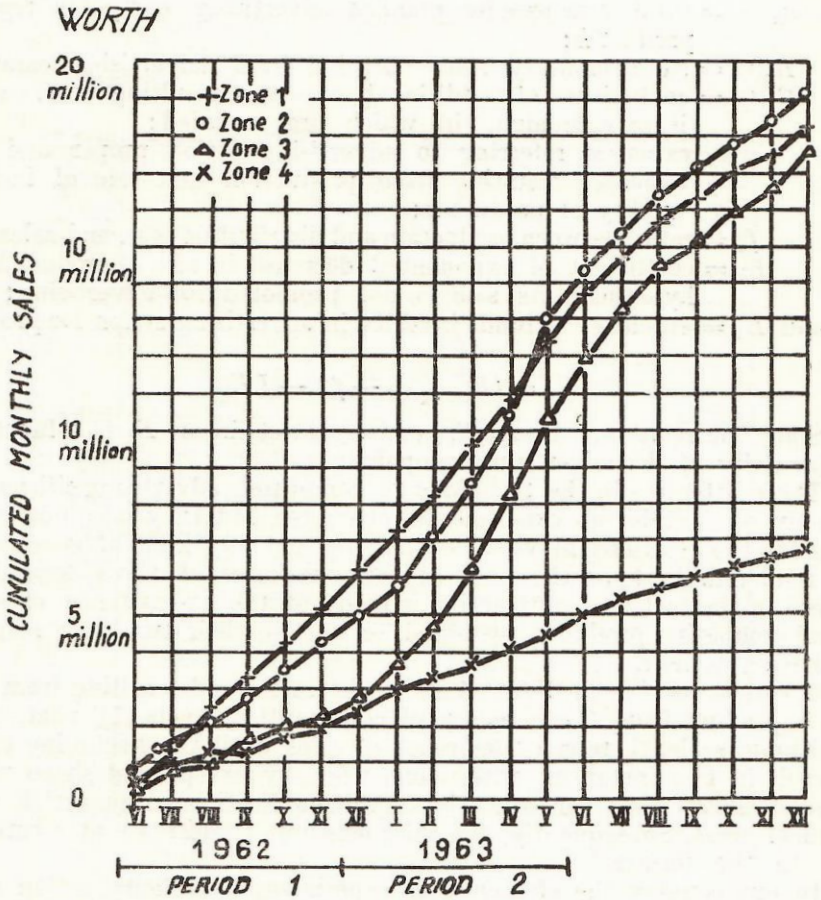

Fig. 3

The above diagram, obtained consequently to such an experiment achieved by M. L. Vidale and H. B. Wolfe of the Cambridge University in Massachusetts [5] guides us towards the basic problem: size of advertising expense funds, optimum use of advertising investment.

The problem might arise, whether the funds spent on publicity action would not have been more usefully invested in scientific research or new

equipment for the respective branch of manufacture. Problems that also remain unsolved are the optimum duration for an advertising campaing as a function of the product and of the advertising means, or the optimum intensity of advertising expenses in view of promoting a new product on the market.

Advertising efficiency, computed in proportion with the funds invested in the publicity action is obviously the "key" required for a proper allotment fo the funds required to advertise a product or a service. If

a_k = total expenses for planned advertising campaign regarding product k;

$R_k(t)$ = supplementary sales resulting from advertising campaign;

$C_k(t)$ = coefficient of additional expenses resulting from advertising campaign, in which are included:

a) expenses referring to advertising action proper and

b) expenses resulting from production and sale of increased quality of product k;

f_k = ratio between production and distribution cost, and sales price;

L_k = coefficient of exponential decrease in sale of product k in the hypothesis its sale is not promoted by advertising;

and E_k = efficiency of funds invested in advertising action for product k.

Then

$$E_k = (R_{0k/a_k})(1 - f_k) - LK$$

Since the relation between R_{0k} and a_k is not linear, E_k is a function of the intensity of the advertising campaign.

These attempts in the technique of computing advertising efficiency do not only still exhibit an experimental character, but they may not even be applied to the relations between advertising and any kind of goods or service rendered. Perhaps the most obvious weakness of these formulations consists in the fact that abstraction is made of the co-existence of one or several competing products, sustained or not, in their turn, by respective advertising actions.

It can be seen from figure 4 shown hereafter and resulting from a test with a product from the category of consumption goods [5], that, on the logarithmic scale, there is a "decrease" of sales until the beginning of 1960 followed, in two instances within one year, by exceptional sharp rises of the sales, both times consequently to the publication of an article in the technical press. Subsequently, the sales continue to decrease at a rate identical to the former.

In our country the efficiency of experimental publicity action can be illustrated by numerous examples.

Let us, for instance, show some positive results obtained by the advertising campaign in the period March-April 1965 for tooth paste and tooth brushes (althought this campaign was not accompanied by a regional and district statistical follow-up of the advertising expenses). Though, in fact, the Ministry of Domestic Trade only applied what is called an "advertising injection", the public nevertheless reacted favourably to the advertising action within a short period, a noticeable increase in the sale of tooth paste and tooth brushes being noted.

The effect of advertising on the sale of tooth paste was materialized by a 30 per cent increase in the region of Jassy, 7.6 per cent in the region of Hunedoara and 47 per cent in the region of Ploiești, while in the town of Botoșani the increase was 21.6 per cent and it amounted to 16.2 per cent in the city of Bucharest.

It is obvious that out of these 5 examples the maximum elasticity occurred in the regions of Ploiești and Jassy and the minimum elasticity in the region of Hunedoara and in the city of Bucharest.

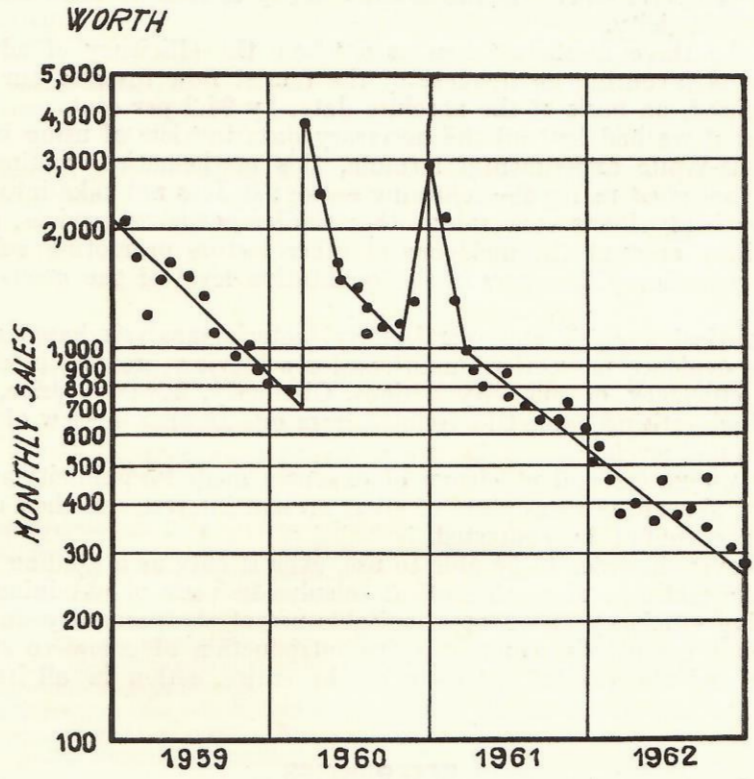

Fig. 4

More numerous statistical data have been placed at our disposal by the National Travel Office, but owing to their insufficient details we did not have the possibility to apply integrally the formulae of the Vidale-Wolfe mathematical model. Operating in the limits of the existing data we nevertheless obtained some interesting results regarding the efficiency of advertising on the flow of tourists from countries X, Y and Z to our country as a function of the indicators: total fund spent, number of tourists attended, total income cashed.

On basis of the absolute data, there results the following:

when $Y_0 = 1$,

$Z_0 = 3.4\ Y_0$

$X_0 = 7.5\ Y_0$

when $Y_1 = 1$,

$Z_1 = 0{,}9\ Y_1$

$X_1 = 6.2\ Y_1$

The optimum pecuniary efficiency of advertising investment therefore keeps up, both in 1964 and in 1965 for the country X, but it decreases below the level reached in the basic year, in the country Z. There still results, that the advertising investments in the country X seem to draw near to the ceiling of elasticity.

On the three markets taken as a whole the efficiency of advertising investments favouring an increase of the tourist flow towards our country has increased, on basis of the absolute data, by 94.2 per cent.

Even if we had had all the necessary data in view of using integrally the Vidale-Wolfe experimental formula, this mathematical method would not have satisfied us for the following reasos: it does not take into account the advertising effects concerning other similar goods or services, nor does it take into account the incidence of other factors promoting sales, such as price reductions, diferences in the qualitative level of the goods offered, etc.

Statistical analysis and particularly factorial analysis based on more complete evidence may offer conditions for achieving progress in determining the efficiency of publicity actions. Obviously, however, price, production and advertising cannot be isolated from one another in view of reaching the optimum levels.

The usefulness and efficiency of expenses made for publicity actions at home and abroad, obviously are of great present interest and their economic importance cannot be contested.

However, in order to be able to use, even if only as a guiding rod, the up-to-date methods of mathematical calculus in view of obtaining greater advertising efficiency and a more suitable use of the pecuniary funds, one of the first conditions required is the introduction of complete statistical evidence and the careful follow-up of the entire action in all its phases.

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APPLICATION DE CERTAINES METHODES MATHEMATIQUES POUR LE CHOIX DES MOYENS PUBLICITAIRES ET POUR LA DETERMINATION DE L'EFFICACITE DE LA RECLAME COMMERCIALE

L'auteur commence par présenter, prenant comme base un exemple concret, l'utilité de la programmation linéaire pour l'utilisation optimum des fonds destinés aux actions publicitaires.

Ensuite, analysant la corrélation entre la publicité et la demande de marchandises ou de services, partant de l'hypothèse d'un coefficient de baisse de la vente non soutenue par la publicité, d'une augmentation constante de la vente sous l'impulsion de la publicité et d'un niveau de saturation, expression du manque d'élasticité atteint, l'auteur se rapproche de la formule de calcul Vidale-Wolfe.

La formule algébrique employée n'est valable que pour la détermination de l'efficacité globale de la publicité. De nouveaux efforts d'investigation seront nécessaires afin de pouvoir déterminer l'efficacité nette.

A l'aide de certaines données ayant trait à l'encouragement publicitaire d'un article de toilette sur le marché interne de Roumanie et à l'action publicitaire sur l'afflux de touristes étrangers à destination de Roumanie, on expose deux cas d'analyse de l'efficacité de la publicité commerciale.

ПРИМЕНЕНИЕ МАТЕМАТИЧЕСКИХ МЕТОДОВ ПРИ ВЫБОРЕ РЕКЛАМНЫХ СРЕДСТВ И ПРИ ОПРЕДЕЛЕНИИ ДЕЙСТВЕННОСТИ КОММЕРЧЕСКОЙ РЕКЛАМЫ

Автор начинает свою работу с изложения — на основе конкретного примера — преимуществ линейного программирования для оптимального использования фондов, предназначенных для осуществления рекламы.

Далее, анализируя соотношения между рекламой и спросом на товары или услуги и исходя из гипотезы коэффициента снижения сбыта товаров, неподдержанных рекламой, и из гипотезы наличия постоянного роста сбыта товаров под импульсом рекламы и уровня насыщенности, являющегося выражением достигнутой неэластичности, автор приближается к исчислению Видаля-Вольфа.

Использованная им алгебраическая формула удовлетворяет только определению общей эффективности рекламы. Для определения и чистой эффективности необходимы усилия в данной области исследовани.

С помощью ряда данных, касающихся рекламирования одного туалетного изделия, сбываемого на внутреннем рынке Румынии и рекламных мероприятий в связи с наплывом туристов из некоторых заграничных стран в нашу страну, демонстрируются два случая анализа эффективности коммерческой рекламы.